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Binary Cyclic Codes in Circular Cliques

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Abstract: In this paper, we study the relationship between circular cliques and binary cyclic codes. We introduced the concept of binary cyclic codes in circular cliques. Every circular clique $K_{k/d}$ clearly admits a (k,d)- vertices. We observed that a circular clique $K_{k/d}$ with gcd(k, d) = 1 if $d \le |i-j| \le k-d$ are Prime circular cliques.
Keywords: Clique number, Circular clique, adjacency matrix, cyclic codes, generator polynomial, generator matrix.

I. INTRODUCTION

Circular Cliques form a natural superclass of perfect graphs, introduced by Zhu[4] almost 20 years ago. Various papers have been written on the theory of circular cliques. Also graphs with circular adjacency matrices is discussed .

Graph coloring theory has a central position in discrete mathematics - for its own interest as well as for the large variety of applications, dating back to the famous four-color problem stated by Guthrie in 1852 Zhu[4] .

Circular cliques have important applications to the theory of designs and error correcting codes.

As generalization of circular cliques, Zhu [4] introduced the class of circular cliques based on a more general coloring concept introduced by Vince [10]. This variation of colorings is based on the observation that odd holes C_{2k+}satisfy $\omega(C_{2k+1}) = 2$ and $\chi(C_{2k+1}) = 3$, whereas their chromatic number should be intuitively about 2, as only one vertex needs to receive the third color. Vince aimed at refining the usual coloring concept taking this intuition into account. He defined the star chromatic number $\chi*(G)$ of a graph in such a way that indeed $\chi*(C_{2k+}) = 2+\frac{1}{k}$ is satisfied for each odd hole, whereas $\chi*(K_k) = k$ still holds for every complete graph on k vertices.

The combinatorial concept being dual to colorings corresponds to cliques in a graph[8]. In a set of k pairwise adjacent vertices, called clique K_k, all k vertices have to be colored differently. Thus the size of a largest clique in G, the **clique number ω(**G), is a trivial lower bound on $\chi(G)$. This bound can be arbitrarily bad and is in general hard to evaluate as well. From the definition, we immediately obtain $\chi c(G) \le \chi(G)$ because a usual k-coloring of G is (k,1)-circular coloring.

Numerous works have been devoted to the circular-chromatic number and we refer to the two surveys by X. Zhu for a detailed overview [2].

In view of the max-min relation between clique and chromatic number and to obtain a lower bound on $\chi c(G)$, Bondy and Hell [1] generalized cliques as follows. Let $K_{k/d}$ with $k \ge 2d$ denote the graph with the k vertices 0, . . . , k−1 and edges i j if and only if $d \le |i-j| \le k-d$. Such graphs $K_{k/d}$ are called circular cliques (note that they are also known as antiwebs in the literature, see [1]). A circular clique $K_{k/d}$ with gcd(k, d) = 1 is said to be prime. Prime circular cliques include all cliques $K_k = K_{k/1}$ as well as all odd antiholes $C_{2k+1}= K_{2k+1/2}$ and all odd holes $C_{2k+1}= K_{2k+1/k}$, see Figure 1.1.

II. PRELIMINARIES

Definition 2.1: A graph G is a pair G = (V,E) consisting of a finite set V and a set E of 2-element subsets of V. The elements of V are called vertices and the elements of E are called edges. Two vertices u and v of G are said to E.∈be adjacent if there is an edge e = (u,v) Two edges are said to be adjacent if they have a common vertex.

Definition 2.2: An undirected graphs do not show the direction which must be taken between nodes. Instead, travel between nodes is allowed along an edge in either direction. There are no loops or multiple edges in undirected graphs.

Definition 2.3: Circular clique is the graph with vertex set and edges between elements at distance. It is denoted by $K_{k/d}$,

Definition 2.4: Adjacency Matrix: The adjacency matrix of a directed graph X is the matrix (X) with rows and columns indexed by vertices of X. Each entry ij is equal to the number of times the arc (i, j) appears in X.

The adjacency matrix of a circulant graph has a pleasing nature : each row is the cyclic shift of the preceding row. If (a_1, a_2, ………… a_n) is the first row, then (a_n, a_1, ………… a_{n-1}) is the second row (a_{n-1}, a_n, ………… a_{n-2}) is the third row and finally (a_2, a_3 , ………, a_1) is the nth row.

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Definition 2.5: If F represents the binary field, then F^n the set of all n-tuples of F is an n-dimensional vector space over F. A k-dimensional subspace of Fn is called an [n,k] binary linear code C. A basis of C consists of k linearly independent binary n-tuples. The matrix G formed by the basis vectors is called a generator matrix of C.The elements of C are called code words and are linear combinations of the rows of the generator matrix G. Since a vector space can have many basis, a code C has many generator matrices.

