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A Manifold Approach to Superconductivity

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Abstract: Although, it's been only a 113 years since the discovery of superconductors and superconductivity the development in condensed matter physics is impeccable. Today we know a lot about them, their types and the phenomenon associated with them but the cause of superconductivity in type-II superconductors is still debatable. Although there have been profound theories like Ginzburg-Landau theory, London penetration effect describing flux pinning, spin fluctuations, quantum vortices in super fluids and so on, they aren't completely able to describe the phenomenon in cuprates, non-phononic mechanisms in superconductors etc. In this paper we approach electrons as superfluid in a complex manifold and topological variations in the manifold due to application of magnetic field to justify the two critical field strengths in type two superconductors and the cause of superconductivity in type-II superconductors. We find that in type-II superconductors, they can superconduct with the presence of non-zero resistance i.e. superconductivity is more fundamental than zero resistance in type-II superconductors, the actual case being the energy gap function created by overlapping of electron wave functions in complex n- dimensional manifolds.
Keywords: Superconductors, Superconductivity, Condensed Matter Physics, Ginzberg-Landau Theory, London Penetration Effect, Flux Pinning, Spin Fluctuations, Quantum Vortices, Superfluid's, Phononic Mechanisms , Manifold, Topology

I. INTRODUCTION

A superconductor is a type of material that conducts electricity with zero energy loss or resistance when cooled to a certain temperature, called the critical temperature (θ_c). Superconductivity is a set of properties observed in certain materials where electrical resistance vanishes and magnetic fields are either partially or totally expelled from the material. On the basis of expulsion of magnetic field i.e. magnetic penetration, there are two types of superconductors i.e. type-I and type-II superconductors. A superconductor can be type-I, meaning it has a single critical field above which all superconductivity is lost and below which the magnetic field is completely expelled from the superconductor; or type-II, meaning it has two critical fields, between which it allows partial penetration of magnetic field through isolated points. These isolated points are called quantum vortices. Most of pure materials like Mercury (Hg), Lead (Pb), Aluminum (Al) etc. are type-I superconductors. The only alloy known to exhibit type-I superconductivity is Tantalum Silicide ($TaSi_2$). Most type of type-II superconductors include metal alloys like Yttrium Barium Copper Oxide (YBCO), complex oxide ceramics but some elemental type-II superconductors also exist like Niobium, Vanadium and Technetium. In the past decade much of research has been focused on higher temperature superconductors and the cause of superconductivity in type-II superconductors. Although Barden-Cooper-Schrieffer (BCS) theory explains superconductivity in type-I superconductors, the cause of superconductivity in type-II superconductors remains unclear.

BCS theory fails to explain higher temperature and complex superconductors. The leading theories trying to explain this problem are: i) Resonating Valance Bond (RVB) Theory and ii) Spin Fluctuation Theory. In this study we diverge from these two theories and dive at a more profound theory. We consider electron pairs on a complex manifolds of n-dimensions $(\mathbb{C}^n)$ and analyze its topological properties and how manifolds topologically evolves under action of magnetic field and search for the cause of superconductivity there.

II. LITERATURE REVIEW

A. Ginzburg-Landau (G-L) theory

The first theory that could phemenologically describe the type-I superconductivity was the Ginzburg-Landau (G-L) theory, named after Vitaly Ginzburg and Lev Landau. A later version of G-L theory was derived from the Bardeen-Cooper-Schrieffer (BCS) theory where the implications of G-L theory even extend to Quantum Field Theory (QFT) and String Theory. According to this theory, the free energy density (f_s) of a superconductor near the superconducting transition can be expressed in terms of a complex order parameter field i.e.$\boldsymbol{\psi(r) = |\psi(r)|e^{i\phi(r)}}$, then

$$f_s = f_n + \alpha(T)|\psi|^2 + \frac{1}{2}\beta(T)|\psi|^4 + \frac{1}{2m^*}\left|\left(-i\hbar\nabla - \frac{e^*}{c}A\right)\psi\right|^2 + \frac{B}{8\pi}$$

Where,

- f_n is the free energy density of normal phase
- $\alpha(T)$ and $\beta(T)$ are phenomological parameter that are functions of absolute temperature (T)
- $m^*$is effective mass
- e^* is effective charge
- **A** is magnetic vector potential
- **B**= **∇ ×A** is magnetic field

A central feature of this theory is the energy of the interaction between the superconducting and normal state. This energy changes from positive to negative when the dimensionless parameter 'k' exceeds $1/\sqrt{2}$

where, $\mathbf{k}=\frac{\textit{ratio of superconducting penetration length "}\lambda\textit{"}}{\textit{coherence length "}\xi\textit{"of superconductingstate itself}}$.

