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Application of First Order differential Equations in R L Circuits

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Abstract: The application of first order differential equation in Growth and Decay problems will study the method of variable separable and the model of Malthus (Malthusian population model), where we use the methods to find the solution to the population problems which are of use in mathematics, physics and biology especially in dealing with problems involving growth and decay problems that requires the use of Malthus model. Differential equations are fundamental importance in engineering mathematics because any physical laws and relations appear mathematically in the form of such equations. In this paper we discussed about first order linear homogeneous equations, first order linear non homogeneous equations and the application of first order differential equation in electrical circuits.
Keywords: Differential Equations, Linear Homogeneous Equations, Linear non Homogeneous Equations, Electrical circuits, RL circuits.

I. INTRODUCTION

The application of first order differential equation in Growth and Decay problems will study the method of variable separable and the model of Malthus (Malthusian population model), where we use the methods to find the solution to the population problems which are of use in mathematics, physics and biology especially in dealing with problems involving growth and decay problems that requires the use of Malthus model. Applications of differential equations also abound in mathematics itself, especially in geometry and harmonic analysis and modeling. Differential equations occur in economics and systems science and other fields of mathematical science. Many physical and engineering problems when formulated in the mathematical language give rise to partial differential equations. Besides these, partial differential equations also play an important role in the theory of elasticity, hydraulics etc. Since the general solution of a partial differential equation in a region R contains arbitrary constants or arbitrary functions, the unique solution of a partial differential equation corresponding to a physical problem will satisfy certain other conditions at the boundary of the region R. These are known as boundary conditions. When these conditions are specified for the time t = 0, they are known as initial conditions. Differential equations are fundamental importance in engineering mathematics because any physical laws and relations appear mathematically in the form of such equations. In this paper we discussed about first order linear homogeneous equations, first order linear non homogeneous equations and the application of first order differential equation in electrical circuits.

II. REVIEW OF LITERATURE

Berezman *et al.* (1986) [3] had published an article "Calculation of the Eigen values of Mathieu's equation with a complex parameter". In this article he gave an effective numerical algorithm suggested for calculating the Eigen values of Mathieu's differential equation when the parameter of the equation takes complex values from a fairly wide range of variation. The algorithm is based on using the theory of continued fractions. The efficiency of the algorithm is verified by a series of numerical experiments and by comparing them with known numerical data. McCoy and Boersma (1986) [22] stated that the axial growth of plant tissue obeys the physical laws of energetic during deformation of a continuous medium. The concept of biological energy conservation was employed to formulate a mathematical model of axial plant growth. The model was derived from a statement of the exchange of the thermodynamic potential energy with the kinetic energy of deformation. This derivation does not invoke a force balance analogy with simple mechanical systems and has no turgor dependence. The derivatives with respect to tissue strain of the turgor, osmotic potential and extent of the biosynthetic reactions, therefore, all participate in the performance of the work of growth. The model formulation is unique to plant growth studies since it combines principles of mechanical energy conservation during deformation with a chemical thermodynamic description of the potential energy.

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The concept that the change of the thermodynamic potential energy performs the work of deformation is more general and applicable to biological systems than the currently employed force balance approach.

J.C. Butcher (1992) [6] proposed the role of orthogonal polynomials in numerical ordinary differential equations. Orthogonal polynomials have many applications to numerical ordinary differential equations. Some of these, especially those connected with the quadrature formulae on which many differential equation methods are based, are to be expected. On the other hand, there are many surprising applications, quite unlike traditional uses of orthogonal polynomials. This paper surveys many of these applications, especially those related to accuracy and equation (PDE). In this way, a much improved reconstruction of partial derivatives was obtained, resulting in significantly improved accuracy in many cases. The implementation algorithm was described, and was validated via three convection diffusion- reaction problems, for steady and transient situations. A Crank-Nicolson implicit time stepping technique was used for the time-dependent problems. A form of 'analytical upwinding' was implicitly implemented by the use of the partial differential operator of the governing equation in the interpolation function, which included the desired information about the convective velocity field.

Abbas et al. (2010) [26] described "Darboux problem for impulsive partial hyperbolic differential equations of fractional order with variable times and infinite delay". He dealt with the existence of solutions to impulsive partial functional differential equations with impulses at variable times and infinite delay, involving the Caputo fractional derivative. This works was considered by using the non- linear alternative of Leray-Schauder type. Wen *et al.* (2010) [32] published a paper "Dissipativity and asymptotic stability of non-linear neutral delay integro- differential equations". It was concerned with the dissipativity and asymptotic stability of the theoretical solutions of a class of non-linear neutral delay integro- differential equations (NDIDEs). They first gave a generalization of the Halanay inequality which played an important role in the study of dissipativity and stability of differential equations. Then, they applied the generalization of the Halanay inequality to NDIDEs and the dissipativity and the asymptotic stability results of the theoretical solution of NDIDEs. From a numerical point of view, it was important to study the potential of numerical methods in preserving the qualitative behavior of the analytical solutions. It provided the theoretical foundation for analyzing the dissipativity and the asymptotic stability of the numerical methods when they were applied to these systems was validated.