Definition 2.6: Code 4.1: A code is a set X such that for all x ∈ X, x is a codeword.

A binary code is a code over the alphabet {0, 1}.

Examples of codes: C_1 = {00, 01, 10, 11} C_2 = {000, 010, 101, 100} C_3 = {00000, 01101, 10111, 11011}

Cyclic Code Definition 2.7: A binary code is cyclic if it is a linear [n, k] code and if for every codeword $(c_1, c_2, \ldots, c_n) \in C$ we also have that $(c_n, c_1, \ldots, c_{n-1})$ is again a codeword in C.

Definition 2.8 : An error correcting code (ECC) is an encoding scheme that transmits messages as binary numbers, in such a way that the message can be recovered even if some bits are erroneously flipped[7].

III. MAIN SECTION

Let $K_{k/d}$ with $k \geq 2d$ denote the graph with the k vertices 0, . . . , k−1 and edges i j if and only if $d \leq |i- j| \leq k-d$. Such graphs $K_{k/d}$ are called circular cliques (note that they are also known as antiwebs in the literature, see [1]). A circular clique $K_{k/d}$ with gcd(k, d) = 1 is said to be prime. Prime circular.

From[6] If C is a binary cyclic code of length n, then C corresponds to a on A. Conversely if $X=\omega(G) \leq \omega c(G)$ coresponds clrcular cliques on G, then X corresponds to a cyclic code.

Let C be a cyclic code of length n. If g(x) is its generator polynomial, then $g(x)/x^n - 1$.

Let $g(x) = g_0 + g_1x + g_2x^2 + \ldots\ldots + g_{n-k}x^{n-k}$. To the kxn generator matrix G, adjoin the remaining n-k cyclic shifts to get the nxn matrix. Choose any row rj having its first element is 0.

$$A=\begin{bmatrix} g0 & g1\,g2 & gn-k & 0 & 0 \\ 0 & g0\,g1 & gn-k-1 & gn-k & 0 \\ \vdots & \vdots\vdots & \vdots & \vdots & \vdots \\ g1 & g2\,g3 & 0 & 0 & g0 \end{bmatrix}$$

Let B be the nxn matrix formed with rj as the first row and remaining rows are the n-1 cyclic shifts of r_j . Since each row of A is a row of B and vice versa, and that C consists of linear sums of rows of A, it can be generated by B. Now B is a cyclic n x n binary matrix with leading element 0, hence form the adjacency matrix of a circular cliques.

Conversely, let there be a circular cliques $X=(K_{k/d})$. If A is the adjacency matrix of X, then A is a cyclic nxn matrix with leading element 0. Let C be the row space of A. The first row r_1 corresponds to a polynomial k(x) of degree ≤ n-1. The remaining rows are xk(x), x^2 k(x),………x_{n-1}k(x). We can prove that C is a cyclic code. Let s(x)∈ C. Then s(x) is a linear combination of these polynomials

$s(x) = a_0k(x) + a_1xk(x) + a_2 x^2 k(x) + \ldots\ldots\ldots + an-1\ xn-1k(x)$.

Then $xs(x) = a_0\ xk(x) + a_1\ x^2k(x) + \ldots\ldots\ldots + a_{n-1}\ k(x)\ (\text{mod}\ (x_{n-1})) = a_{n-1}k(x) + a_0xk(x) + \ldots\ldots\ldots + a_{n-2}x_{n-1}k(x)$

This means that the first cyclic shift of s(x) can be generated by the rows of A. Consequently the second cyclic shift and all the remaining cyclic shifts can be generated using the rows of A. Thus s(x) and all its cyclic shifts belong to C, hence C is a cyclic code.

Here we find the generator polynomials of the cyclic codes, dimensions and error correcting codes in circular cliques.

Example 3.1: Consider the circular cliques $X= K_{k/d}$, gcd(k, d) = 1

$X=(K_{7/1})$, the adjacency matrix of X is

$$\begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

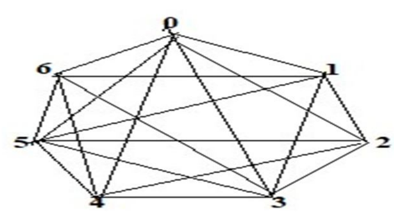

Figure1.1, $K_{7/1}$

The polynomial represented by X is $k(x)=x+x^2+x^3+x^4+x^5+x^6$

$$=x(1+x+x^2+x^3+x^4+x^5)$$

$$=x(1+x^3)\,(x^3+x+1)$$

We know that $\mathbf{x^7 - 1 = (1 + x)\,(1 + x + x^3)(\,1 + x^2 + x^3\,)}$

Therefore gcd $(k(x), x^7 - 1) = x+1$. Hence X corresponds to the cyclic code $C = \langle x \rangle$. Since the degree of the generator polynomial k(x) is 3, dimension of the code is 6 and has no error correcting codes.

