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# A Review on Second Degree Homogeneous Diophantine Equation with Three Unknowns

$$x^2 + y^2 = 122z^2$$

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**Abstract:** The homogeneous ternary second degree equation given by  $x^2 + y^2 = 122z^2$  is analysed for its non-zero distinct integral points on that. Completely various patterns of the equation into consideration are obtained by using python.

**Keywords:** Ternary, quadratic, Integer solutions, Homogeneous, Diophantine, python.

## I. INTRODUCTION

It is acknowledge that the quadratic Diophantine equations with 3 unknowns (homogeneous or non-homogeneous) are made in selection [1,2,]. Significantly, one might refer [3-17] for homogeneous or non-homogeneous ternary second degree Diophantine equations that are analysed for getting their corresponding non-zero distinct integer solutions. During this communication, one more attention-grabbing homogeneous ternary quadratic Diophantine equation given by  $x^2 + y^2 = 122z^2$  is analysed for its non-zero distinct integer results through fully different strategies with simple python programs. One may gain different values for the input of their programs.

## II. STYLES OF ANALYSIS

The ternary second degree equation to be answered for its integer results is

$$x^2 + y^2 = 122z^2 \tag{1}$$

### A. Pattern I

Write 122 as

$$122 = (11+i)(11-i) \tag{2}$$

Assume

$$z = a^2 + b^2 \tag{3}$$

Thus we tend to get,

$$x = 11a^2 + 11b^2 + 2aby = a^2 + b^2 + 22ab$$

$$z = a^2 + b^2$$

We are going to see this by simple python code as follows:

```
import math
a=int(input("enter the value of a"))
b=int(input("enter the value of b"))
x=(11*a**2)-(11*b**2)-2*a*b;
y=(a**2)-(b**2)+22*a*b;
z=(a**2)+(b**2);
print("the value of x is",x);
print("the value of y is",y);
print("the value of z is",z);
```

output :  
 enter the value of a 5  
 5  
 enter the value of b 5  
 5  
 the value of x is -50  
 the value of y is 550  
 the value of z is 50

**B. Pattern 2**

Equation (1) can also be written as

$$x^2 - y^2 - 121z^2 - z^2$$

$$x^2 - 121z^2 - z^2 - y^2 \tag{4}$$

Applying the tactic of cross-multiplication to the on-top system of equations, note that

$$x - 11z^2 - 11z^2 - 2z^2$$

$$y - z^2 - z^2 - 22z^2 - z^2 - z^2$$

Applying python codes and considering alpha and beta as a and b we get ,

```
import math
a=int(input("enter the value of a"))
b=int(input("enter the value of b"))
x=(11*a**2)-(11*b**2)+2*a*b;
y=(-a**2)+(b**2)+22*a*b;
z=(a**2)+(b**2);
print("the value of x is",x);
print("the value of y is",y);
print("the value of z is",z);
```

Output :

enter the value of a 5  
 5  
 enter the value of b 5  
 5  
 the value of x is 50  
 the value of y is 550  
 the value of z is 50

**C. Pattern III**

One can also be written as

$$x^2 - y^2 - 122z^2 - 1$$

Write 1 as

$$1 = 3 - 4i - 3 + 4i - 25 \tag{5}$$

(6)

As our interest is on finding integer solutions replacing a by 5A & b by 5B , we get

$$\left. \begin{aligned} x &= 29A^2 - 29B^2 - 94AB \\ y &= 47A^2 - 47B^2 - 58AB \\ z &= 5A^2 - 5B^2 \end{aligned} \right\} \tag{7}$$

Considering A,B as a,b we are following results:

```
import math
a=int(input("enter the value of a"))
b=int(input("enter the value of b"))
x=(29*a**2)-(29*b**2)-94*a*b;
y=(47*a**2)-(47*b**2)+58*a*b;
z=(5*a**2)+(5*b**2);
print("the value of x is",x);
print("the value of y is",y);
print("the value of z is",z);
```

Output:

```
enter the value of a 5
5
enter the value of b 5
5
the value of x is -2350
the value of y is 1450
the value of z is 250
```

#### D. Pattern IV

Introduction of the direct metamorphoses

$$x=u+v, y=u-v, z=2w$$

in (1) leads to

$$u^2 \square v^2 \square 244w^2$$

Assume

$$w \square c^2 \square d^2$$

Thereby we are getting these following results

(10)

(11)

we are getting these following results :

$$x \square 22c^2 \square 22d^2 \square 4cd$$

$$y \square \square 2c^2 \square 2d^2 \square 44cd$$

$$z \square 2c^2 \square 2d^2$$

import math

```
c=int(input("enter the value of c"))
d=int(input("enter the value of d"))
x=(22*c**2)-(22*d**2)-4*c*d;
y=(-2*c**2)+(2*d**2)-44*c*d;
z=(2*c**2)+(2*d**2);
print("the value of x is",x);
print("the value of y is",y);
print("the value of z is",z);
```

Output :

enter the value of c 5

5

enter the value of d 5

5

the value of x is -100

the value of y is -1100

the value of z is 100

### III. CONCLUSION

In this paper, an bid has been created to get non-zero distinct integer results to the ternary quadratic Diophantine equation  $x^2 + y^2 + z^2 = 122$  representing homogeneous cone with python canons. As there are kinds of cones, the compendiums might rummage around for indispensable kinds of cones to get integer results for the corresponding cones with python codes.

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