Although G-L theory provides bright insight on superconductivity, its failure to describe the order of phase transition is evident in some layered bulk materials, in quasi two-dimensional systems, a new field solution is observed at non-zero temperature.

B. *The BCS Theory of Superconductivity*

The first microscopic theory of superconductivity were laid by papers of Bardeen, Cooper and Schrieffer in 1957. The BCS theory of superconductivity has a wide range of applicability from liquid helium to type-I metallic superconductors. The major postulates of BCS theory are as follows:

1) An attractive interaction between electrons can lead to a ground state separated from excited states by an energy gap. The critical field, the thermal properties, and most of the electromagnetic properties are consequences of the energy gap.
2) The electron-phonon-electron interaction leads to energy gap of the given magnitude. The indirect interaction proceeds when one electron interacts with the lattice and deforms it; a second electron sees the deformed lattice and adjusts itself to take advantage of the deformation to lower its energy. Thus the second electron interacts with the first electron via lattice deformation.
3) The penetration depth and the coherence length emerge as natural consequences of the BCS theory. The London equation is obtained for magnetic fields that vary slowly in space. Thus the central phenomenon in super conductivity, the Meissner effect is obtained in natural way.

Now, we dive into the current research being done on the title of our study i.e. Cause of Superconductivity in type-II superconductors, current research includes two major approaches i.e. Resonance Valance Bond (RVB) Theory and Spin Fluctuation Theory.

C. *Resonance Valance Bond Theory*

The RVB Theory was proposed by American Physicist P.W. Anderson, This theory states "In copper oxides lattices, electrons form neighboring copper atoms interact to form a valance bond." With doping, these electrons can act as mobile cooper pairs and are able to superconduct. RVB theory builds on Hubbard and t-J models. The more accepted model is the Hubbard model, the Hamiltonian of Hubbard model sates the following:

$$\boldsymbol{H} = -\boldsymbol{t}\sum_{\langle ij\rangle}\left(\boldsymbol{C}_{i\sigma}^{+}\,\boldsymbol{C}_{j\sigma} + \boldsymbol{h.c.}\right) + \boldsymbol{U}\sum_{i}\boldsymbol{n_i}\uparrow\boldsymbol{n_i}\downarrow$$

The Hamiltonian also describes the physics of Mott insulators i.e. class of materials that are accepted to conduct electricity according to conventional band theories, but turn out to be insulators (particularly at low temperature). Anderson suggested that, it can have non-degenerate ground state composed of disordered spin states

$$|\boldsymbol{RVB}\rangle = \sum_{\boldsymbol{C}}|\boldsymbol{C}\rangle$$ Where, C= covering of a lattice by nearest neighbor atoms

However, a rigorous proof for the existence of a superconducting ground state in either the Hubbard or t-J Hamilton is yet not known. Further the stability of the RVB ground state has not yet been conformed.

D. *Spin Fluctuation Theory*

Now, we dive into the theory of superconductivity which has most of scientific community working into it right now. The theory of spin fluctuation, in a high temperature (Tc)-superconductor, the mechanism is extremely similar to a conventional superconductor except in this case, phonons virtually play no role and their role is done by spin density waves. Spin density waves are low-energy ordered states of solids.

They occur at low temperature in anisotropic, low-dimensional metals or metals that have high densities of states at Fermi-level. When an electron moves in a high temperature superconductor, its spin creates a spin-density wave around it. This pin density wave in turn causes a nearby electron to fall into the spin depression created by the first electron. Hence, Cooper pair is formed. When the temperature is lowered more spin density waves and cooper pairs are formed eventually leading to superconductivity. There is strong coulomb repulsion between electrons. This repulsion prevents pairing of the Cooper pairs on the same lattice site.
Considering the spin fluctuation-mediated interaction between the electrons, the Hamiltonian is given by,

$$H = \sum_{\sigma k} \xi_k \, a^{+}_{k\sigma} \, a_{k\sigma} + \frac{1}{2N} \sum_{\sigma} \sum_{k.k'} V_{kk'} \, a^{+}_{k\sigma} \, a^{+}_{-k-\sigma} \, a_{-k'-\sigma} \, a_{k'\sigma'}$$

