



# IJRASET

International Journal For Research in  
Applied Science and Engineering Technology



---

# INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

---

**Volume:** 11    **Issue:** II    **Month of publication:** February 2023

**DOI:** <https://doi.org/10.22214/ijraset.2023.49130>

[www.ijraset.com](http://www.ijraset.com)

Call:  08813907089

E-mail ID: [ijraset@gmail.com](mailto:ijraset@gmail.com)

# An Integral Solution of Negative Pell Equation $x^2 = 5y^2 - 9^t$

S. Vidhya<sup>1</sup>, B. Amala<sup>2</sup>

<sup>1</sup>Assistant Professor, PG and Research Department of Mathematics, Cauvery College for Women (Autonomous), Affiliated to Bharathidasan University, Trichy – 18.

<sup>2</sup>PG Student, PG and Research Department of Mathematics, Cauvery College for Women (Autonomous), Affiliated to Bharathidasan University, Trichy – 18.

**Abstract:** We look for non-trivial integer solution to the equation  $x^2 = 5y^2 - 9^t, t \in N$  for the singular choices of particular by (i)  $t = 2k$  (ii)  $t = 2k+1, \forall k \in N$ . Additionally, recurrence relations on the solutions are obtained.

**Keywords:** Negative Pell equations, Pell equations, Diophantine equations, Integer solutions.

## I. INTRODUCTION

It is well known the Pell equation  $x^2 - Dy^2 = 1$  ( $D > 0$  and square free) has at all times positive integer solutions. When  $N \neq 1$ , the Pell equation  $x^2 - Dy^2 = -N$  possibly will not boast at all positive integer solutions. In favour of instance, the equations  $x^2 = 3y^2 - 1$  and  $x^2 = 7y^2 - 4$  comprise refusal integer solutions.

This manuscript concerns the negative Pell equation  $x^2 = 5y^2 - 9^t$ , where  $t > 0$  and infinitely numerous positive integer solutions are obtained for the choices of  $t$  known by (i)  $t = 2k$  (ii)  $t = 2k+1$ . Supplementary recurrence relationships on the solutions are consequent.

## II. PRELIMINARY

The Pell equation is a Diophantine equation of the form  $x^2 - dy^2 = 1$ . Given  $d$ , we would like to find all integer pairs  $(x, y)$  that satisfy the equation. Since any solution  $(x, y)$  yields multiple solutions  $(\pm x, \pm y)$ , we may restrict our attention to solutions where  $x$  and  $y$  nonnegative integer. We usually take  $d$  in the equation  $x^2 - dy^2 = 1$  to be a positive non square integer. Otherwise, there are only uninteresting solutions: if  $d < 0$ , then  $(x, y) = (\pm 1, 0)$  in the case  $d < -1$ , and  $(x, y) = (0, \pm 1)$  or  $(\pm 1, 0)$  in the case  $d = -1$ ; if  $d = 0$ , then  $x = \pm 1$  ( $y$  arbitrary); and if a nonzero square, then  $dy^2$  and  $x^2$  are consecutive squares, implying that  $(x, y) = (\pm 1, 0)$ . Notice that the Pell equation always has trivial solution  $(x, y) = (1, 0)$ . We now investigate an illustrate case of Pell's equation and its solution involving recurrence relations.

Let  $p$  be a prime. The negative Pell's equation  $x^2 - py^2 = -1$  is solvable if and only if  $p = 2$  (or)  $p \equiv 1 \pmod{4}$ .

## III. METHOD OF ANALYSIS

Consider the negative Pell equation  $x^2 = 5y^2 - 9^t, t \in N$

1) Choice 1:  $t = 2k, k > 0$

The Pell equation is

$$x^2 = 5y^2 - 9^{2k}, k > 0 \tag{1}$$

Let  $(x_0, y_0)$  be the initial solution of (1) specified by

$$x_0 = 18.9^k; y_0 = 9.9^k$$

Applying Brahma Gupta Lemma Connecting  $(x_0, y_0)$  and  $(\tilde{x}_n, \tilde{y}_n)$ , the progression of non-zero separate solution to (1) are obtained as

$$x_{n+1} = \frac{1}{2}[18.9^k fn + 9.9^k \sqrt{5}gn] \tag{2}$$

$$y_{n+1} = \frac{1}{2\sqrt{5}}[9.9^k gn + 18.9^k \sqrt{5}fn] \tag{3}$$

The recurrence relationship fulfilled by the solutions of (1) are specified by

$$\begin{aligned} x_{n+3} - 18x_{n+2} + x_{n+1} &= 0 \\ y_{n+3} - 18y_{n+2} + y_{n+1} &= 0. \end{aligned}$$

