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Analysis of MOFTP using Modern Zero Suffix Method

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Abstract: In this paper, a new algorithm is introduced for dealing with the Multi-Objective Fuzzy Transportation Problem (MOFTP) where all parameters such as transportation cost, demand and supply are in triangular fuzzy numbers. The approach involves modern zero suffix method using Harmonic Mean. First the Multi-objective fuzzy transportation problem is converted into a crisp value by using a Robust's Ranking Method. Then the crisp value is solved by Modern Zero Suffix method. The effectiveness of the data is examined by a numerical example.
Keywords: Transportation, Triangular fuzzy numbers, Multi-objective Fuzzy Transportation, Harmonic Mean, Modern Zero Suffix method, Robust's Ranking.

I. INTRODUCTION

The transportation problem is a special type of linear programming problem, where the objective is to minimize the cost of distributing a product from a number of sources to a number of destinations. To obtain the initial basic feasible solution, there are several methods as North West Corner method (NWC), Least Cost method (LC) and Vogel's Approximation method (VAM). The transportation problem in this case is aimed as a single objective. But often in real life problem, there are multiple objectives needed to achieve while making the transportation operation. The transportation problem typically arises from a singular objective, whether its minimizing transportation time or cost, as developed by Hitchcock [10]. Diaz [3] developed an alternative algorithm to obtain all non-dominated solutions for MOTP and it depends on the satisfaction level regarding how closely a compromise solution aligns with the ideal solution. Diaz [4] expanded a procedure to obtain all non-dominated solution for MOTP. Ringuest et. al.,[14] developed two iterative algorithms to solve MOTP. Bit et.al., [2] examined a k-objective transportation problem incorporating fuzzy numbers and used α-cut to formulate the fuzzy transportation problem in linear programming terms. The first foraging ant system was developed using the notation in Maniezzo et.at., [12]. Dorigo et. al., [5] described ACO as a probabilistic method for locating optimum pathways. Shyu et.al., [15] presented a real-world problem by utilizing both existing ant traits and brand-new ones. Kaur et.al., [11] presented to obtain the best compromise solution of linear MOTP. To solve the minimum spanning tree problem and the TP, we use modified ACA which was introduced by Ekanayake et.al., [6]. Ambadas Deshmukh et.al., [1] presented a new ranking method to order any two fuzzy triangular number. Hebasayed et.al [9] presented a new summation method for solving MOTP. Goel et.al., [8] developed a new row maxima method to solve MOTP using C programme. K.P.O. Niluminda et.al., [13] developed a novel alternative algorithm that uses geometric mean and penalty technique to address MOTP. E.M.U.S.B. Ekanayake [7] expanded an ant colony technique to solve a MOTP.

II. PRELIMINARIES

A. Triangular Fuzzy Number

A fuzzy number $\tilde{A} = (a_1, a_2, a_3)$ is a Triangular fuzzy number. Where a_1, a_2, a_3 are real number. Its membership function is defined as follows

$$\mu_{\tilde{A}}(x) = \begin{cases} \left(\dfrac{x - a_1}{a_2 - a_1}\right), & for\ a_1 \le x \le a_2 \\ 1, & for\ x = a_2 \\ \left(\dfrac{a_3 - x}{a_3 - a_2}\right), & for\ a_2 \le x \le a_3 \end{cases}$$

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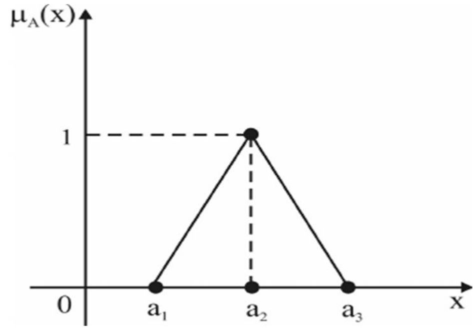

Figure 2.1.1: Triangular Fuzzy Number

B. *Operations on Triangular Fuzzy number:*

Addition, Subtraction and Multiplication of any two triangular fuzzy numbers are also triangular fuzzy number. Suppose triangular fuzzy numbers $\tilde{A}$ and $\tilde{B}$ are defined as,

$$\tilde{A} = (a_1, a_2, a_3) \text{ and } \tilde{B} = (b_1, b_2, b_3) \text{ then}$$