With, $V_{kk'} = \frac{3}{2} I^2 [\chi(k - k' + \chi(k + k')]$

Where, $\xi_k = \varepsilon_k - \mu$ is the single particle energy, measured for chemical potential. Using BCS mean field approximation we have equation for gap function as,

$$\Delta(K) = -\sum_{K} V_{KK'} \langle a_{K'\uparrow} a_{-K'\downarrow} \rangle$$

From this equation we arrive at the following eigenvalue problem:

$$\lambda \Delta(K) = \rho(0) \langle V_{KK'} \Delta(K') \rangle_{FS}$$

Where, $\rho(0)$ is the density of state at the fermi level and the average $\langle \quad \rangle_{FS}$ is taken over fermi surface. From largest value of eigenvalue of λ the transition temperature is given by:

$$T_c = 1.13 \omega_c e^{-1/\lambda}$$

Where, ω_c is the cutoff frequency.

III. METHODOLOGY

The basic physical mechanism responsible for the high critical temperature isn't yet clear. However, it is clear that two electron pairing is involved, although nature of pairing remains controversial. On this paper, we approach type-II superconductivity with a more fundamental approach. We approach a manifold way to superconductivity. We consider Cooper-pairs as holomorphic interactions caused due to overlapping of complex manifolds ($\mathbb{C}^n$). On a non-superconducting temperature each electron in lattice is expressed as:

$$i\hbar \frac{\partial}{\partial t} \Psi(x,t) = \left[\frac{-\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x,t) \right] \Psi(x,t) \ldots \ldots \ldots \ldots \ldots \ldots (3.01)$$

And their lattice interactions have to be considered independently. The lattice interaction equation is given by: $\Psi_c(r) = C_c \cos(ax) K_0(x) \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (3.02)$

Where, C_c is normalized constant, K_0 is zero order modification of Bessel function and $a = \pi K_F \xi_0$, where K_F is Fermi wave vector and ξ_0 is coherence length.

A. *Cooper Pairs in Complex Manifolds*

For simplicity, let us consider the superconducting material to be a continuous differential topological manifold **'M'** defined by open sets $\{U_a\}_{a \in A}$, where U_a has a corresponding coordinate map $\mathbb{Z}_a: U_a \to \mathbb{C}^n$ to an open subset of $\mathbb{C}^n$. Let the superconducting material also have only one cooper pair of two electrons (say **'a'** and **'b'** with wave functions Ψ_1 **and** Ψ_2 corresponding to complex planes $\mathbb{C}^a$ ***and*** $\mathbb{C}^b$ respectively).

On trivial interactions i.e. $>> T_c$, $U_a \cap U_b = \phi$

$$\Rightarrow \mathbb{Z}_a . \mathbb{Z}_b^{-1} : \mathbb{Z}_b(U_a \cap U_b) \neq \mathbb{Z}_a(U_a \cap U_b) \ldots \ldots \ldots \ldots \ldots \ldots \ldots (3.11)$$

I.e. they are non-holomorphic. So, at trivial interactions complex open spaces U_a *and* U_b there is no overlapping of complex manifold and no energy gap due to manifold pairing is seen. Considering, non-trivial interactions i.e. $\leq T_c$, the properties of 'M' as a continuous differential manifold doesn't change but at non-trivial interactions there is overlapping of complex manifold i.e. $U_a \cap U_b \neq \phi$

$$\Rightarrow \mathbb{Z}_a . \mathbb{Z}_b^{-1} : \mathbb{Z}_b(U_a \cap U_b) \rightarrowtail \mathbb{Z}_a(U_a \cap U_b) \ldots \ldots \ldots \ldots \ldots \ldots \ldots (3.12)$$

Are holomorphic transition functions. This shows, overlapping of complex manifolds creates holomorphic transition functions. If ΔE is the superconducting energy gap created by holomorphic transition then for superconducting nature,

$\Delta E < \hbar\omega_{\vec{p}}$ ……………………. (3.13) Where $\omega_{\vec{p}}$ is lattice vibration frequency of momentum $\vec{p}$.

As, $\mathbb{C}^n \cong \mathbb{R}^{2n}$, the superconducting energy gap caused due to overlapping complex manifold can be generalized. The use of complex manifold helps to generalize those dimensions. From **3.13** the dimensionless parameter '**K**' is calculated and found to be:

$$K = \frac{\lambda}{\xi} \approx \frac{\Delta E\left[\ln(T_c) - \ln\left(1.13\omega_{\vec{p}}\right)\right]}{2\hbar^2 m_e}\left(\frac{3n}{\pi}\right)^{\frac{1}{3}} \ldots\ldots\ldots\ldots\ldots\ldots (3.14)$$

Where, ΔE is superconducting energy gap, T_c is critical temperature and n is free electron density.

