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# An Integral Solution of Negative Pell Equation $x^2 = 5y^2 - 9^t$

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**Abstract:** We look for non-trivial integer solution to the equation  $x^2 = 5y^2 - 9^t, t \in N$  for the singular choices of particular by (i)  $t = 2k$  (ii)  $t = 2k+1, \forall k \in N$ . Additionally, recurrence relations on the solutions are obtained.

**Keywords:** Negative Pell equations, Pell equations, Diophantine equations, Integer solutions.

## I. INTRODUCTION

It is well known the Pell equation  $x^2 - Dy^2 = 1$  ( $D > 0$  and square free) has at all times positive integer solutions. When  $N \neq 1$ , the Pell equation  $x^2 - Dy^2 = -N$  possibly will not boast at all positive integer solutions. In favour of instance, the equations  $x^2 = 3y^2 - 1$  and  $x^2 = 7y^2 - 4$  comprise refusal integer solutions.

This manuscript concerns the negative Pell equation  $x^2 = 5y^2 - 9^t$ , where  $t > 0$  and infinitely numerous positive integer solutions are obtained for the choices of  $t$  known by (i)  $t = 2k$  (ii)  $t = 2k+1$ . Supplementary recurrence relationships on the solutions are consequent.

## II. PRELIMINARY

The Pell equation is a Diophantine equation of the form  $x^2 - dy^2 = 1$ . Given  $d$ , we would like to find all integer pairs  $(x, y)$  that satisfy the equation. Since any solution  $(x, y)$  yields multiple solutions  $(\pm x, \pm y)$ , we may restrict our attention to solutions where  $x$  and  $y$  nonnegative integer. We usually take  $d$  in the equation  $x^2 - dy^2 = 1$  to be a positive non square integer. Otherwise, there are only uninteresting solutions: if  $d < 0$ , then  $(x, y) = (\pm 1, 0)$  in the case  $d < -1$ , and  $(x, y) = (0, \pm 1)$  or  $(\pm 1, 0)$  in the case  $d = -1$ ; if  $d = 0$ , then  $x = \pm 1$  ( $y$  arbitrary); and if a nonzero square, then  $dy^2$  and  $x^2$  are consecutive squares, implying that  $(x, y) = (\pm 1, 0)$ . Notice that the Pell equation always has trivial solution  $(x, y) = (1, 0)$ . We now investigate an illustrate case of Pell's equation and its solution involving recurrence relations.

Let  $p$  be a prime. The negative Pell's equation  $x^2 - py^2 = -1$  is solvable if and only if  $p = 2$  (or)  $p \equiv 1 \pmod{4}$ .

## III. METHOD OF ANALYSIS

Consider the negative Pell equation  $x^2 = 5y^2 - 9^t, t \in N$

1) Choice 1:  $t = 2k, k > 0$

The Pell equation is

$$x^2 = 5y^2 - 9^{2k}, k > 0 \tag{1}$$

Let  $(x_0, y_0)$  be the initial solution of (1) specified by

$$x_0 = 18.9^k; y_0 = 9.9^k$$

Applying Brahma Gupta Lemma Connecting  $(x_0, y_0)$  and  $(\tilde{x}_n, \tilde{y}_n)$ , the progression of non-zero separate solution to (1) are obtained as

$$x_{n+1} = \frac{1}{2}[18.9^k fn + 9.9^k \sqrt{5}gn] \tag{2}$$

$$y_{n+1} = \frac{1}{2\sqrt{5}}[9.9^k gn + 18.9^k \sqrt{5}fn] \tag{3}$$

The recurrence relationship fulfilled by the solutions of (1) are specified by

$$\begin{aligned} x_{n+3} - 18x_{n+2} + x_{n+1} &= 0 \\ y_{n+3} - 18y_{n+2} + y_{n+1} &= 0. \end{aligned}$$

2) *Choice 2:  $t = 2k+1, k > 0$*

The Pell equation is

$$x^2 = 5y^2 - 9^{2k+1}, k > 0 \tag{4}$$

Let  $(x_0, y_0)$  be the initial solution of (4) given by

$$x_0 = 6.9^k; y_0 = 3.9^k$$

Applying Brahma Gupta Lemma Connecting  $(x_0, y_0)$  and  $(\tilde{x}_n, \tilde{y}_n)$ , the progression of non-zero dissimilar integer way out to (4) is obtained as

$$x_{n+1} = \frac{1}{2}[6.9^k fn + 3.9^k \sqrt{5}gn] \tag{5}$$

$$y_{n+1} = \frac{1}{2\sqrt{5}}[3.9^k gn + 6.9^k \sqrt{5}fn] \tag{6}$$

The recurrence relationships fulfilled employing the solutions of (4) are convinced utilizing

$$\begin{aligned} x_{n+3} - 18x_{n+2} + x_{n+1} &= 0 \\ y_{n+3} - 18y_{n+2} + y_{n+1} &= 0. \end{aligned}$$

#### IV. CONCLUSION

In this paper, we have presented infinitely many integer solutions for the represented by the negative Pell equation  $x^2 = 5y^2 - 9^t$ . As the binary quadratic Diophantine equations are rich in variety, one may search for the other choices of negative Pell equations and determine their integer solutions along with suitable properties.

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