1) Addition $\tilde{A} (+) \tilde{B} = (a_1, a_2, a_3) + (b_1, b_2, b_3)$
   $= (a_1 + b_1, a_2 + b_2, a_3 + b_3)$
2) Subtraction $\tilde{A} (-) \tilde{B} = (a_1, a_2, a_3) - (b_1, b_2, b_3)$
   $= (a_1 - b_3, a_2 - b_2, a_3 - b_1)$
3) Multiplication: $\tilde{A} \times \tilde{B}$= (min $(a_1b_1, a_1b_3, a_3b_1, a_3b_3)$, a_2b_2 ,max $(a_1b_1, a_1b_3, a_3b_1, a_3b_3)$)
4) Symmetric image: $-\tilde{A} = (-a_3, -a_2, -a_1)$

C. *Robust's Ranking Method:*

Robust's ranking method which satisfy compensation, linearity and additive properties and provides results which are consistent with human intuition. If $\tilde{A}$ is a fuzzy number then the Robust's ranking is defined by

$$R(\tilde{A}) = \frac{1}{2}\int_0^1 (a_\alpha{}^L, a_\alpha{}^U)\, d\alpha$$

Where $(a_\alpha{}^L, a_\alpha{}^U)$ is the α level cut of fuzzy number $\tilde{A}$,

$(a_\alpha{}^L, a_\alpha{}^U) = \{ (a_2 - a_1)\alpha + a_1,\ a_3 - (a_3 - a_2)\alpha\}$

$R(\tilde{A}) = \frac{1}{2}\int_0^1 \{ (a_2 - a_1)\alpha + a_1,\ a_3 - (a_3 - a_2)\alpha\}\, d\alpha$

D. *Harmonic Mean:*

The harmonic mean is a type of average calculated by dividing the number of observations by the reciprocal of each number in the series. The formula for the harmonic mean of n numbers $(x_1, x_2, x_3, \dots\dots\dots, x_n)$ is:

$$HM = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \dots\dots\dots + \frac{1}{x_n}}$$

III. MATHEMATICAL FORMULATION OF FUZZY TRANSPORTATION PROBLEM

The mathematical models of fuzzy multi-objective transportation problem is to minimize the total transportation cost, time, distance from m sources to n destinations is as follows

$$Minimize\ \bar{z} = \sum_{i=1}^{n}\sum_{j=1}^{n} \bar{c}_{ij}{}^1\ \bar{x}_{ij}$$

$$Minimize\ \bar{z} = \sum_{i=1}^{n}\sum_{j=1}^{n} \bar{c}_{ij}{}^k\ \bar{x}_{ij}$$

Subject to $\sum_{j=1}^{n} \bar{x}_{ij} = \bar{a}_{ij}$ i = 1,2, 3………., m

$\sum_{i=1}^{m} \bar{x}_{ij} = \bar{b}_{ij}$ j = 1,2, 3………., n

$\sum_{i=1}^{m} \bar{a}_{ij} = \sum_{j=1}^{n} \bar{b}_{ij}$ i = 1,2, 3………., m; j = 1,2,3……….,n

and $x_{ij} \geq 0$ for all i and j

where $\bar{c}_{ij}$ is the fuzzy unit transportation cost, time, distance from i^{th} source to j^{th} destination.

Destination / Source	D_1	D_2		D_n	Supply
S_1	$(c_{11}{}^1, c_{11}{}^2 \dots \dots, c_{11}{}^k)$	$(c_{12}{}^1, c_{12}{}^2 \dots \dots, c_{12}{}^k)$	..	$(c_{1n}{}^1, c_{1n}{}^2 \dots \dots, c_{1n}{}^k)$	a_1
S_2	$(c_{21}{}^1, c_{21}{}^2 \dots \dots, c_{21}{}^k)$	$(c_{22}{}^1, c_{22}{}^2 \dots \dots, c_{22}{}^k)$	..	$(c_{2n}{}^1, c_{2n}{}^2 \dots \dots, c_{2n}{}^k)$	a_2
	. . . .	. . . .	. . . .		. . . .
S_m	$(c_{m1}{}^1, c_{m1}{}^2 \dots \dots, c_{m1}{}^k)$	$(c_{m2}{}^1, c_{m2}{}^2 \dots \dots, c_{m2}{}^k)$	..	$(c_{mn}{}^1, c_{mn}{}^2 \dots \dots, c_{mn}{}^k)$	a_m
Demand	b_1	b_2	..	b_n	

