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A Problem based on Lagrange Relaxation Approach & Solving Optimization on Bi-level Model Programming Problem

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Abstract: In this paper, a salutation method based on Lagrange relaxation for discrete-continuous bi-level problems, with binary variables in the leading problem, considering the optimistic approach in bi-level programming. Linearized taking advantage of the structure of the leading problem, an algorithm is solving two-dimensional bi-level linear programming problems, classification of constrains, algorithm removes all redundant constraints, cycling and solution of problem.
Keywords: Lagrange Relaxation, Optimization Problem, Approach Method, Bi-level model

I. INTRODUCTION

Optimization is the process of making something as good as possible. In operation research optimization problem can be defined as the problem of finding the best solution of better than previously known the best solution among a set of feasible solution by trying different variations of the input [1]. Optimization problems can be categorized as either continuous discrete, based on the nature of the variables in the optimization function. The original formulation for Bi-level programming problem appeared in 1973, in a paper authored by J. Bracken and J. McGill [2], W. Candler and R. Norton [3]. It used the designation bi-level or multilevel programming. It was not until the early eighties that these problems started receiving the attention they deserve [4] [5] [6] [7]. Several authors studies bi-level programming problem intensively and contributed to its proliferation in the mathematical programming community. In general, a bi-level programming problem is difficult to solve [8], proved that even the simplest case, as the linear bi-level problem. Since 1980, a significant efforts have been devoted to understanding the fundamental concepts associated with bi-level programming various version of the linear bi-level programming problem are present by [9] [10] [11] [12]. At the same time, various algorithms have been proposed for solving these problems. Techniques inherent of extreme point algorithms and has been largely applied to the linear bi-level programming problems because for this problem, for a solution, then there is at least one global minimizer that is an extreme point [13]. In many bi-level programming problem, a subset of variable is restricted to taking discrete values. These characteristics [14] [15] [16] proposed branch and bound algorithms for mixed and binary integers [17] developed a branch and bound algorithm to solve binary non-linear mixed integer problem. It is pointed out [18] that the branch and bound methods require linear or non-linear convex functions at the lower level of the bi-level problem to be functional. Proposed fundamental properties to solution in bi-level linear programs with binary variables. Its penalty function method to solve discrete bi-level problems [19]. A reformulation and linearization algorithm for the whole bi-level mixed-integer general problem with continuous variables in the follower using the representation of its convex Hull [20]. Two others classes of Algorithms using multi parametric programming to solve bi-level integer problems with the integer variable controlled by first level [21]. An algorithms based on the same strategy for bi-level mixed integer problems where the follower solves an integer problem [22]. A survey on the linear bi-level programming problem has been written by O. Ben–Ayed [23]. The complexity of the problem has been addressed by a number of authors [24] [25] [26]. It has been proved that even the linear bi-level programming problem [27] [28]. An algorithm for the global optimization of bi-level mixed-integer nonlinear problems consisting of generating a convergent lower bounded and an optimal upper bound [29]. An exact algorithm for the linear mixed-integer bi-level problem with some simplification. Considered integer bi-level problems with the leader objective function being linear-fractional and linear the follower, it proposed an iterative algorithm of cut generation to solve the problem [24]. Using decomposition techniques, an algorithm based on benders decomposition to solve the linear mixed integer binary problem, with the method, the target value bounded, are used to transform the problem into problems of one level [25]. Based on the use of benders decomposition with a continuous sub problems [26, 30].

A solution method for discrete continuous bi-level problem based on Lagrange relaxation is presented. The nonlinear problem can be linearized by taking advantage of the structure of the binary variables of the leader problem. An attempt has been made a develop a method in which contractions are analysed, and used for solved two-dimensional linear bi-level programming problems our proposal allows us to find a global optimal through a decomposition technique, taking advantage of some characteristics of the problems involved. This paper is organized as follow: the definition and formulation of the discrete continuous bi-level problems.

