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A Review of Prime Numbers, Squaring Prime Pattern, Different Types of Primes and Prime Factorization Analysis

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Abstract: ***The study of prime numbers and their properties has always been an intriguing and fascinating topic for mathematicians. Primes can be considered the "basic building blocks," the atoms, of the natural numbers. They play a significant role in number theory. Also, prime numbers, in this current world of computers and digitalization, have paramount significance for the computer programmers and scientists to tackle relevant real-life problems. Since long time, many studies and researches have been conducted regarding prime numbers pattern. In this paper, a squaring prime pattern is presented. Moreover, fifteen different types of primes with their Python code to generate them is included within. In cryptographic encryption system, prime numbers play a major role for security systems in which prime factorization is necessary. Therefore, prime factorization of composite numbers using Sieve of Eratosthenes algorithm on different platforms and time analysis based on that has been presented in the paper. Also, factorization analysis of primes by five different algorithms has been shown and comparison of prime factorization of composite numbers vs time taken graph has been plotted. Two major applications of primes are also covered.***
Keywords: Prime numbers, Composite numbers, Python, Java, Ruby, R, Cicada, Cryptography, Encryption, Decryption, Prime Factorization, Factorization Algorithm and NP-problem.

I. INTRODUCTION

A prime number, or simply a prime, is a natural number greater than 1 that has no positive divisors other than 1 and itself. Symbolically, a number p is said to be prime if
(i) $p > 1$
(ii) p has no positive divisors except 1 and p.
A prime number is an integer greater than 1, only having two factors: 1 and itself. the fundamental theorem of arithmetic, also called the unique factorization theorem and prime factorization theorem, states that every integer greater than 1 can be represented uniquely as a product of prime numbers, up to the order of the factors [3]. For example, 21 is an integer greater than 1, which is not a prime number, but it can be represented using the product of primes, i.e., 3 x 7. "1 is a prime number or not" remains confusing for people. This can be understood by the fundamental theorem of arithmetic. One can also be written as 1 = 1 x 1 x …… x 1 x 1. This doesn't obey the fundamental theorem of arithmetic, so it is not a prime.
Another definition for prime numbers has been given by Borevich and Shafarevich as an element $\mathbb{p}$ of the ring **D**, nonzero and not a unit, that cannot be decomposed into factors $\mathbb{p}$=ab, neither of which is a unit in **D**, in their classic text "Number Theory". It is well known that 2, 3, 5, 7, 11, 13, etc. are the first few primes. A natural number greater than 1 that is not a prime is called a composite number. The property of being a prime is known as primality. And two numbers, **a** and **b**, are said to be relatively prime if they have no prime factors in common [2].
Prime numbers and their pattern are a subject of fascination to the mathematicians since the very earliest time. There are numerous publications and different theories made by different mathematicians all over the world, seeking out ways to find to find the exact patterning. The inability to find an order has been eloquently documented, such as in Havil's book:
"The succession of primes is unpredictable. We don't know if they will obey any rule or order that we have not been able to discover still. For centuries, the most illustrious minds tried to put an end to this situation, but without success. Leonhard Euler commented on one occasion. Mathematicians have tried in vain to this day to discover some order in the sequence of prime numbers, and we have reason to believe that it is a mystery into which the human mind will never penetrate. In a lecture given by D.
Zagier in 1975, he said, "There are two facts about the distribution of prime numbers that I hope to convince you so overwhelmingly that they will be permanently engraved in your hearts. The first is that [they are] the most arbitrary and ornery objects studied by

mathematicians: they grow like weeds among the natural numbers, seeming to obey no other law than that of chance, and nobody can predict where the next one will sprout. The second fact is even more astonishing, for it states just the opposite: that the prime numbers exhibit stunning regularity, that there are laws governing their behaviour, and that they obey these laws with almost military precision." (Havil 2003) [1]