Table 3.1 Multi-objective transportation

IV. PROPOSED ALGORITHM

To solve MOTP, it is obvious to find the right efficient solution which should be very close to an optimal solution (or ideal solution). It works well for both balanced and unbalanced MOTP and providing a reliable solution. Steps of novel method are as following:

1) *Step 1:* Test whether the given multi-objective fuzzy transportation problem is a balanced one or not. If it is a balanced one, then go to step 3. If it is an unbalanced one, then go to step 2.
2) *Step 2:* Introduce dummy rows (or columns) with zero fuzzy costs to form a balanced one.
3) *Step 3:* Convert the multi-objective fuzzy transportation problem into a crisp problem using Robust's ranking method.
4) *Step 4:* Then the harmonic mean value is using the below formula,

$$HM = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \dots \dots \dots + \frac{1}{x_n}}$$

5) *Step 5:* Subtract each row entries of the transportation table from the minimum row. Then, subtract each column entries of the transportation table on a certain minimum column. From concentrated matrix, every row and every column has no less than one zero.
6) *Step 6:* Select one zero and compute the number of zeros in the corresponding row and column expect the selected zero, and mark the sum of the number of zeros in suffix.
7) *Step 7***:** Select every zero and mark the suffix as the way of step 6. Select the lowest suffix and allocate the conforming cell. Each allocation is in rising order of suffices.
8) *Step 8:* Sometime suffix values are identical; select the minimum cost cell of the conforming suffix values.
9) *Step 9:* Repeat the procedure for the resulting reduced transportation table until all the rim requirements are satisfied.

V. NUMERICAL EXAMPLE

Consider the following multi-objective fuzzy transportation problem.

A company has three sources S_1, S_2, S_3 and three destinations D_1, D_2, D_3 the multi-objective fuzzy transportation cost for unit quantity of the product form i^{th} source to j^{th}destination is C_{ij}

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Where, $C_{ij}=\begin{pmatrix} 4,5,6 & 6,7,8 & 3,4,5 \\ 7,8,9 & 5,6,7 & 6,7,8 \\ 6,7,8 & 1,2,3 & 8,9,10 \\ \\ 3,4,5 & 2,3,4 & 7,8,9 \\ 2,3,4 & 8,9,10 & 4,5,6 \\ 7,8,9 & 3,4,5 & 2,3,4 \\ \\ 5,6,7 & 4,5,6 & 1,2,3 \\ 3,4,5 & 1,2,3 & 2,3,4 \\ 8,9,10 & 5,6,7 & 6,7,8 \end{pmatrix}$

Cost value, supplies and demands are triangular fuzzy number. And fuzzy supply are (4,5,6), (3,4,5), (7,8,9) respectively and fuzzy demand are (1,2,3), (8,9,10), (5,6,7) respectively.

Solution:

Step 1:

Construct the Multi-objective fuzzy transportation table for the given multi-objective fuzzy transportation problem and then, convert it into a balanced one, if it is not.

Destination / Source	D_1	D_2	D_3	Supply
S_1	4,5,6 7,8,9 6,7,8	6,7,8 5,6,7 1,2,3	3,4,5 6,7,8 8,9,10	4,5,6
S_2	3,4,5 2,3,4 7,8,9	2,3,4 8,9,10 3,4,5	7,8,9 4,5,6 2,3,4	3,4,5
S_3	5,6,7 3,4,5 8,9,10	4,5,6 1,2,3 5,6,7	1,2,3 2,3,4 6,7,8	7,8,9
Demand	1,2,3	8,9,10	5,6,7	17