2) *Choice 2:  $t = 2k+1, k > 0$*

The Pell equation is

$$x^2 = 5y^2 - 9^{2k+1}, k > 0 \tag{4}$$

Let  $(x_0, y_0)$  be the initial solution of (4) given by

$$x_0 = 6.9^k; y_0 = 3.9^k$$

Applying Brahma Gupta Lemma Connecting  $(x_0, y_0)$  and  $(\tilde{x}_n, \tilde{y}_n)$ , the progression of non-zero dissimilar integer way out to (4) is obtained as

$$x_{n+1} = \frac{1}{2}[6.9^k fn + 3.9^k \sqrt{5}gn] \tag{5}$$

$$y_{n+1} = \frac{1}{2\sqrt{5}}[3.9^k gn + 6.9^k \sqrt{5}fn] \tag{6}$$

The recurrence relationships fulfilled employing the solutions of (4) are convinced utilizing

$$\begin{aligned} x_{n+3} - 18x_{n+2} + x_{n+1} &= 0 \\ y_{n+3} - 18y_{n+2} + y_{n+1} &= 0. \end{aligned}$$

#### IV. CONCLUSION

In this paper, we have presented infinitely many integer solutions for the represented by the negative Pell equation  $x^2 = 5y^2 - 9^t$ . As the binary quadratic Diophantine equations are rich in variety, one may search for the other choices of negative Pell equations and determine their integer solutions along with suitable properties.

#### REFERENCES

- [1] Carmichael R.D, History of Theory of numbers and Diophantine Analysis, Dover Publication, New York, 1959.
- [2] Nagell T, Introduction to Number Theory, Chelsea publishing company, New York, 1982.
- [3] Mordell L.J, Diophantine equations, Academic press, London, 1969.
- [4] Janaki G, Vidhya S (2016), On the integer solutions of the Pell equation  $x^2 - 79y^2 = 9^k$ , International Journal Of Scientific Research in Science, Engineering and Technology,2(2),1195-1197.
- [5] Janaki G, Vidhya S (2016), On the integer solutions of the Pell equation  $x^2 = 20y^2 - 4^t$ , International Journal Multidisciplinary Research and Development,3(5), 39-42.
- [6] Janaki G, Vidhya S (2016), On the negative Pell equation  $y^2 = 21x^2 - 3$ , International Journal Of Applied Research,2(11),462-466.



- [7] Vidhya S, Janaki G (2017), Observation on  $y^2 = 6x^2 + 1$ , International Journal Of Statistics and Applied Mathematics, 2(3), 04-05.
- [8] Vidhya S, Janaki G (2018) An integral solution of negative Pell's equation involving two-digit sphenic numbers, International Journal Of Computer Sciences and Engineering, 6, 444-445.
- [9] Vidhya S, Janaki G (2019), Observation on Remarkable Diophantine Equation, Compliance Engineering Journal, 10(12), 667-670.
- [10] Vidhya S, Janaki G (2019), Observation on  $y^2 = 11x^2 + 1$ , International Journal for science and Advance Research in Technology, 5(12), 232-233.
- [11] Janaki, G., & Vidhya, S. (2016). Rectangle with area as a special polygonal number. International Journal of Engineering Research, 4, 88-91.
- [12] Janaki, G., & Vidhya, S. Special pairs of rectangles and Sphenic number. International Journal for Research in Applied Science & Engineering Technology (IJRASET), 4, 376-378.



10.22214/IJRASET



45.98



IMPACT FACTOR:  
7.129



IMPACT FACTOR:  
7.429



# INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Call : 08813907089  (24\*7 Support on Whatsapp)