A Discussion on the Concept of Similarity

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Abstract: In this paper a concept of relative measurement of similarity is proposed. With the approach presented we can quantify degree and percentage of similarities between two objects. Objects may be anything, a geometrical shape, a domestic article, a machine part, a piece of art, etc. In this paper, similarity is defined as a set of answers as True or False to a set of questions on attributes of these objects. The paper also discusses the properties of the proposed concept of similarity.
Keywords: Transitivity, Similarity, Intersection of Sets, Union of Sets, Universal Sets

I.   INTRODUCTION

A	B	C
Figure 1	Figure 2	Figure 3

A and B possess the same shape. B and C have different shapes but cover the same area.

We can pose the following questions:

Is B and A more similar than B and C?

Is A and C more similar than B and C?

The answer to these questions will require an understand of what we mean by "similarity". To measure similarity, we need to define similarity first. We are not dealing with "absolute similarity", but a concept which we define as relative similarity.

We begin by proposing a statement which expresses an attribute of the objects which are under comparison. This would be a logical statement must which elicits necessary answers to compare the objects.

For example, the attributes could be the areas of geometrical figures or the number of corners they possess, or any other thing which could describe them.

Now we define a set S_i = {Objects with a given logical attribute} which contains all elements which conform to the Statement/Attribute defining the set.

Examples of such sets are:

   S_1: Set of elements with area equal to α units

   S_2: Set of elements with number of corners greater than β

   S_3: Set of elements with number of concave edges less than γ

   S_4: Set of elements with number of convex edges greater than δ

   And so on.

Say, figure B has 1 concave edge, and figure C has 3 concave edges. These two figures will be elements of a set S_p of elements having at least one concave edge.

However, the set S_q "The shape has 3 concave edges" will only have shape C as an element and not shape B.

Therefore, in terms of Statement of set S_p, the shapes B and C are similar. But in terms of Statement of set S_q B and C are not similar.

If we consider a set S_r consisting of elements whose "Area enclosed by shapes is α units" both shapes B and C having an area of α units will be elements of set S_r.
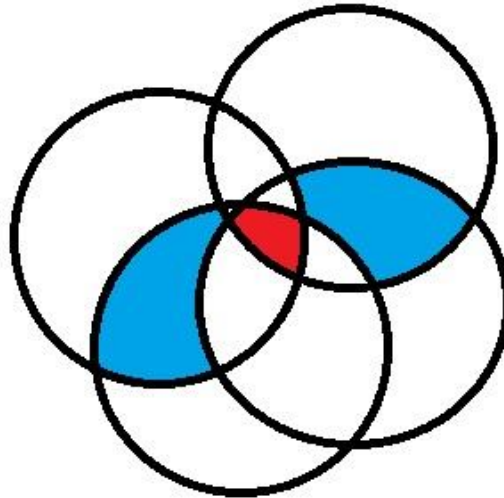

Figure 4 Similarity Venn diagram

We now depict the set of objects through circles; each circle representing a set of objects possessing a particular attribute. The Venn diagram so produced is depicted in Figure 4.

The intersection of the circles is the measure of the similarity.

In Figure 4, each blue region is an overlap of two circles, or sets, so the elements are similar to 2nd degree.

There is one red region where there is an overlap of four circles, so the elements in the red region are similar to the 4th degree.

However, the 2 blue intersections are from completely different sets, we still call the elements of both, similar to 2nd degree.

The number of overlaps has been denoted by Ω. Degree of similarity is defined as,

$$Degree\ of\ similarity = \bigcap_{i=1}^{N} S_i$$

Where N is the total number of attributes or sets.

Two or more shapes are more similar when,

$$\bigcap_{i=1}^{N} S_i > \bigcap_{i=1}^{n} S_i$$

Where N > n

Percentage of similarity is defined as number of overlaps by total number of sets, that is $\frac{\Omega}{N} \times 100\%$

II.   SIMILARITY TO SAME

Having defined similarity, we can now move onto to define "same".

A question may be: Are figure A and figure B the same?

The answer to this may be either 'Yes' or 'No'

But if we ask the question "Is figure A similar to figure B?

It would be followed by another question "Similar according to which attribute?"

A figure is said to be same if and only if, all possible questions with different attributes will return an answer "yes".

On rephrasing the question "Are figures A and B similar in respect to every possible attribute?"

If the answer to this question is "yes", then we conclude "Figures A and B are same".
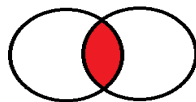
III.   SUPERPOSITION
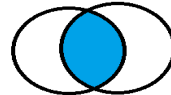

Figure 5   Figure 6

In Figure 5, red portion is the overlap between sets S_1 and S_2, satisfying particular attributes. In Figure 6, blue is the overlap of sets S_1 and S_2, satisfying another set of attributes. In this case both the intersection is counted. If all the intersections of two sets are taken into account, the total number of overlaps will be the total number of intersections.

For example:

A set S_1 contains figures that are made of thick lines and closed figures. A set S_2 contains figures with thick line and thin line and some open figures and some closed figures. Now we define similarity by defining the intersection. Say similarity is of "thin lines".

Hence there will be the overlap, between set S_1 and set S_2. We now consider another intersection of set S_1 and S_2 defining similarity as "closed figures". We have thus considered two different intersections of the same two sets based on two different attributes. We may call it superposition of the two intersections each based on one attribute. Thus, number of superposition equals the degree for a given two sets.

IV.   ERROR

If we ask "Do the sets "asofih" (or any random sequence of characters that convey no meaning)?"

"Asofih" being a meaningless 'word', making it a meaningless attribute, the answer to the question can neither be "yes" nor a "no". the answer therefore is an error.

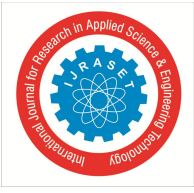Here "error" shall be regarded as the result. Error is one of the limitations of assessing similarity.

V.   UNIVERSAL SIMILARITY SET

$$\Sigma_0 = \{Set\ of\ dissimilar\ sets\}$$
$$\Sigma_1 = \{Set\ of\ similar\ sets\ with\ 1st\ degree\ similarity\}$$
$$\Sigma_n = \{Set\ of\ similar\ sets\ with\ nth\ degree\ similarity\}$$
$$\Sigma^* = \bigcup_{i=0}^{K} \Sigma_i$$

Thus Σ^* is the universal set that contains all similar sets. It is defined as a set whose elements are sets with every possible degree of similarity.

VI. TRANSITIVITY

If set A is n^{th} degree similar to set B and set A is n^{th} degree similar to set C, then set B is n^{th} degree similar to set C, if and only if the attributes considered are the same. Thus, property of transitivity is not universal.

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