To provide a bit of a historical context, prime numbers are not exactly known to be discovered at the exact time. There is speculation that some of the very earliest human civilizations had some concept of primes, but the first concrete evidence appears to be some Papyrus writings of the ancient Egyptians from over 3500 years ago [5]. The ancient Greeks were the first to include prime numbers in their academic curriculum. Since then, a lot of mathematicians have contributed in this field of mathematics. In 1640, Pierre de Fermat stated (without proof) Fermat little theorem (later proved by Leibniz and Euler) also investigated the primality of the Fermat numbers $2^{2^n} + 1$, and Marin Mersenne studied the Mersenne primes, prime numbers of the form $2^p - 1$ with p itself a prime number. Goldbach formulated Goldbach's conjecture, that every even number is the sum of two primes in a 1742 letter to Euler [7]. Likewise, Euler, Gauss, Chebyshev, and Reimann also contributed heavily, particularly in the distribution of primes. Still, study of prime numbers is very intriguing and challenging and there are numerous unsolved problems regarding prime numbers such as The Twin Prime Conjecture, Goldbach's Conjecture, infinitely many primes of the form n# -1?, infinitely many primes of the form $n^2 + 1$? [6] and many more.

Prime numbers have a huge number of applications, both within mathematics and in the wider world. In fact, these days, primes are used by all of us on an almost daily basis, even if we're not aware of it at the time. For mathematicians, primes are important because they are the atoms of multiplication [5], so many abstract problems involving multiplication can be solved if we know enough about prime numbers. In the wider world, the main uses of prime numbers are related to computers. In security system such as online transactions and communications, prime numbers are used. So, the analysis of prime generation has been carried out, and factorization of the composite numbers using Traditional, Fermat Theorem, Pollard Rho and other methods.

II. SQUARING PRIME PATTERN

Here are the prime numbers up to 151:

2	3	5	7	11	13
17	19	23	29	31	37
41	43	47	53	59	61
67	71	73	79	83	89
97	101	103	107	109	113
127	131	137	139	149	151

1	(2)	(3)	4	(5)	6	(7)	8	9	10
(11)	12	(13)	14	15	16	(17)	18	(19)	20
21	22	(23)	24	25	26	27	28	(29)	30
(31)	32	33	34	35	36	(37)	38	39	40
(41)	42	(43)	44	45	46	(47)	48	49	50

These are the numbers from 1 to 151 that are only divisible by 1 and themselves. On looking at these numbers superficially, we find no peculiar patterns between them. While looking at these, we won't instantly search for patterns and we won't find it easily as well. But guess what? There are several patterns among the prime numbers that were found by different mathematicians. In this paper, we will specifically be talking about the squaring primes pattern and proving it as well.

Consider the prime number p. Now, relying on this squaring prime pattern, we have, i.e., the square of any prime number minus one is the multiple of 24. And this pattern can be applied to all prime numbers, exclusive of 2 and 3, which can be considered sub-primes. For instance, let us take any two prime numbers, 19 and 73. Then,

$19^2 - 1 = 360$

$= 24 * 15$

$73^2 - 1 = 5328$
$= 24 * 222$

Here, we can see the pattern followed by these numbers.

Basically, this pattern is derived from another simple pattern which includes multiples of 6, which says that the prime numbers lie above or below the multiples of 6.

From the above table, we can clearly see that the numbers on either side or even both sides of the multiples of 6 are primes. We can find that numbers one less or one more than a multiple of 6 are prime for smaller numbers, and this applies to either of the numbers as the number increases.

Let's take the numbers 10 and 20.

10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20

It's not magic that the primes are only on either side of multiples of 6, but those are the only possible places they can be. Between 12 and 18, they can't be at the even points because primes cannot be even. And the number at the centre of multiples of 6 is a multiple of 3. Thus, primes can only be above or below a multiple of 6. Now let's move on to proving the squaring primes pattern. Consider a prime number p, which can be represented as:

$p = 6k + 1$

$p = 6k - 1$

where k is an integer that can be either even (2m) or odd (2m + 1).

Now the primes can be represented as:

1. $p = 6(2m) + 1 = 12m + 1.$
2. $p = 6(2m + 1) + 1 = 12m + 7.$
3. $p = 6(2m) - 1 = 12m - 1.$
4. $p = 6(2m + 1) - 1 = 12m + 5.$

Let us square the primes. Then we have:

1. $p^2 = 144m^2 + 24m + 1 = 24(6m^2 + m) + 1 = 24 * k_1 + 1$
2. $p^2 = 144m^2 + 168m + 49 = 24(6m^2 + 7m + 2) + 1 = 24 * k_2 + 1$
3. $p^2 = 144m^2 - 24m + 1 = 24(6m^2 - m) + 1 = 24 * k_3 + 1$
4. $p^2 = 144m^2 + 120m + 25 = 24(6m^2 + 5m + 1) + 1 = 24 * k_4 + 1$

From the above observations, we can conclude that $p^2 - 1$ is a multiple of 24. This pattern can also be proved in a better way, which is described below:

$p^2 - 1 = (p - 1)(p + 1)$

On representing the numbers p – 1, p and p + 1 on a number line, we can say that p – 1 and p + 1 are even as p cannot be even, which implies that one of them is a multiple of 2 and the other is a multiple of 4.