Table 5.1 Multi-objective Fuzzy Transportation

Step 3

Using Robust's ranking method the multi-objective transportation problem is converted into a crisp transportation problem as

$$R(a) = \frac{1}{2}\int_0^1 ({a_\alpha}^l, {a_\alpha}^u)\,d\alpha$$

The $\alpha - cut$ of the fuzzy number R(4,5,6) is $(\alpha + 4, 6 - \alpha)$

$$R(a) = \frac{1}{2}\int_0^1 (\alpha + 4, 6 - \alpha)\,d\alpha$$

$$= \frac{1}{2}(10)$$

R(4,5,6) = 5

Similarly,

R(7,8,9)=8 , R(6,7,8)=7 , R(6,7,8)=7 , R(5,6,7)=6 , R(1,2,3)=2, R(3,4,5)=4, R(6,7,8)=7, R(8,9,10)=9 , R(3,4,5)=4 , R(2,3,4)=3 , R(7,8,9)=8, R(2,3,4)=3 , R(8,9,10)=9 , R(3,4,5)=4, R(7,8,9)=8 , R(4,5,6)=5 , R(2,3,4)=3 , R(5,6,7)=6 , R(3,4,5)=4 , R(8,9,10)=9 , R(4,5,6)=5 , R(1,2,3)=2 , R(5,6,7)=6 , R(1,2,3)=2 , R(2,3,4)=3 , R(6,7,8)=7.

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Therefore, the crisp transportation table is

Source \ Destination	D_1	D_2	D_3	Supply
S_1	5 8 7	7 6 2	4 7 9	5
S_2	4 3 8	3 9 4	8 5 3	4
S_3	6 4 9	5 2 6	2 3 7	8
Demand	2	9	6	17

Table 5.2: Crisp Transportation Table

Step 4-8 : (Harmonic mean of objectives (Cost, Time, Distance))

Using harmonic mean (HM) the above table value is converted into a single value.

$$HM = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \dots\dots\dots + \frac{1}{x_n}}$$

$$HM(5,8,7) = \frac{3}{\frac{1}{5} + \frac{1}{8} + \frac{1}{7}} = \frac{3}{0.2 + 0.125 + 0.143} = 6.41$$

Similarly,

HM (7,6,2) = 3.70, HM (4,7,9) = 5.94, HM (4,3,8) = 4.23, HM (3,9,4) = 4.31, HM (8,5,3) = 4.55, HM (6,4,9) = 5.67, HM (5,2,6) = 3.46, HM (2,3,7) = 3.07.

Source \ Destination	D_1	D_2	D_3	Supply
S_1	6.41	$\boxed{5}$ 3.70	5.94	~~5~~
S_2	$\boxed{2}$ 4.23	$\boxed{2}$ 4.31	4.55	~~4~~ 2
S_3	5.67	$\boxed{2}$ 3.46	$\boxed{6}$ 3.07	~~8~~ ~~6~~ 2
Demand	~~2~~	~~9~~ ~~4~~ 2	~~6~~	17