For our type-II superconductor like YBCO,T_c $\omega_{\vec{p}}$ and n are known so the value of 'K' is calculated and found to be **approx. 1.75** i.e. $= \frac{\lambda}{\xi} > 1$, which verifies one of the properties of type-II superconductor. Hence, pairing of complex dimensions is a definitive way to explain type-II superconductivity as it is more generalized way of explaining it and isn't limited to specific type-II superconductors and definitively satisfies one of the basic prerequisites of type-II superconductivity i.e.$K > 1$.

Following the equation of phononic interactions for gap voltage we have;

$$V = \frac{\left|M_{\vec{q}}\right|^2}{\left(E_{\vec{k}} - E_{\vec{k}+\vec{q}}\right) - (\hbar\,\omega_{\vec{p}})} \ldots\ldots\ldots\ldots\ldots\ldots.. (3.15)$$

Following non phononic mechanisms the equation for gap function is given by:

$$\Delta(K) = -\sum_n \sum_{k.k'} \frac{\langle a_{k'\uparrow} | a_{-k'\downarrow}\rangle}{\lambda(\mathbb{Z}_a.\mathbb{Z}_b{}^{-1})_n - \left(\hbar\,\omega_{\vec{p}}\right)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.16)$$

B. Two Critical Fields

The overlapping of complex manifolds suggests presence of stable configurations of superconductor in magnetic field with regions where a mixed state i.e. Vortex state is formed where external magnetic fields will penetrate the thin normal regions uniformly, and the field will also penetrate somewhat into the superconducting material. The vortex state describes the circulation of superconducting currents in vortices throughout the bulk specimen. If Φ_0 be the fluxoid i.e. quantum of flux in a superconductor then, If dU_n be the area of overlapping complex manifold i.e. $\mathbb{Z}_a(U_a \cap U_b)$ then in a vector field **A**,

$$\oiiint_{\mathbb{Z}_a}^{\mathbb{Z}_b} A.dl = \iiint_{\mathbb{Z}_a}^{\mathbb{Z}_b{}^{-1}} (\nabla \times A).\,dU_n = \iiint_{\mathbb{Z}_a}^{\mathbb{Z}_b{}^{-1}} B.dU_n = \Phi \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. (3.21)$$

$$\text{Or, } \Phi = \mathbb{Z}_a.\mathbb{Z}_b{}^{-1}{}_{\rightarrowtail}\left[\sum_{\mathbb{R}^n}\sum_{\mathbb{C}^n}(2\pi\hbar c/q')_{S(\mathbb{R}^n,\mathbb{C}^n)}\right] \ldots\ldots\ldots\ldots\ldots (3.22)$$

Where, $q'$is the charge of a cooper pair i.e. **2e**$^-$. So more specifically,

$$i.e.\ \Phi = \mathbb{Z}_a.\mathbb{Z}_b{}^{-1}{}_{\rightarrowtail}\left[\sum_{\mathbb{R}^n}\sum_{\mathbb{C}^n}(\pi\hbar c/e)_{S(\mathbb{R}^n,\mathbb{C}^n)}\right] \ldots\ldots\ldots\ldots\ldots (3.23)$$

Considering magnetic fields as $\overline{\mathbb{Z}_a.\mathbb{Z}_b{}^{-1}}$ at non trivial interactions then, the converse of equation **(3.23)** must provide two solutions for magnetic fields (**B**) i.e. two critical magnetic fields say, $\mathbf{B_1}$ and $\mathbf{B_2}$ then,

$$\Phi = \mathbb{Z}_a.\mathbb{Z}_b{}^{-1}{}_{\rightarrowtail}\left[\sum_{\mathbb{R}^n}\sum_{\mathbb{C}^n}\left(\frac{\pi\hbar c}{e}\right)_{S(\mathbb{R}^n,\mathbb{C}^n)}\right] \text{ and } \iiint_{\mathbb{Z}_a}^{\mathbb{Z}_b{}^{-1}} B.dU_n = \Phi$$

Equating, two given equations

$$\iiint_{\mathbb{Z}_a}^{\mathbb{Z}_b{}^{-1}} B.dU_n = \mathbb{Z}_a.\mathbb{Z}_b{}^{-1}{}_{\rightarrowtail}\left[\sum_{\mathbb{R}^n}\sum_{\mathbb{C}^n}(\pi\hbar c/e)_{S(\mathbb{R}^n,\mathbb{C}^n)}\right]$$