1) Therefore, (p – 1)(p + 1) is a multiple of 8. Also, on a set of three consecutive numbers, one of them must be a multiple of 3. But p cannot be a multiple of 3, which implies either (p – 1) or (p + 1) is a multiple of 3.

2) Therefore, (p – 1)(p + 1) is a multiple of 3. From the above points, we can say (p – 1) (p + 1) is definitely a multiple of 24. Now, we know that every prime, excluding 2 and 3, could be represented in a pattern as the square of the prime is one less than a multiple of 24. This is the Squaring Prime Pattern [4].

III. TYPES OF PRIMES

There are in total 76 types [8] of primes as of now. But the study of prime numbers is an ongoing field of research. New types of primes may be discovered or defined as mathematical exploration continues. In this paper, fifteen most important primes are illustrated.

A. Mersenne Prime

A Mersenne prime [9] is a prime number that is one less than a power of two. That is, it is a prime number of the form $M_n = 2^n - 1$ for some integer n [9]. As of 2023, 51 Mersenne primes are known. This class of prime also holds the largest known prime, $M_{82589933}$.

PythonCode:

https://colab.research.google.com/drive/1kPZg2BdNDUk0BcvB7EgZgi7mofv5UEKU#scrollTo=BvgihA4JoG6P&line=3&uniqifier=1

B. Fermat Prime

A Fermat prime [8] is a prime number that is in the form of $2^{2^n} + 1$, where n is a positive integer. As of 2023, the only known Fermat primes are $F_0 = 3$, $F_1 = 5$, $F_2 = 17$, $F_3 = 257$, and $F_4 = 65537$.
Python Code: https://colab.research.google.com/drive/1Gg3U76FCdMkaQ89S-aXt9Uw_s1_AkPbA#scrollTo=rEGBGvI6Nb_bo&line=10&uniqifier=1

C. Sophie German Prime

A prime number p is a Sophie Germain prime [9] if $2p + 1$ is also prime. The number $2p + 1$ associated with a Sophie Germain prime is called a safe prime. For example, 11 is a Sophie Germain prime and $2 \times 11 + 1 = 23$ is its associated safe prime.
Python Code: https://colab.research.google.com/drive/1kV4eQwfUCR53z2XKTu41ltXOHIGDsxw-#scrollTo=7xq0Ub1fmh-m&line=2&uniqifier=1

D. Wilson Prime

A Wilson Prime [10] is a prime p > 1 such that if and only if
$(p - 1)! \equiv -1 \pmod{p^2}$
In other words, it is a prime p such that p^2 divides $(p – 1)! + 1$. Only known Wilson primes are 5, 13, 563. Another Wilson prime is greater than $2 * 10^{13}$.
PythonCode:
https://colab.research.google.com/drive/1iVFODLy0_FPpmgChtRHVcCaroU8NNDdn#scrollTo=I76FzVJBo0ui&line=3&uniqifier=1

E. Twin Primes

Twin Prime is the prime in the form of (p, p + 2), where p is a prime number. In other words, twin prime is two less or more than the other prime – for example, either pair of (17, 19) or (41, 43). Every twin prime except (3, 5) is in the form of (6n – 1, 6n + 1), where n is a natural number.
PythonCode:
https://colab.research.google.com/drive/1r3y9ejuq77FrOEbsHkjW9jc_M7MqJWwI#scrollTo=jo87DITgp1YE&line=3&uniqifier=1

F. Palindromic Primes

In general, a palindrome is a word, sentence, verse, or number that reads the same backward and forward. So, a prime number which is palindrome is palindromic prime. Except 11, all palindromic primes have odd number of digits.
Python Code: https://colab.research.google.com/drive/1cYJ6GM36hsl3ERTQ-OppYLJPLLmEHUQd#scrollTo= u1ClhtbetB_HE_&line=17&uniqifier=1

G. Pseudo Prime

A Pseudoprime [10] is an integer which shares a common property with all prime numbers but is not actually a prime. Pseudoprimes are classified according to which property of primes they satisfy.