Han et al. (2013) [17] published an article "A partial differential equation formulation of Vickrey's bottle-neck model: methodology and theoretical analysis". The continuous-time Vickrey model can be described by an ordinary differential equation (ODE) with a right- hand side which is discontinuous in the unknown variable. Such a formulation induced difficulties with both theoretical analysis and numerical computation. It was widely suspected that an explicit solution to this ODE does not

In Mathematics, a partial differential equation (PDE) is a differential equation that contains unknown multivariable functions and their partial derivatives. Partial differential equations (PDEs) are used to formulate problem involving functions of several variables, and are either solved by hand, or used to create a relevant computer model. PDEs can be used to describe a wide variety of phenomena such as sound, heat, electrostatics, electrodynamics, fluid flow, or elasticity. These seemingly distinct physical phenomena can be formalized identically in terms of PDEs, which shows that they are governed by the same underlying dynamic. Just as ordinary differential equations often model one-dimensional dynamical systems, partial differential equations often model multidimensional systems. PDEs find their generalization in stochastic partial differential equations

A. Linear Differential Equations

A linear differential equation is any differential equation that can be written in the following form.

$$a_n(t)y^{(n)}(t) + a_{n-1}(t)y^{(n-1)}(t) + \cdots + a_0(t)y(t) = g(t) \quad (3)$$

The important thing to note about linear differential equations is that there are no products of the function, y(t), and its derivatives and neither the function or its derivatives occur to any power other than the first power. The coefficients $a_0(t), \ldots\ldots\ldots, a_n(t)$ and g(t) can be zero or non-zero functions, constant or non-constant functions, linear or non-linear functions. Only the function, y(t), and its derivatives are used in determining if a differential equation is linear.

Non-Linear Differential Equations:

If a differential equation cannot be written in the form, (3) then it are called a non-linear differential equation.

A linear ordinary differential equation of order *n* is said to be homogeneous if it is of the form and there is no term that contains a function of *x* alone.

Solution Method of First Order ODEs -------- (1)

Solution of Linear (Homogeneous Equation)Function v(x) =the velocity of fluid flowing a straight channel with varying cross-section

typical form of the equation:

$$\frac{du(x)}{dx} + p(x)u(x) = 0$$

The solution $u(x)$) in Equation (1) is

$$u(x) = \frac{K}{F(x)} \quad \text{-------- (2)}$$

Where K = constant to be determined by given condition and the function $F(x)$ has the form:

$$F(x) = e^{\int p(x)\,dx} \quad \text{-------- (3)}$$

In which the function $p(x)$ is given in the differential equation in Equation

B. Solution of Linear (Non-homogeneous Equations)

Typical form of the differential equation:

$$\frac{du(x)}{dx} + p(x)u(x) = g(x) \quad \text{-------- (4)}$$

The appearance of function $g(x)$ in Equation (4) makes the DE Non-homogeneous.

The solution of ODE in Equation (4) is similar by a little more complex than that for the homogeneous equation in (1):

$$u(x) = \frac{K}{F(x)}\int F(x)g(x)\,dx + \frac{K}{F(x)} \quad \text{---- (5)}$$

Where function $F(x)$ can be obtained from Equation (3) as: $F(x) = e^{\int p(x)\,dx}$

Example

Solve the following differential equation $\frac{du(x)}{dx} - (\sin x)u(x) = 0$ (a), with condition $u(0) = 2$

Solution

By comparing terms in Equation (a) and (4), we have: $p(x) = -\sin x$ & $g(x) = 0$

Thus by using Equation (5), we have the solution $u(x) = \frac{K}{F(x)}$

Where the function F(x) is: $F(x) = e^{\int p(x)\,dx} = e^{\cos x}$ leading to the solution

$u(x) = Ke^{-\cos x}$

Since the given condition is u (0) = 2, we have:

$$2 = Ke^{-\cos(0)} = K\left(e^{-1}\right) = \frac{K}{e} = \frac{K}{2.7183}$$

(Or) K = 5.4366.