II. PROBLEM DEFINITION AND FORMULATION

We formulate the discrete linear binary problem, where optimistic approach is assumed [31]. If it has alternative optimal solution, choose the one that is the best for the leader.

$$\min_{m,n} \; e_1 m + f_1 n$$

Subject to: $C_1 m + D_1 n \leq h_2$

$$\min_{n} e_2 m + f_2 n$$

Subject to: $e_2 m + f_2 n$

$$m \in \{0,1\}, n \geq 0$$

Where, $e_{1,} f_1 \in \mathbb{Z}$, $e_2, f_2 \in \mathbb{Z}^{t2}$, $h_1 \in \mathbb{Z}^u$, $h_2 \in \mathbb{Z}^v$

$$C_1 \in \mathbb{Z}^{u \times t}, D_1 \in \mathbb{Z}^{u \times t_2}, C_2 \in \mathbb{Z}^{v \times t_1}, D_2 \in \mathbb{Z}^{v \times t_2}$$

When the lower-level problem is linear, then it reformulated by replacing the lower-level problem with its karush-kuhn-Tucker condition.

$$\min_{m,n} \; e_1 m + f_1 n$$

Subject to:

$$C_1 m + D_1 n \leq h_1$$

$$C_2 m + D_2 n \leq h_2$$

$$\lambda^S (h_2 - C_2 m - D_2 n) = 0$$

$$(f_2 + \lambda^S D_2) n = 0$$

$$m \in \{0, 1\}, n, \lambda \geq 0$$

Where,

$$f_2^s n \geq -\lambda^S D_2 n \geq \lambda^S C_2 m - \lambda^S h_2$$

This problem can be reformulated by relating the prime and dual constraints and requiring the duality gap to be zero.

By including the equation:

$$f_2^T n = \lambda^S C_2 m - \lambda^S h_2$$

Then, The problem given as:

$$\min_{m,n} \; e_1 m + f_1 n$$

Subject to:

$$C_1 m + D_1 n \leq h_1$$

$$C_1 m + D_2 n \leq h_2$$

$$-\lambda^S D_2 \leq f_2$$

$$f_2 n \ = \lambda^S C_2 m - \lambda^S h_2$$

$$m \in \{0,1\}, n, \lambda \geq 0$$

The problem: $\lambda m = \ \mu \left[\mu \ldots \ldots \ldots . \mu_{t_1}\right]$,

Where μ_1 is the *i*-th column of μ constraint. It can be obtain the following equivalent problem:

$$\min_{m,n} e_1 m + \ f_1 n$$

Subject to:

$$C_1 m + D_1 n \leq h_1$$

$$C_1 m + \ D_2 n \ \leq h_2$$

$$-\lambda^S D_2 \leq \ f_2$$

$$f_2 n \ = \ \Sigma_{j=1}^{t_1} \mu_j^S \ C_{2i} - \lambda^S h_2$$

$$\mu_{k_j} \ \geq \ \lambda_k - L(1 - m_j) \quad \text{k = 1………v, j= 1……….} t_1$$

$$\mu_j \ \leq \ \lambda \quad \text{j=1……….} t_1$$

$$\mu^S \leq Lm \quad \text{k=1……..v}$$

$$\mu \geq 0$$

$$m \in \{0, 1\}, n, \lambda \geq 0$$

where L is positive number

It is ensured that variables μ_{kj} take value zero

If $m_j \ = 0$ and λ_k

If $m_j \ = 1$, j=1………t_1 , *k*…1…..*v*,

This problem can be transformed into following model equivalent and avoid the use of binary variables.

$$\min_{m,n} e_1 m + f_1 n$$

Subject to

$$C_1 m + \ D_1 n \leq h_1$$

$$C_1 m + \ D_2 n \leq h_2$$

$$\lambda^S D_2 \leq f_2$$

$$f_2 n \ = \ \Sigma_{j=1}^{t_1} \mu_j^S \ C_{2j} - \lambda^S h_2$$

$$\mu_{kj} \geq \lambda_k - L(1 - m_j) \ \text{k= 1......v; j=1…..} t_1$$

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$\mu \leq \lambda$ j=1…..t_1