1) Fermat Pseudoprime to Base a: For an integer a > 1, if a composite integer n satisfies $a^{n-1} \equiv 1 \pmod{n}$, then n is said to be a Fermat pseudoprime to base a and is denoted by psp(a).

Python Code: https://colab.research.google.com/drive/14_YQEiOyw2cDF7LV-49Y_FHWingPcj1l#scrollTo=i_NCT_v7huf_Gr&line=18&uniqifier=1

2) Euler Pseudoprime to Base a: For an integer a > 1, if an odd composite integer n which is coprime to a satisfies the congruence relation

$\left(\frac{a}{n}\right) \equiv a^{(n-1)/2} \pmod{n}$,

Where $\left(\frac{a}{n}\right)$ is the Jacobi symbol, then n is called *Euler pseudoprime to base a* and denoted by epsp(a).
PythonCode:
https://colab.research.google.com/drive/1RoXtCzG2dp3w_9CqibwJKuFjQYW4e4BF#scrollTo=NSF2kcA4xcv4&line=38&uniqifier=1

3) Strong Pseudoprime to Base a: Let n = 2sd + 1 where s and d are positive integers and d is odd. Also, let a > 1 be a positive integer coprime to n such that one of the following conditions holds:

$a^d \equiv 1 \pmod n$, or

$a^{2^r d} \equiv -1 \pmod n$, for some integer $0 \leq r < s$.

Then n is called a *strong pseudoprime to base a* and is denoted by spsp(a).

PythonCode:

https://colab.research.google.com/drive/1Lnk4JEmxlsuyP6OzFuxHIP5E8qlWrxw#scrollTo=AwLALahcNu7n&line=54&uniqifier=1

H. Circular Prime

Prime that remains prime upon cyclic rotation of its digits is circular prime. In other words, it is a prime with the property that the number generated at each intermediate step while cyclically permuting its (base 10) digits will be prime. Examples include 1193. 1931, 9311, and 3139 are also primes.

Python Code: https://colab.research.google.com/drive/10SJeY8hw9B10x7b-FrG2XhL5FC4xTaPc#scrollTo=tTC3A-AS8rbQ&line=12&uniqifier=1

I. Gaussian Prime

Gaussian primes are a specific type of prime numbers that arise in the context of complex numbers. The concept of Gaussian primes is an extension of the notion of prime numbers from the real number system to the complex number system. In the complex plane, a Gaussian integer is a number of the form a + bi, where a and b are integers, and **i** is the imaginary unit √(-1). A Gaussian integer is considered a Gaussian prime if it has the following properties:

1. One of *a or b* is zero & the absolute value of the complex number is a prime number of the form *4n + 3* (*n* is an integer).
2. Both *a and b* is nonzero and $|a^2 + b^2|$ i.e., modulo of Gaussian number is a prime number, which is not in the form of *4n + 3*.

To put it simply, a Gaussian prime is a Gaussian integer that has no nontrivial divisors within the set of Gaussian integers. For example, some of the Gaussian primes include: - 1 + i, - 1 – i, 2 + i, etc.

Python Code: https://colab.research.google.com/drive/1LRfqaI7DasGTCdhN48wlzdTrRKZJSvBs#scrollTo=8VjVz8UYARPz&line=27&uniqifier=1

J. Cousin Primes

Cousin primes [9] are the primes that differ by four. The only prime belonging to two pairs of cousin primes is 7, (3, 7) and (7, 11).

Python Code: https://colab.research.google.com/drive/1FHeCkgUWUzFkG_IWhF74lVf33VByDxb7#scrollTo=79iu0XQiB_xPO&line=15&uniqifier=1

K. Wagstaff Prime

A Wagstaff prime is a prime number of the form $\frac{2^p+1}{3}$, where *p* is an odd prime number. The first three Wagstaff primes are 3, 11, and 43.

PythonCode:

https://colab.research.google.com/drive/1tbx60sWhLGi2cXXQQNYmrfBwg7bGUlxc#scrollTo=rpYPVgnQDC_F&line=5&uniqifier=1

L. Wall-Sun-Sun Prime

A prime p > 5 [8], if p^2 divides the Fibonacci number $F_{p-\left(\frac{p}{5}\right)}$, where the Legendre symbol $\left(\frac{p}{5}\right)$ is defined as

$$\left(\frac{p}{5}\right) = \begin{cases} 1 & if\ p \equiv \pm 1\ (mod\ 5) \\ -1 & if\ p \equiv \pm 2\ (mod\ 5) \end{cases}$$

As of now, no Wall-Sun-Sun primes are known.