Hence the solution of Equation (a) is

$$u(x) = 5.4366\, e^{-\cos x}$$

III. APPLICATION OF FIRST ORDER DIFFERENTIAL EQUATIONS ARISE IN THE MODELING OF ELECTRICAL CIRCUITS (RL CIRCUITS)

A. Electrical Circuits Transformed Form of ElementsResistor

If a current $i(t)$ is flowing through a resistor R ohms, then the voltage $v_R(t)$ across the resistor is given by $v_R(t) = Ri(t)$

Inductor

For an inductance of L Hentry, the voltage – current relationship is given by

$$v_L(t) = L\frac{d}{dt}i_L(t)ds$$

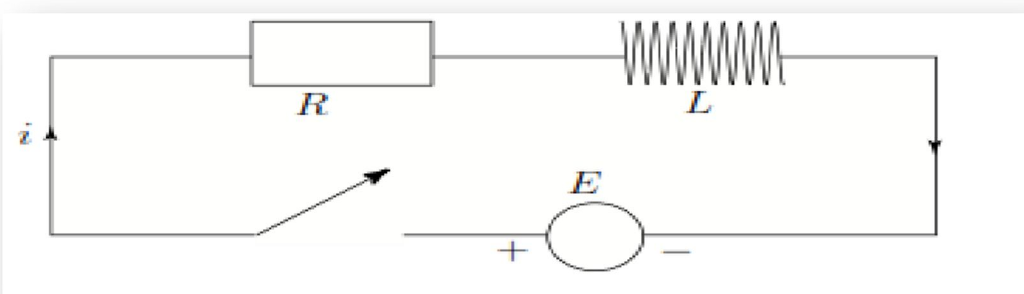
Capacitor

For a capacitance of C farad, the voltage current relationship is given by

$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i(t)dt$$

Example

Finding the optimal current of the below electrical circuits (RL Circuits) in which the initial condition is $i = 0$ at $t = 0$



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B. Solution

By Kirchhoff voltage law (KVL) method, we get

The differential equation for the RL circuit of the figure above was shown to be $L\frac{di}{dt} + Ri = E$ in which the initial condition is $i = 0$ at $t = 0$

Write this equation in standard form $\frac{du(x)}{dx} + p(x)u(x) = g(x)$ and obtain the integrating factor.

Divide the differential equation through by L to obtain

$$\frac{di}{dt} + \frac{R}{L} i = \frac{E}{L}$$

This is now in standard form

The integrating factor is $e^{\int \frac{R}{L} dt} = e^{\frac{Rt}{L}}$

Multiplying the equation in standard form by the integrating factor gives

$$e^{\frac{Rt}{L}} \frac{di}{dt} + e^{\frac{Rt}{L}} \frac{R}{L} i = e^{\frac{Rt}{L}} \frac{E}{L}$$

or, rearranging $\frac{d}{dt}\left(e^{\frac{Rt}{L}} i \right) = e^{\frac{Rt}{L}} \frac{E}{L}$

Now integrate both sides and apply the initial condition to obtain the solution

$$\int d\left(e^{\frac{Rt}{L}} i \right) = \int e^{\frac{Rt}{L}} \frac{E}{L} dt$$

$$e^{\frac{Rt}{L}} i = \frac{E}{L} e^{\frac{Rt}{L}} \frac{L}{R} + C$$

$$e^{\frac{Rt}{L}} i = \frac{E}{R} e^{\frac{Rt}{L}} + C$$

$$i = \frac{E}{R} + Ce^{\frac{Rt}{L}}$$

Applying the initial condition $i = 0$ at $t = 0$ gives

$$0 = \frac{E}{R} + C$$

$$C = -\frac{E}{R}$$

Now

$$i = \frac{E}{R} + Ce^{-\frac{Rt}{L}}$$

$$i = \frac{E}{R} - \frac{E}{R} e^{-\frac{Rt}{L}}$$

$$i = \frac{E}{R}\left(1 - e^{-\frac{Rt}{L}}\right)$$

Note that as

$$t \to \infty, \qquad i \to \frac{E}{R}$$

IV. CONCLUSION

Finding the current of the RL circuits is $i = \frac{E}{R}\left(1 - e^{-\frac{Rt}{L}}\right)$ by the method of solution of first order

It has been observed that differential equations can describe any phenomena and the given conditions are completely solvable to find various resultsordinary differential equation. This same procedure is often utilized in several types of electrical circuits and signals and systems. This application is useful for solving several different types of networking circuits and Fluid Mechanics Analysis. Fundamentally, it consists of finding optimal solution of first order ordinary linear homogeneous equations and first order ordinary linear non homogeneous equations.

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