$\mu_k^S \leq Lm \quad k=1........v$

$\mu \geq 0$

$m \leq m^2$

$0 \leq m \leq 1, n, \lambda \geq 0$

It is considered as a complicated constraint

III. PROBLEM FORMULATION OF ALGORITHMS

A. Lagrange Relaxation Approach Method

It's complicated constraint given as:

$y \geq 0.$

A Larangian relaxation is defined by:

$$\min_{m,n} e_1 m + f_1 n + y(m - n^2)$$

Subject to: $C_1 m + D_1 n \leq h_1$

$$C_1 m + D_2 n \leq h_2$$

$$-\lambda^S D_2 \leq f_2$$

$$f_2 n = \Sigma_{j=k}^{t_1} \mu_j^S C_{2j} - \lambda^S h_2$$

$$\mu_{kj} \geq \lambda_k - 2(1 - m_k) \text{ k=1……v; j=1……….}t_1$$

$$\mu_j \leq \lambda \quad \text{j=1………}t_1$$

$$\mu_k^S \leq Lm \ k = 1.........v,$$

$$\mu \geq 0$$

$$0 \leq m \leq 1,\ y,\ \lambda \geq 0$$

Where the dual function ω(y) is defined as the Lagrange sub-problem:

$$\omega\ (y) = \min_{m,n}\ e_1 m + f_1 n + y\ (m - m^2)$$

Subject to: $C_1 m + D_1 n \leq h_1$

$$C_1 m + D_2 n \leq h_2$$

$$-\lambda^S D_2 \leq f_2$$

$$f_2 n = \Sigma_{j=k}^{t_1} \mu_j^S C_{2j} - \lambda^S h_2$$

$$\mu_{kj} \geq \lambda_k - L\ (1 - m_k) \quad \text{k=1…….v; j=1……….}t_1$$

$$\mu_j \leq \lambda \ \text{j=1……..}t_1$$

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$\mu_k^S \leq Lm$ k=1…….v,

μ≥0

0≤m≤1, y, λ≥0

Where the dual function ω (y) is defined as the Lagrange sub problems:

$$\omega(y) = \min_{m,n}\ e_{1m} + f_1 n + y(m - m^2)$$

Subject to:

$$C_1 m + D_1 n \leq\ h_1$$

$$C_2 m + D_2 n \leq h_2$$

$$-\lambda^S D_2 \leq f_2$$

$$f_2 n\ =\ \Sigma_{j=1}^{t_1}\ \mu_j^S\ C_{2j}\text{-}\lambda^S h_2$$

$\mu_{kj} \geq\ \lambda_k\text{-}L\,(1 - m_j)$ k=1……..v; j=1…..t_1

$\mu_j \leq \lambda$ j=1………t_1

$\mu_k^S \leq Lm$ k=1…………v

μ ≥ 0 0≤m≤1, n, λ≥0

From the previous solution, for all y≥0

$$\omega(y) \leq e_1 m + f_1 n$$

The value of the dual function are lower bonds of the optimal value. It has duality gap

$$C_1 m + f_1 n - \omega(y) \geq 0$$

We try to solve this problem and reduce the duality gap, the duality gap usually greater than zero. For the convex problem the duality gap disappears [24].

To find the vector of multipliers y for which the lower bounded given by dual function is maximum. $\min_y\ \omega(y)$

Subject to y≥0

The feasible solution could be in the feasible region this problems could solved by listing all of then as:

$$\omega(y) = \min_{m,n}\ e_1\, m^S + y\,(m^s - (m^m)^2),\ S=1\ldots\ldots\omega$$

A dual decomposition algorithm is proposed, an approximation of the dual function is maximized and has convergence properties Similar to those of the sub gradient method