M. Good Prime

Primes p_n for which $p_n^2 > p_{n-i}\, p_{n+i}$ for all $1 \leq i \leq n-1$, where p_n is the *n*th prime.

In other words, good prime is the prime whose square is greater than the product of any two primes at the same number of positions before and after it in the sequence of primes [8] [9].

For instance,
we take a series of prime numbers like 11, 13, 17, 19 and 23 then $17^2 > 13 * 19$ and $17^2 > 11 * 23$. The required condition is fulfilled, so 17 is a good prime.
PythonCode:
https://colab.research.google.com/drive/1bNkcQGVT7LFaCQVBXcvhTmct0VsE6TcY#scrollTo=2ubMSHsSEF0c&line=5&uniqifier=1

N. Balanced Prime

The prime formed by taking out arithmetic means of prime numbers above and below a specific number than that and if the arithmetic mean value itself is a prime number and that value is known as balanced prime. It is given by;

$$p_k = \frac{\sum_{i=1}^{n}(p_{k-i} + p_{k-i})}{2n}$$

where p_{k-i} and p_{k+i} are also primes and p_k is i^{th} mode Balanced Prime, k is the index of ordered prime. For example
PythonCode:
https://colab.research.google.com/drive/1bksAcJSSJ4r_zUJbtikHRy9OWuPh6Jnp#scrollTo=BiVGRJIKFzCC&line=13&uniqifier=1

O. Primorial Prime

A primordial prime is a prime number of the form $p_n\# \pm 1$, where $p_n\#$ is the primordial of p_n. The primordial of p_n is defined as;

$$p_n\# = \prod_{k=1}^{n} p_k$$

Here, $p_n\# + 1$ is also known as Euclid Number (E_n) and $p_n\# - 1$ is also known as Kummer number (E_n).
PythonCode:
https://colab.research.google.com/drive/1BfaAyci5i1IaNK3oQAnE2dV95c2WJPhg#scrollTo=uccKfpgaHSlw&line=12&uniqifier=1

Language	Execution Time taken by different OS in milliseconds.		
	MacOS	Window 11	Linux-Xubuntu
R	664839	604779	965214
Ruby	1903.06	1185.84	2283.98
Python	1315.23	1678.21	2651.10
C	155.19	93	362.81
C++	425.70	612	1400.8
Java	57	72	366

IV. TIME ANALYSIS OF PRIME GENERATION

Here, in the given table, the analysis of time required to generate prime numbers using Sieve of Eratosthenes algorithm on six different programming languages on different operation systems is given. This table includes the execution time to complete the generation of prime numbers from 1 to 10^7. From the table below, it can be seen that Java is the most efficient and less time-consuming language in almost all the operating systems.
Here, Python, one of the most popular languages, seem to be significantly slower except R. But still, Python remains the topmost priority for most of the developers because of the its extremely short line of codes and easier trouble shooting capability.

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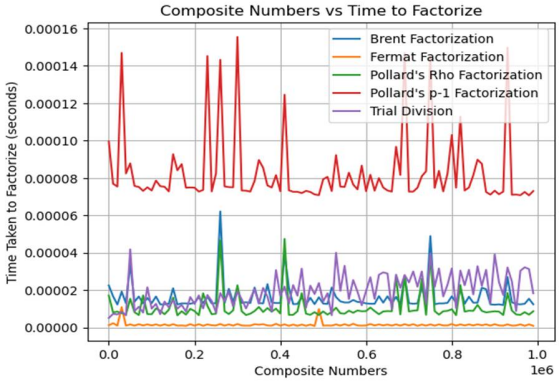
V. PRIME FACTORIZATION ANALYSIS

Since prime factorization is an NP-problem, many cryptographic systems are based and built on it. A number of methods are available for prime factorization. The paper of Connelly Barnes on Integer Factorization Algorithms [11] presents the graph of decimal digits in prime factors and number of steps required to factorize a number.

In this paper, comparison of composite numbers vs time taken to factorize that composite number using Trial Division Factorization, Fermat Factorization, Pollard Rho Factorization, Pollard P1 Factorization and Brent Factorization method [11].