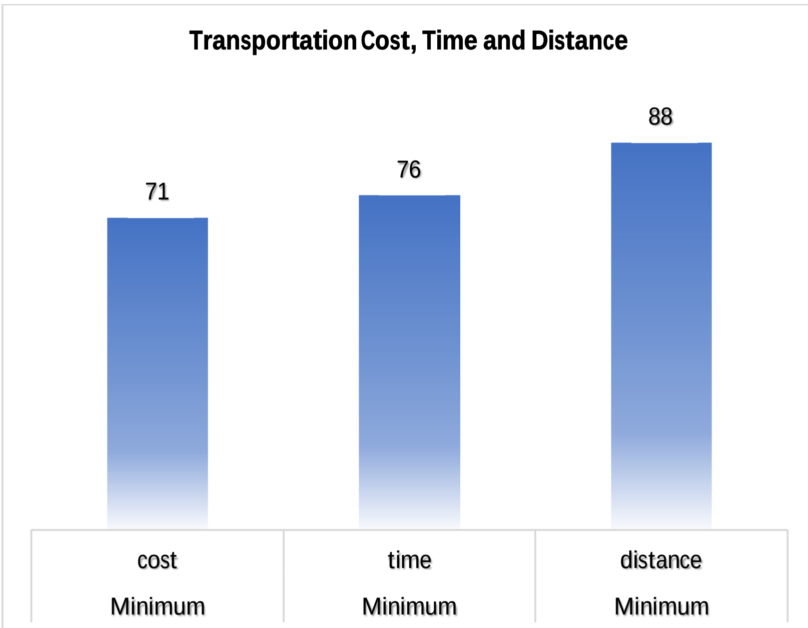
Table 5.3 Final Transportation Table

Therefore,
Minimum transportation cost, time, distance = 5[7,6,2] + 2[4,3,8] + 2[3,9,4] + 2[5,2,6] + 6[2,3,7]
= [71,76,88]

