



# IJRASET

International Journal For Research in  
Applied Science and Engineering Technology



---

# INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

---

**Volume: 10    Issue: VII    Month of publication: July 2022**

**DOI: <https://doi.org/10.22214/ijraset.2022.44310>**

**[www.ijraset.com](http://www.ijraset.com)**

**Call:  08813907089**

**E-mail ID: [ijraset@gmail.com](mailto:ijraset@gmail.com)**

# Conditions for Inscribed Triangle to Be Similar to Given Triangle

Bishal Panthi

Kathmandu Model College

**Abstract:** In this research paper, I will derive the conditions for similarity between a triangle and its inscribed triangle. I will go through the inverse similarity conditions of an inscribed triangle for acute angled triangle and right angled triangle. I will use the basic laws of geometry to deduce the conclusion. Apart from mid-point triangle, we will analyze the conditions of similarity for other inscribed triangle.

**Keywords:** Inscribed triangle, Inverse Similarity, Mid-point triangle

## I. INTRODUCTION

When we select one point on each sides of triangle and join all those points, the triangle thus formed is called inscribed triangle. We know that mid-point triangle is similar to the triangle in which it is inscribed. Here, I will explain the existence of an inscribed triangle similar to a given triangle except mid-point triangle.

### A. Inverse Similarity

Inverse similarity means the inscribed triangle is similar to the given triangle where corresponding vertices are not collinear.

On the basis of number of mid-point included by inscribed triangle, I will further go into detail.

As shown in figure 1, P and R are the mid-points of AB and AC respectively while Q is not mid-point of BC.

Condition when inscribed triangle includes two mid-points of a side of triangle

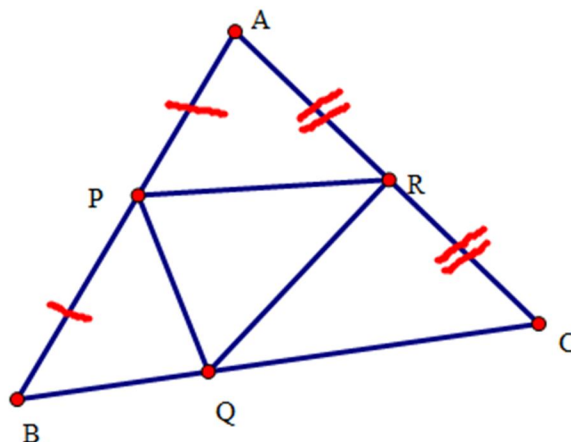


FIGURE 1:

As shown in figure 1, P and R are the mid-points of AB and AC respectively while Q is not mid-point of BC.

### First case:

When  $\triangle ABC \sim \triangle QRP$

Given:

$AP = PB$  &  $AR = RC$

From mid-point theorem,

$PR \parallel BC$

Now,

$\angle ABC = \angle PRQ$ ..... (1) (Corresponding angle of similar triangle)

$\angle APR = \angle ABC$ ..... (2) (Being corresponding angle, since  $PR \parallel BC$ )

From (1) and (2):

$\therefore \angle PRQ = \angle APR$

$\therefore AB \parallel QR$

According to mid-point theorem,

If  $AR = RC$  and  $AB \parallel QR$ , then  $BQ = QC$  which contradicts our assumption that  $Q$  is not the mid-point of  $BC$ . Hence, it is not possible for an inscribed triangle being inversely similar to the given triangle when only two mid-points are included.

Condition when inscribed triangle includes only one mid-point of a side of a triangle

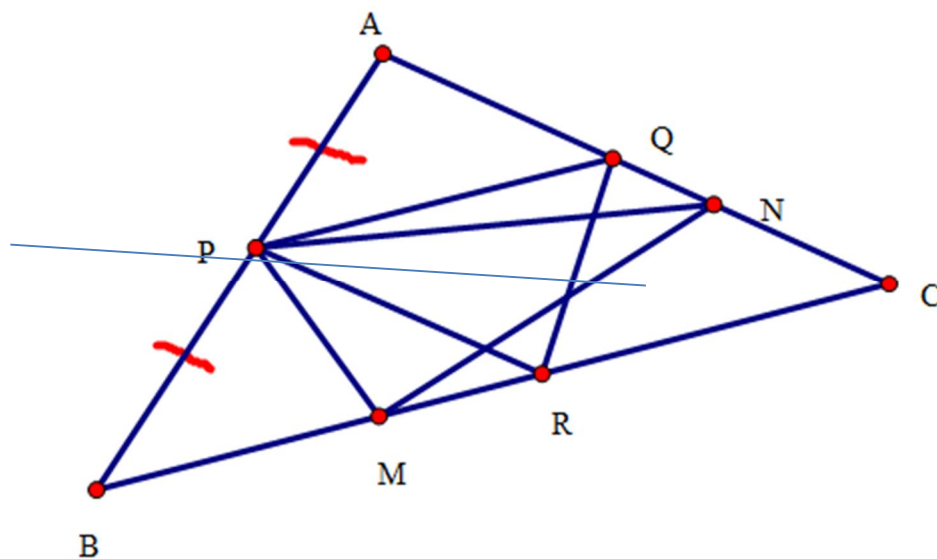


Figure 2:

As shown in the **figure 2**,  $P$ ,  $Q$  and  $R$  are the mid-points of  $AB$ ,  $AC$  and  $BC$  respectively. Similarly,  $M$  and  $N$  (both are not mid-points of  $BC$  and  $AC$  respectively) are the points on  $BC$  and  $AC$  respectively. Similarly, points  $C$  and  $P$  are joined.

We have,

$\triangle MNP \sim \triangle ABC$

For the condition of inverse similarity, when  $M$  lies between  $B$  and  $R$ ,  $Q$  had to lie between  $A$  and  $N$  and vice versa.

Proof:

$\triangle ABC \sim \triangle MNP \sim \triangle RQP$

Here:

$\angle MPN = \angle RPQ$  (Corresponding angle of similar triangle)

$\angle MPN - \angle RPN = \angle RPQ - \angle RPN$  (Subtracting common angle in both sides)

$\therefore \angle MPR = \angle NPQ$ ..... (3)

$\therefore \triangle MNP \sim \triangle RQP$

$\frac{PM}{PN} = \frac{PR}{PQ}$  (Ratio of corresponding sides of similar triangle is equal) ..... (4)

From equation (3) and (4), we can conclude that:

$\triangle PMR \sim \triangle PNQ$

And,  $\angle PMR = \angle PNQ$  (Corresponding angle of similar triangle are equal)

$\therefore P, M, C$  and  $N$  are the cyclic points (Exterior angle of a cyclic quadrilateral is equal to the interior opposite angle)

$\angle PMN = \angle PCN$  ..... (5) (Angle subtended on the same arc chord of cyclic quadrilateral)

$\angle PMN = \angle PAC$  ..... (6) ( $\because \triangle ABC \sim \triangle MNP$ , so corresponding angle of similar triangle are equal)

From (5) and (6), we have:

$$\angle PAC = \angle PCN$$

Hence, PCA is isosceles triangle  $PA = PC$ .

We have,  $PA = PB$ , since P is the mid-point of side AB

$$\therefore PA = PB = PC$$

Hence,  $\triangle ABC$  is right angle with  $\angle ACB = 90^\circ$

Similarly,  $\angle MPN = 90^\circ$  (Corresponding angle of congruent triangle)

From the above expression, we can conclude that if only one vertex of the inscribed triangle similar to given triangle includes the mid-point of the side of given triangle, then both the inscribed triangle and given triangle are right-angled triangle.

## II. CONVERSE PROOF

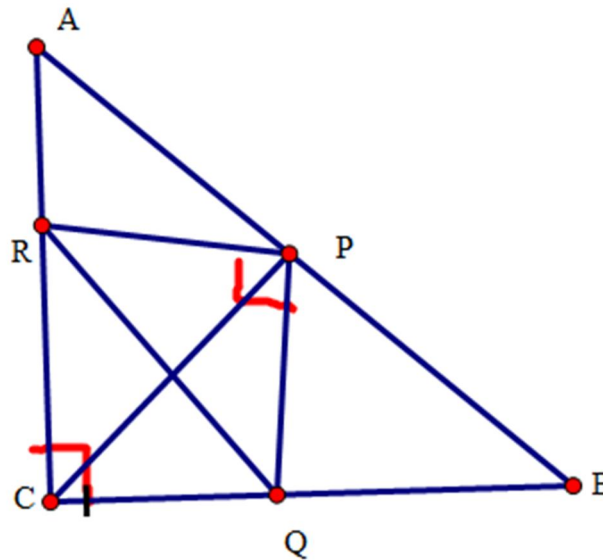


FIGURE 3:

In the figure 3,  $\triangle ABC \sim \triangle QRP$

And, points C and P are joined.

Similarly, both  $\triangle ABC$  and  $\triangle QRP$  are right angled triangle.

From the given figure:

$$\angle QCR + \angle QPR = 180^\circ$$

$\therefore$  Q, C, R and P are cyclic points.