Python Code: https://colab.research.google.com/drive/1M2djYjhtKy4Q9jtL5vdoKx6jk4cSMLdx#_crollTo=78NEyiDgy_YWa&line=4&uniqifier=1

VI. APPLICATIONS

A. Primes in Nature

The prime numbers have various applications in different fields, and one intriguing application can be found in the life cycle of periodical cicadas. Periodical cicadas are a fascinating group of insects that spend the majority of their lives underground and emerge in large numbers at specific intervals, often in prime number years i.e., 7, 13 or 17. Note that, 7,13,17 are prime numbers. The reason [2] behind this 7, 13, 17-year period is that to avoid predators. Assume that a cicada' appeared at non-prime number intervals, say every 10 years, then predators appearing every 2, 5, or 10 years would be sure to meet them. The prime-numbered life cycles of different broods may help with resource partitioning. For example, broods with 13-year cycles and 17-year cycles will emerge in different years, reducing competition for resources (food, space, etc.) between them.

In 20th century, a great mathematician Stanislaw Ulam discovered amazing design considering density of prime numbers which is known as Ulam's rose or prime number spiral. It is similar to some pattern that can be seen in nature, simply a rose [12].

B. Cryptography

Prime numbers [13] play a fundamental role in encryption and cryptography, especially in asymmetric encryption algorithms like RSA (Rivest-Shamir-Adleman). The steps of the RSA algorithm involve the use of prime numbers to generate the public and private keys. Here's a overview of the RSA algorithm and its application of prime numbers:

RSA Algorithm

1) Key Generation

a) Choose two large distinct prime numbers, p and q.

b) Calculate their product, n = p * q. The number n is used as the modulus for both the public and private keys.

c) Compute the Euler's totient function of n: φ(n) = (p - 1) * (q - 1).

2) Public Key

a) Choose an integer e (1 < e < φ(n)) such that e and φ(n) are co-prime (i.e., gcd(e, φ(n)) = 1).

b) The public key consists of the pair (e, n).

3) Private Key

a) Calculate the modular multiplicative inverse of e modulo φ(n). Let's call this d.

b) The private key consists of the pair (d, n).

4) Encryption

To encrypt a message M, convert it into an integer representation (usually using a reversible encoding scheme), and then apply the encryption formula:

$C \equiv M^e \pmod n$

C is the ciphertext.

5) Decryption

To decrypt the ciphertext C, apply the decryption formula:

$M \equiv C^d \pmod n$

M is the original message.

The security of RSA heavily relies on the difficulty of factoring large composite numbers into their prime factors. The keys' strength is directly related to the size and security of the chosen prime numbers (p and q). Larger prime numbers increase the difficulty of factorization and improve the security of RSA.