Proposed method: Transportation Cost, Time and Distance

Method	Minimum cost	Minimum time	Minimum distance
Proposed method	71	76	88

Table 5.4 Transportation Cost, Time and Distance



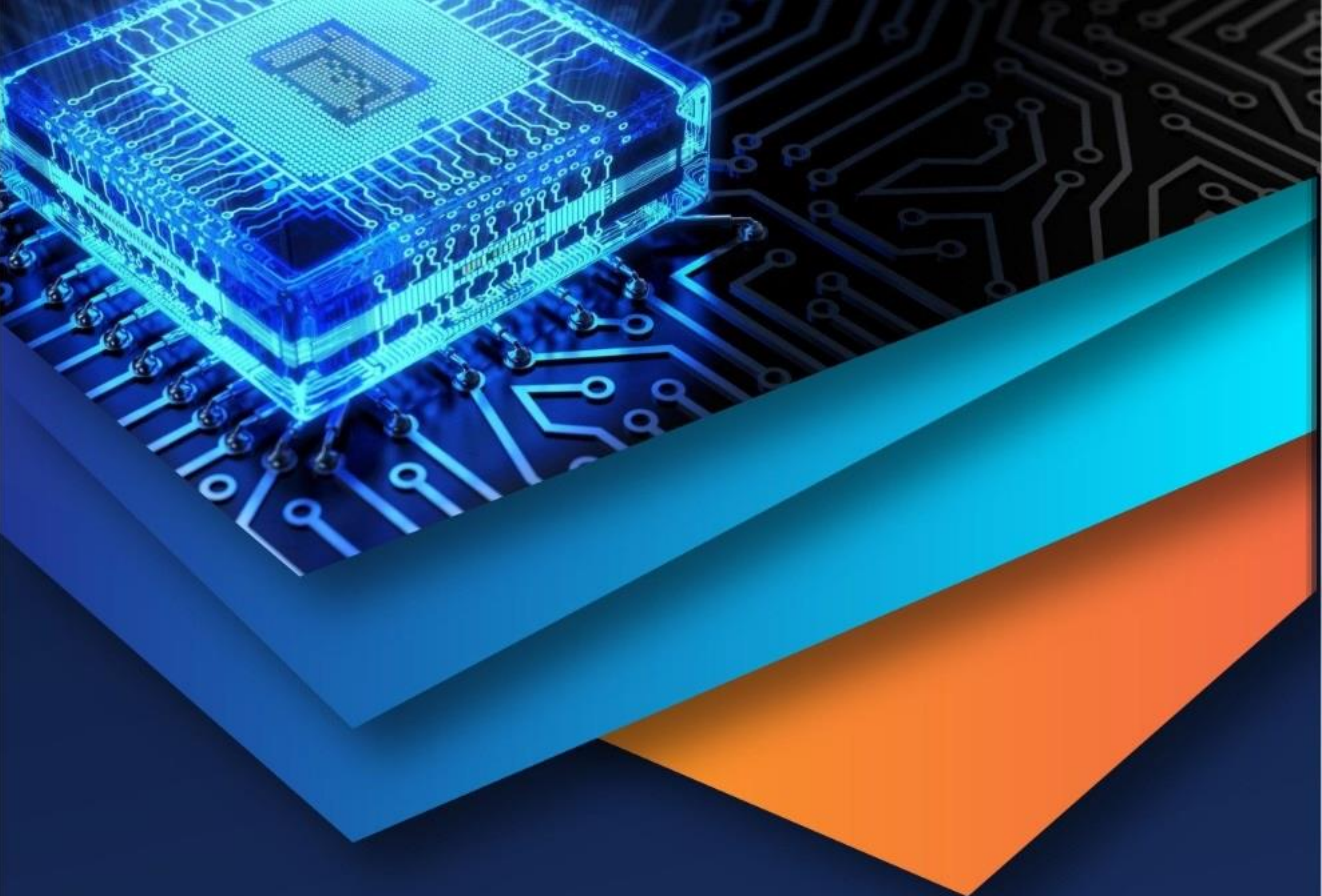
VI. CONCLUSION

In this paper we have analysed a transportation problem using an alternative method for applying fuzzy MOTP, namely modern zero suffix method, which provides the best solution of the multi-objective transportation system as often as possible. Multi-objective Transportation Problems are those where more than one objective needs to be optimized. Several methods have been put forward in the literature to solve MOTP. Instead of utilizing traditional methods, the harmonic mean with the modern zero suffix method is applied in this study to solve a MOTP. This method is also simple and easily understandable for effective solutions.

REFERENCES

[1] Ambadas Deshmukh, Dr. Arun Jadhav, Ashok D., Mhaske, K.L.Bondar (2021). Fuzzy transportation problem by using triangular fuzzy numbers with ranking using area of trapezium, rectangle and centroid at different level of α-cut.Turkish Journal of Computer and Mathematics Education, vol.12 No.12, 3941-3951.
[2] Bit,A.K.,Biswal,M.P., and Alam, S.S.(1992). Fuzzy programming approach to multicriteria decision making transportation problem. Fuzzy Sets Syst, 50, 135-141.
[3] Diaz, J.A.(1978). Solving multiobjective transportation problems. Ekon Math Obzor, 14,267-274.
[4] Diaz, J.A.(1979). Finding a complete description of all efficient solutions to a multiobjective transportation problem. Ekon Math Obzor, 15,62-73.
[5] Dorigo,M., Caro, G. D., & Gambardella, L.M. (1999). Ant algorithms for discrete optimization. Artificial Life, 5(2), 137-172.
[6] Ekanayake, E.M.U.S.B., Daundasekara, W.B., & Perera, S.P.C.(2020a). An approach for solving minimum spanning tree problem and transportation problem using modified ant colony algorithm. American Academic Research,3(9),28-45.
[7] Ekanayake, E.M.U.S.B. (2023). To use an ant colony technique to solve a crispy type bi- and tri-objective transportation problem. Journal of Computational and Cognitive Engineering,Vol.2(4), 294-303.
[8] Goel, P. (2021). New row maxima method to solve multi-objective transportation problem using C-programme and fuzzy technique. International Journals of Engineering, Science & Mathematics, 10(4), 27-36.

[9] Heba Sayed, Prof.Hegazy Zaher, Samia Mohammed (2021). Solving Multi-objective transportation problem using summation Penalty method. Journal of University of Shanghai for Science and Technology, vol.23, 464-469
[10] Hitchcock. (1941). The distribution of a product from several sources to numerous localities, J. Math. Phy., Vol.20, pp. 224-230.
[11] Kaur, L., Rakshit, M., & Singh, S. (2018). A new approach to solve multi-objective transportation problem. Applications and Applied Mathematics: An International Journal, 13(1), 150-159.
[12] Maniezzo, V., Dorigo, M., & Colorni, A. (1996). The ant system: Optimization by a colony of cooperating agents. IEEE Transactions on Systems, Man, and Cybernetics-Part B, 26(1), 29-41.
[13] Niluminda, K.P.O., Ekanayake, E.M.U.S.B. (2023). The Multi-objective transportation problem solve with geometric mean and penalty method. Indonesian Journal of Innovation and Applied Sciences (IJIAS), 3(1), 74-85
[14] Ringuest, J. L., & Rinks, D.B. (1987). Interactive solutions for the linear multi-objective transportation problem. European Journal Operations Research, 32(1), 96-106.
[15] Shyu, S. J., Lin, B. M. T., & Hsiao, T. S. (2006). Ant colony optimization for the cell assignment problem, in pcs networks. Computers & Operations Research, 33(6), 1713-1740.



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