For the converse, we consider disoverlapping of complex manifold, then reversing the limits we get:

$$\iiint_{\mathbb{Z}_b}^{\mathbb{Z}_a{}^{-1}} B.dU_n = \overline{\mathbb{Z}_a.\mathbb{Z}_b{}^{-1}}.\ {}_{\rightarrowtail}\left[\sum_{\mathbb{R}^{-n}}\sum_{\mathbb{C}^{-n}} -(\pi\hbar c/e)_{S(\mathbb{R}^{-n},\mathbb{C}^{-n})}\right] \ldots\ldots\ldots (3.24)$$

As, magnetic field deforms the overlapping of complex manifolds, the overlapping decreases which decreases the phenomological constant 'K' i.e. $\frac{\partial K}{\partial T} = decreasing.$

So as it decreases the boundary conditions i.e. $\boldsymbol{\lambda}$ *and* $\boldsymbol{\xi}$ remain constant. As,

$$\iiint_{\mathbb{Z}_b}^{\mathbb{Z}_a^{-1}} \boldsymbol{B}.\boldsymbol{U_n} = \bar{\boldsymbol{\phi}}$$

Or, $\boldsymbol{B}_{(\mathbb{R}^n,\mathbb{C}^n)} \cong \bar{\boldsymbol{\phi}} \left[\frac{\partial K}{\partial T}\right]_{\lambda}^{\xi}$ from the partial differentiation, solving for boundary conditions we obtain

Or, $\boldsymbol{B}_{(\mathbb{R}^n,\mathbb{C}^n)} \cong \bar{\boldsymbol{\phi}} \left[\frac{1}{\pi k^2}\right]_{\lambda}^{\xi}$

Or, $\boldsymbol{B}_{(\mathbb{R}^n,\mathbb{C}^n)} \cong \frac{\bar{\phi}}{\pi\lambda^2}$ and $\boldsymbol{B}_{(\mathbb{R}^n,\mathbb{C}^n)} \cong \frac{\bar{\phi}}{\pi\xi^2}$ this, gives two critical magnetic fields. Let us say $\mathbf{B_1}$ be lower and $\mathbf{B_2}$ be upper critical fields then, $\boldsymbol{B}_{\boldsymbol{1}(\mathbb{R}^n,\mathbb{C}^n)} \cong \frac{\bar{\phi}}{\pi\lambda^2} \dots\dots\dots\dots(\mathbf{3.25})$ and $\boldsymbol{B}_{\boldsymbol{2}(\mathbb{R}^n,\mathbb{C}^n)} \cong \frac{\bar{\phi}}{\pi\xi^2} \dots\dots(\mathbf{3.26})$. These equations provide a good approximation for us to determine the two critical magnetic fields for type-II superconductors.

IV. RESULTS

A. *The Origin of Magnetic Vortices*

The presence of superconducting material between two critical fields $\mathbf{B_1}$ and $\mathbf{B_2}$ suggests the presence of vortex state in a superconductor where discrete (quanta) of magnetic field penetrates through the superconducting material. As initially we have considered the superconducting material as a continuous, differentiable manifold **'M'** open by sets$\{\boldsymbol{U_a}\}_{\boldsymbol{a\in A}}$. Then, by this analogy we can consider magnetic vortices as Homotopical defect in the complex manifolds of the superconductor which allows the passage of magnetic field lines but as the magnetic vortices are inly homotopical defects, non-homological, the manifold is still a continuous, differential, topological manifold. The thickness of vortex depends upon the type of type-II superconductor, for YBCO they are about 200nm thick. In this way it represents quantized flux circulation of the magnetic field lines. The circulation of quantum vortex in a superfluid was found to be:

$$\oint_C \boldsymbol{V}.\boldsymbol{dl} = \frac{\hbar}{\boldsymbol{m}}\oint_c \boldsymbol{\nabla}_{\boldsymbol{\phi_v}}.\boldsymbol{dl} = \frac{\hbar}{\boldsymbol{m}}\Delta^{tot.}\boldsymbol{\phi_v} \dots\dots\dots\dots(\mathbf{4.11})$$

As, Circulation is quantized $\oint_C \boldsymbol{V}.\boldsymbol{dl} = \frac{2\pi\hbar}{m}\boldsymbol{n} \dots\dots\dots\dots\dots\dots\dots\dots(\mathbf{4.12})$