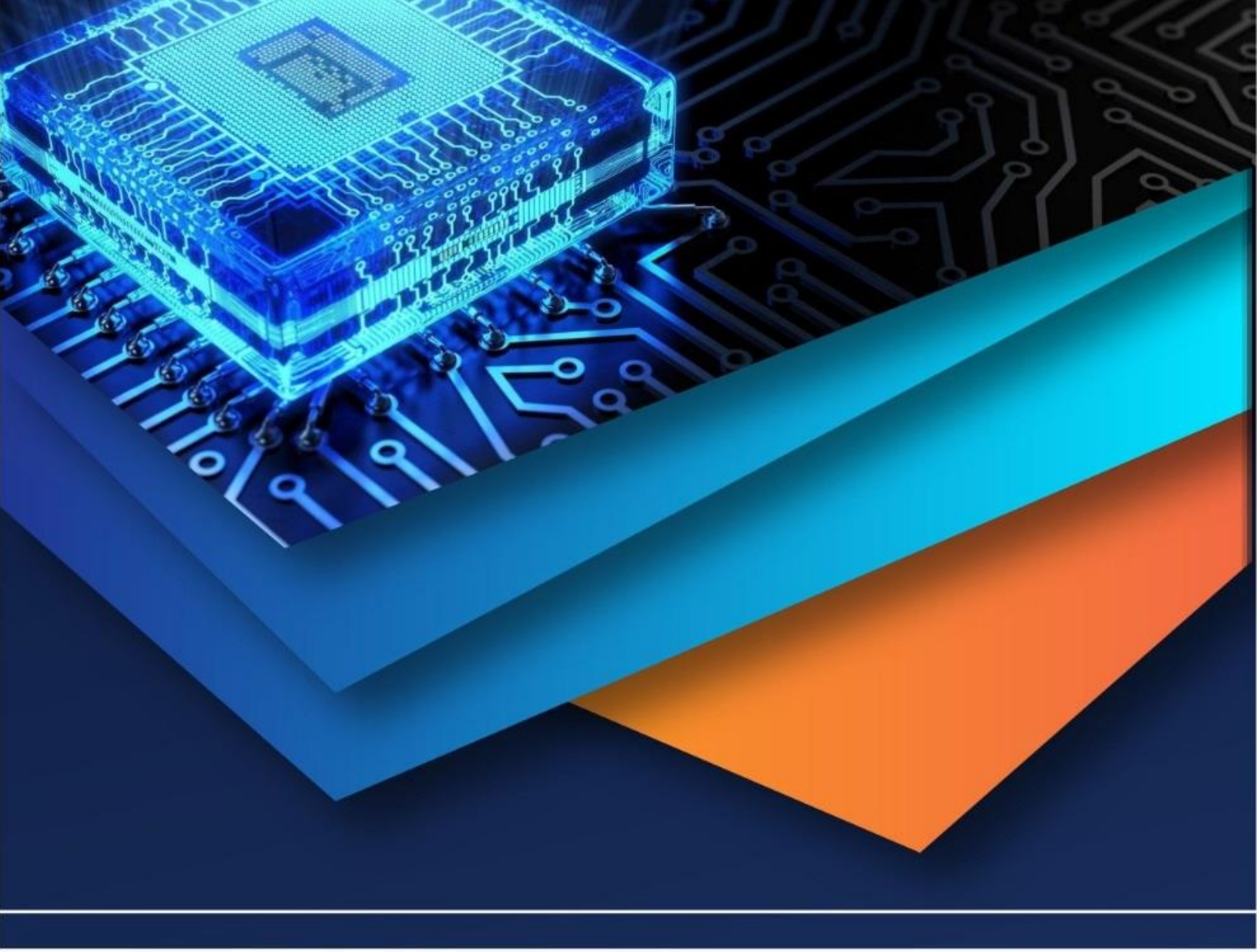
The public key (e, n) is designed for encryption and is distributed openly to anyone who wants to send encrypted messages to the owner of the private key. The private key (d, n) is kept secret and is used for decryption. The use of prime numbers in generating the keys ensures that the encryption and decryption processes are mathematically connected but computationally challenging to reverse without knowing the prime factors of n.

VII. CONCLUSIONS

This is a deep and detailed study about prime numbers. In this paper, there is a thorough introduction of different types of primes with the python codes embedded, squaring prime pattern, time analysis of generation of primes and comparison of different factorization methods. Moreover, important applications of prime numbers are also included.

REFERENCES

[1] Porras Ferreira, J.W. (2017) The Pattern of Prime Numbers. Applied Mathematics, 8, 180-192
[2] A.R.C.De Vas Gunasekara, A.A.C.A.Jayathilake and A.A.I.Perera (2015) Survey on Prime Numbers. Elixir Appl. Math. 88 (2015) 36296-36301
[3] .https://en.wikipedia.org/wiki/Fundamental_theorem_of_arithmetic
[4] https://youtu.be/ZMkIiFs35HQ
[5] https://serious-science.org/prime-numbers-6114
[6] J J O'Connor and E F Robertson, Prime Numbers, MacTutor. School of Mathematics and Statistics; University of St Andrews, Scotland
[7] https://en.wikipedia.org/wiki/Prime_number
[8] .https://en.wikipedia.org/wiki/List_of_prime_numbers
[9] https://en.wikipedia.org/wiki/Category:Classes_of_prime_numbers
[10] Masum Billal, Amir Hossein Parvardi (August 28, 2018), Topics in Number Theory: An Olympiad-Oriented Approach
[11] .https://www.researchgate.net/deref/http%3A%2F%2Fconnellybarnes.com%2Fdocuments%2Ffactoring.pdf
[12] Mathematics of Computation, Vol. 64, No. 209 (Jan., 1995), pp. 397-405, Published by: American Mathematical Society, Article Stable URL: http://www.jstor.org/stable/2153343.
[13] A Gentle Introduction To Number Theory And Cryptography [Notes For The Project Grad 2



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