Example 3.2:Consider the circular cliques $X= K_{k/d}$, gcd(k, d) =1

$X=(K_{7/2})$, the adjacency matrix of X is

$$\begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

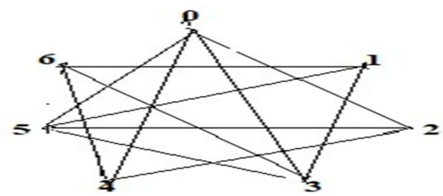

Figure1.2, $K_{7/2}$

The polynomial represented by X is $k(x)= x^2+x^3+x^4+x^5=x^2(1+x)(1+x)^2$

Therefore gcd $(k(x), x^7 - 1) = x+1$. Hence X corresponds to the cyclic code $C = \langle x \rangle$. Since the degree of the generator polynomial k(x) is 2, dimension of the code is 6 and has no error correcting codes.

Example3.3: Consider the circular cliques $X= K_{k/d}$, gcd(k, d) =1

$X=(K_{7/3})$, the adjacency matrix of X is

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

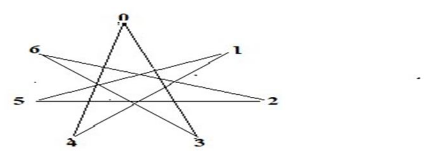

Figure1.3, $K_{7/3}$

The polynomial represented by X is $k(x)= x^3+x^4=x^3(x+1)$

Therefore gcd $(k(x), x^7 - 1) = x+1$. Hence X corresponds to the cyclic code $C = \langle x \rangle$. Since the degree of the generator polynomial k(x) is 1, dimension of the code is 6 and has no error correcting codes.

Example3.4:Consider the circular cliques $X= K_{k/d}$, gcd(k, d) =1

$X=(\mathbf{K_{11/1}})$, the adjacency matrix of X is

$$\begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

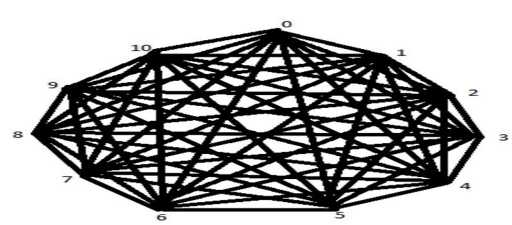

Figure1.4, $K_{11/1}$

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The polynomial represented by X is $k(x)=x+x^2+x^3+x^4+x^5+x^6+x^7+x^8+x^9+x^{10}$
$=x(1+x+x^2+x^3+x^4+x^5+x^6+x^7+x^8+x^9)$
$=x^8(x^2+x)(x+1)$

Therefore gcd $(k(x), x^{11} - 1) = x+1$. Hence X corresponds to the cyclic code C =<x> . Since the degree of the generator polynomial k(x) is 1, dimension of the code is 10 and has no error correcting codes.

IV. CONCLUSION

In this paper, we introduced the concept of binary cyclic codes in circular cliques. The circular cliques and a binary cyclic codes are presented and proved. Every circular clique $\mathbf{K_{k/d}}$ clearly admits a (k,d)- vertices. We observed that a circular clique $K_{k/d}$ with gcd(k, d) = 1 are Prime circular cliques. The degree of the generating polynomials are decreased.

1) 1.$K_{7/1}$, $K_{7/2}$, $K_{7/3}$......($_{k=0.....n-1}$ vertices) gcd $(k(x), x^7 - 1) = x+1$. Hence X corresponds to the cyclic code C =<x+1> , the degree of the generator polynomial k(x) are 3,2 and 1, dimension of the code is 6 and has no error correcting codes.
2) $K_{11/1}$, $K_{11/2}$, $K_{11/3}$......($_{k=0.....n-1}$ vertices) gcd $(k(x), x^{11} - 1) = x+1$. Hence X corresponds to the cyclic code C =<x> , the degree of the generator polynomial k(x) is 1, dimension of the code is 10 and has no error correcting codes.
   We now seek a condition under which the cyclic code corresponding to one circular cliques becomes a subset of another.

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