Homotopical defects are those ideas which signify structures in a physical system that are stable against perturbations. They won't decay, dissipate, disperse or evaporate in the way ordinary waves might. They work as pinned vortex tubes in type-II superconductors. Let us consider the magnetic vortices to be a homotopical defect in a topological manifold **'M'** open by sets$\{\boldsymbol{U_a}\}_{\boldsymbol{a\in A}}$. Let the defect be a discontinuous set of family of maps i.e. $\boldsymbol{M}_{\{\boldsymbol{U_a}\}} : \boldsymbol{\chi} \mapsto \boldsymbol{\omega}$ indexed by $\boldsymbol{a} \in \boldsymbol{A}$ where, $\boldsymbol{\chi}$ is the family of map and $\boldsymbol{\omega}$ is the vorticity of the magnetic field. As the vorticity depends upon magnetic field at constant temperature and pressure: $\boldsymbol{M}_{\{\boldsymbol{U_a}\}}: \boldsymbol{\chi} \mapsto \boldsymbol{\omega} \approx \boldsymbol{\nabla} \times \Delta\boldsymbol{B}$ where, **B** is the intensity of magnetic field.

I.e. the vortex map depends upon the value of critical magnetic field, foe the lower critical magnetic field the vorticity is low and current is easily circulated in the superconductor but when the intensity of magnetic field increases the vorticity increases as well and finally at upper critical magnetic field vorticity tends to maximum and finally superconductivity is destroyed and circulating current finally experiences some resistance, i.e. at $\boldsymbol{B} = \frac{\phi}{\pi\xi^2}$ the vorticity field is maximum:

$$\text{So, } \boldsymbol{\chi}: \boldsymbol{\omega_{max}} \approx \boldsymbol{\nabla} \times \frac{\phi_0}{\pi\xi^2} \dots\dots\dots\dots\dots\dots\dots\dots(\mathbf{4.13})$$

$$\text{Or, } \boldsymbol{\nabla} \times \frac{\phi_0}{\pi\xi^2} = \boldsymbol{\mu_0\epsilon_0}\frac{\partial \boldsymbol{E}}{\partial \boldsymbol{t}}$$

$$\text{Or, } \boldsymbol{\nabla} \times \frac{\mathbb{Z}_a.\mathbb{Z}_b{}^{-1}{}_{\mapsto}\left[\sum_{\mathbb{R}^n}\sum_{\mathbb{C}^n}(\pi\hbar c/e)_{S(\mathbb{R}^n,\mathbb{C}^n)}\right]}{\pi\xi^2} = \boldsymbol{\mu_0\epsilon_0}\frac{\partial \boldsymbol{E}}{\partial \boldsymbol{t}} \dots\dots\dots\dots\dots\dots(\mathbf{4.14})$$

Equation (4.14) shows that between the upper and lower critical field while the material superconduct there is development of opposing toroid of electric field inside the superconductor itself due to circulation of current inside the superconductor itself due to circulation of current around the magnetic vortices while beyond the lower critical field , the material superconduct, it experiences resistance to an extent but is very much negligible only after the superconductivity is destroyed the resistance becomes measureable. This means that the magnetic vortices provide a way for the material to superconduct while the material experiences some quantity of resistance. If $\mathbb{Z}_0$ be the topology of homotopical defect in manifold **'M'** then,

$$\mathbb{Z}_0\left(\overline{\mathbb{Z}_a.\mathbb{Z}_b{}^{-1}}\right) = \mathbb{Z}_a.\mathbb{Z}_b{}^{-1}[\boldsymbol{\omega} \approx \boldsymbol{\omega_{max}} \mapsto (\mathbb{C}^n,\boldsymbol{T})\boldsymbol{\nabla} \times \boldsymbol{B}] \dots\dots\dots\dots\dots(\mathbf{4.15})$$

Equation (4.15) provides us the topological structure if magnetic vortices evolving with magnetic field and temperature. This equation provides the solutions for stable configurations in dis-overlapping complex manifolds. It predicts the curl of magnetic field entering the superconductor is quantized and circulation of flux is conserved. It suggests that while the manifolds are dis-overlapping in the presence of magnetic field, there are presence of some special topological structures in the homotopical defect which meta-stabilize the manifold and form a vortex which allows the material to still superconduct by circulating current around magnetic vortex and still bear a non-zero resistance. If **'Γ'** be the circulation of vorticity field around a closed manifold **'M'** with unit normal **'n'** is:

$$\boldsymbol{\Gamma} = \int \boldsymbol{B}.\boldsymbol{dr} = \int_{M}^{\cdot} \boldsymbol{\omega}.\boldsymbol{n}\,\boldsymbol{ds} = \oint_{M}^{\cdot} \nabla \times \frac{\mathbb{Z}_a.\mathbb{Z}_b{}^{-1}{}_{\rightarrowtail}\left[\sum_{\mathbb{R}^n}\sum_{\mathbb{C}^n}(\boldsymbol{\pi}\hbar\boldsymbol{c}/\boldsymbol{e})_{\boldsymbol{S}(\mathbb{R}^n,\mathbb{C}^n)}\right]}{\boldsymbol{\pi\xi^2}}\,\boldsymbol{n}.\boldsymbol{ds} \ldots\ldots\ldots.(\mathbf{4.16})$$

$$\textbf{i.e.}\frac{\boldsymbol{D\Gamma}}{\boldsymbol{Dt}} = \boldsymbol{\omega}\int_{\Gamma}^{\cdot}(\Delta\boldsymbol{B}).\boldsymbol{dr} + \int\frac{1}{\boldsymbol{\omega}^2}\,\Delta\boldsymbol{\Psi} \times \nabla\lfloor\boldsymbol{p}\rangle\ \boldsymbol{n}.\boldsymbol{ds} \ldots\ldots\ldots\ldots\ldots\ldots(\mathbf{4.17})$$

On, solving (4.16) and (4.17) we get the required equation for the evolution of wave function with respect to time as:

$$\frac{\partial\boldsymbol{\Psi}(\mathbb{C}^n,\boldsymbol{t})}{\partial\boldsymbol{t}} = \int_{-\infty}^{+\infty}\frac{\mathbf{1}}{\mathbf{4}\boldsymbol{\pi}}\sum_{i=0}^{k}\left[\sum_{n=0}^{M}(\mathbf{2}\boldsymbol{\chi}+\boldsymbol{k})\,\mathbf{exp}\big(\boldsymbol{i}(\boldsymbol{k}^n\boldsymbol{\chi} * \lfloor\boldsymbol{p}\rangle)\big)\ \ [\boldsymbol{S}_i.]\boldsymbol{\chi}_p\right]\boldsymbol{d\chi} \ldots\ldots.(\mathbf{4.18})$$

Where, $\lfloor\boldsymbol{p}\rangle$ defines the momentum operator of cooper pair in the manifold

This equation states that while complex manifolds are dis-overlapping the paired wave functions also evolves and at either **$T>T_c$** or, **$B>B_2$** this wave function collapses as $\boldsymbol{\chi}: \boldsymbol{\omega}_{max} \longrightarrow \mathbf{0}$ i.e. $\frac{\partial\boldsymbol{\Psi}(\mathbb{C}^n,\boldsymbol{t})}{\partial\boldsymbol{t}} \longrightarrow \mathbf{0}$

In summary, as the magnetic field increases the corresponding homotopical defect i.e. vorticity increases and so does the circulation of vorticity. Then when the limit of $\boldsymbol{\omega}_{max}\ \nabla \times \frac{\boldsymbol{\phi}_0}{\boldsymbol{\pi\xi^2}}$ is overcome the vortices are so deep enough that the circulation of current around those gives rise to measureable resistance and destroys superconductivity. It also gives the insight that between **B_1** and **B_2** as there is circulation of current around magnetic vortices, it must give rise to non-zero yet non-measureable resistance i.e. for type-II superconductors , zero resistance isn't the absolute requirement, up to when overlapping energy $(\Delta\boldsymbol{E})$ of complex manifold supersedes the superconducting energy gap the material can superconduct.
I.e. superconductivity in type-II superconductors doesn't depend on presence or absence of resistance but rather is fundamentally dependent on the overlapping energy of electrons on the complex manifold.

V. DISCUSSION

The findings clearly show that apart from given theories, superconductivity in type-II superconductors can be explained by using a more fundamental theory on topological pairing of electron on complex manifold, considering the superconducting material as a continuous differentiable manifold **'M'**. It states that at non-trivial interactions the manifolds overlap creating an energy gap and correspondingly the equation for energy gap function is obtained in equation **(3.16)**. On application of magnetic field the overlapping of complex manifold begins disoverlapping and during these process there is presence of quasi-stable state in the manifold where magnetic field entering a superconductor is quantized and magnetic vortices arise in a superconductor and current circulates through that superconductor where the phenomological constant is decreasing and solving the boundary conditions provide two critical magnetic fields given in the equation **(3.25)** and **(3.26).** The origin of magnetic vortices describe that superconductivity in type-II superconductors doesn't depend on presence or absence of resistance but rather is fundamentally dependent on the overlapping energy of electrons on the complex manifold**.**

The further study is limited due to lack of experimental result, the experimental evidence if vorticity i.e. $\boldsymbol{\omega}$ and the circulation of vorticity i.e. **Γ** causing circulation of current in vortices must be experimentally verified very precisely before continuing further study on this matter. After adequate experimental evidence we can extend this theory and move on to the theoretical prediction of superconductors with very high critical temperatures.