The dual function is concave and the problem can be reformed from Lagrangian relaxation

$$\min_y\ \omega(y)$$

Subject to:

$$\omega \leq e_1 m^1 +\ f_1 n^1 + y(m^1 - (m^1)^{2)}$$

$$\omega \leq e_1 m^1 + f_1 n^w + y(m^w - (m^w)^2)$$

The optimization of the problem consists of the iterative resolution with the solution a new value of the multiplier y is obtained, which evaluated in the Lagrange sub problem; when the multiplier generated non-bounded sub problems, a constraint must be entered on the eliminates the multiplier. It is obtained by the solving the boundary sub-problem.

$$\omega\,(y) = \min_{m,n}\; e_1 m + f_1 n + y(m - m)^2$$

Subject to:

$$C_1 m + D_1 n \leq 0$$

$$C_2 m + D_2 n \leq 0$$

$$-\lambda^S D_2 \leq 0$$

$$f_2 n = \Sigma_{j=1}^{t_1} \mu_j^S C_{2j} - \lambda^S h_2$$

$$\mu_{kj} - \lambda_k - Lm_j \geq 0 \quad k=1\ldots\ldots.v;\ i=1\ldots\ldots..t_1$$

$$\mu \leq \lambda\ j=1\ldots\ldots.t_1$$

$$\mu_k^S \leq Lm\ \ k=1\ldots\ldots.v$$

$$\mu \geq 0$$

$$0 \leq m \leq 1,\ n,\ \lambda \geq 0$$

If the optimal solution is a negative value

$$0 \leq e_1 m^w + f_1 n^w + y(m^w - (m^w)^2)$$

The Lagrange relaxation algorithm iterates formed by Lagrange and boundary sub-problem that evaluates the proposed multipliers.

IV. MATHEMATICAL FUNDAMENTAL PRINCIPLE

We defined two types of constraint classes for the proposed method of the algorithm. Considering the normal to be towards the half plane region not satisfied by constraints, we defined the following:

Concave constraints whose normal make angles with the x-axis in the range [0, π]

Constraints whose normal make angle with the x-axis in the range [0, π]

Constraints and convex constraints defined here are other than non-negative constraints. Variable types of constrains on the basis of the above definition are given:

	Concave						Convex					
x_j	+	+	-	-	+	0	+	+	-	-	-	0
y_j	+	+	+	+	0	+	-	-	-	-	0	-
z_j	+	-	+	-	+	+	+	-	+	-	-	-

The form of bi-level linear programming problem considered here:

$$\min_p \; or \; \min_p \; f_1\,(p,q) \quad \text{Where } q \text{ solve} \quad (1)$$

$$\min_q \; or \; \min_q \; f_2(p,q) \quad (2)$$

$$x_j P + y_j\, Q \leq z_j \quad (3)$$

$$P, q \;\geq 0$$

It can be observed inducible region for the finite solution

Case 1. A part of the line of concave constraints:

Case 2. A part of the line of convex constraints or part of the x-axis. (From 2)

The control it is only on the variable, therefore for a given x, if it is maximized in the positive direction of y-axis, then extreme point will be a point on a line of concave constraint. The method of solving problems we come across two type of redundant concave constraint and type of redundant convex constraint. A concave constraint which is redundant when no convex constraints are considered is one type of redundant concave constraints.

V. CONCLUSION

The proposed method is based on the analysis constraints. The traditionally used of finding optimum such as interior point method or simplex method in which search is made by moving along the boundary of the feasible region the properties of constraints there is a possible of developing a method which solves the problem in finite number.

The algorithm operation solving an application problem. The performance of the algorithm was measured using the execution time and comparing the obtained solutions with those corresponding to the single-level-reformulation. We use a Lagrange relaxation algorithm, it is possible to find a global solution efficiently

We propose an algorithm to solve the discrete continuous bi-level problem with result very close to the optimum. Lagrange relaxation is applied to the reformulation to a single level of the problem considering and the binary variables are relaxed for the construction of the Lagrange sub-problem.

VI. FUTURE WORK & SCOPE

The approach will be used without considering the single level reformulation of the bi-level problem and the consideration of more than one objective in the leaser function. The method can be used for problems involving cooperative decision-making at two level e.g. allocation of resources at minimum cost considering maximisation of the level of service, location of unwanted facilities, among others.

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