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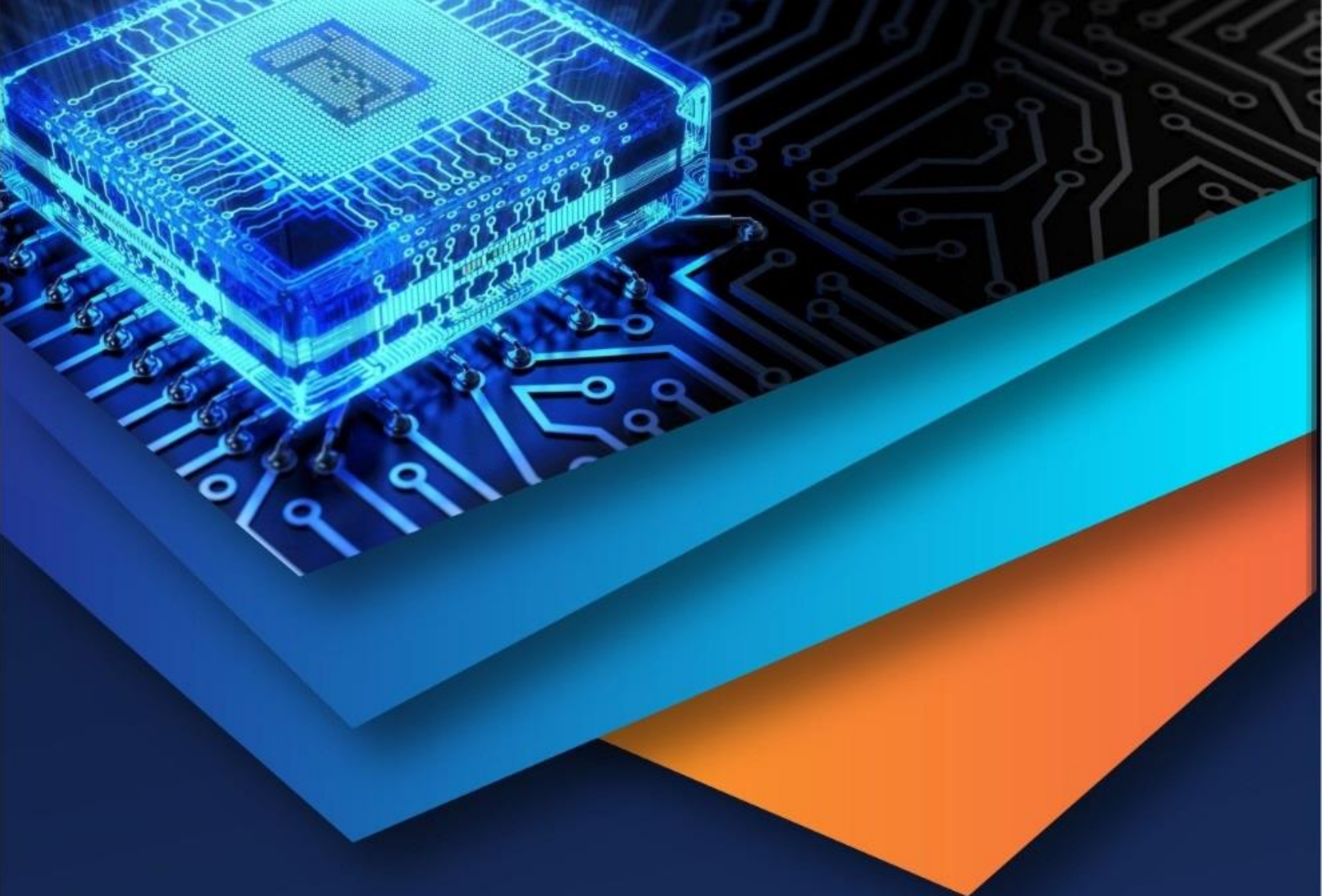

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