Hence,

$$\angle PCR = \angle PQE \dots \dots \dots (7) \quad (\text{Angle subtended on same chord of cyclic quadrilateral})$$

$$\angle PQR = \angle BAC \dots \dots \dots (8) \quad (\text{Corresponding angle of similar triangle})$$

From (7) and (8):

$$\angle PAC = \angle PCA$$

$\triangle PAC$  is isosceles triangle.

$$\therefore PA = PC \dots \dots \dots (I)$$

Again,

$$\angle QCP = \angle QRP \dots \dots \dots (9) \quad (\text{Angle subtended on same chord of cyclic quadrilateral})$$

$$\angle PRQ = \angle ABC \dots \dots \dots (10) \quad (\text{Corresponding angle of similar triangle})$$

From (9) and (10)

$$\angle PBC = \angle PCB$$

$\triangle PBC$  is isosceles triangle.

$$\therefore PB = PC \dots \dots \dots (II)$$

From (I) and (II):

$$PA=PB=PC$$

Hence, we can conclude that if the inscribed triangle is inversely similar to the given triangle, where both the given triangle and inscribed triangle are right angled triangles, the vertex of the right angle of inscribed triangle will be mid-point of the hypotenuse of given triangle.

Condition when inscribed triangle does not include any mid-point of a side of a triangle

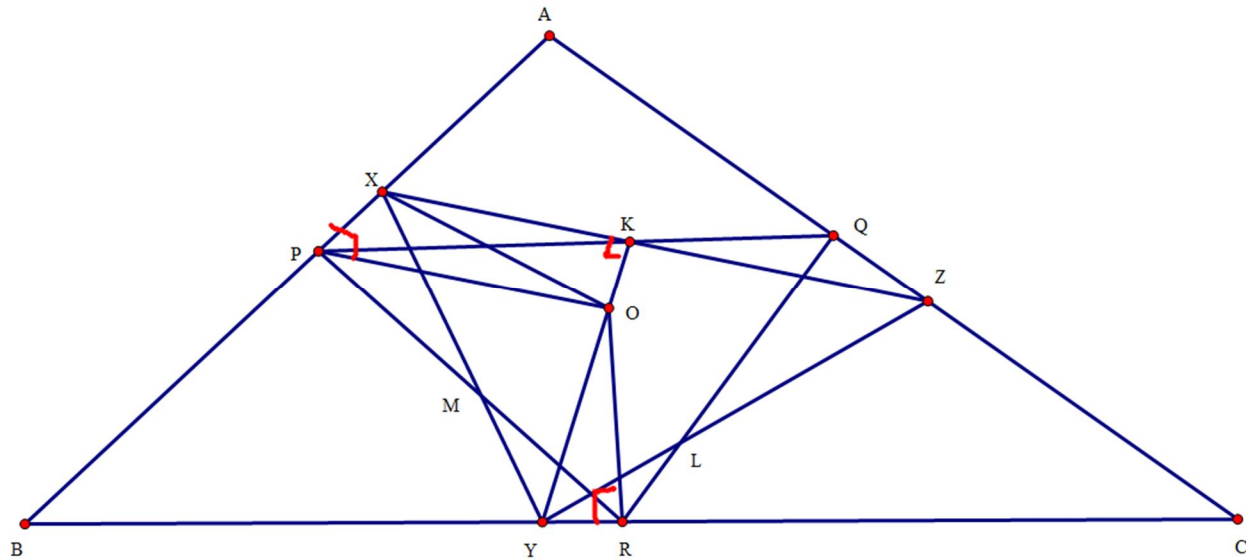


FIGURE 4

In the **figure 4**, K, L and M are the intersections points of corresponding sides of similar triangle. Similarly, O is the circumcenter of  $\triangle ABC$  and  $YK \perp LXZ$ . The reason behind choosing circumcentre O and  $YK \perp LXZ$  is to find several pairs of perpendicular lines and cyclic points. P, Q and R are the mid-points of AB, AC and BC respectively.

**Proof:**

$\angle ORB = 90^\circ$  (Perpendicular drawn from center to the chord bisects the chord)

$\therefore \angle ORY + \angle OYR = 90^\circ$  (Being right-angled triangle)

$PQ \parallel BC$  (From mid-point theorem)

$\angle PKY = \angle KYC$  (Being alternate angle)

Since,  $YK \perp LXZ$

$\angle PKX + \angle PKO = 90^\circ$

Similarly,

$\angle OPX = 90^\circ$  ..... (9)

$\angle OPX + \angle OKX = 90^\circ + 90^\circ = 180^\circ$

$\therefore$  O, P, X and K are cyclic points:

$\angle XKP = \angle XCP$  ..... (10) (Angle subtended on same chord of cyclic

quadrilateral)

From (9) and (10),

$\angle XOP = \angle YOR$

$\angle OPX = \angle OYR = 90^\circ$

$\therefore \triangle ORY \sim \triangle OPX$

$\frac{OY}{OX} = \frac{OR}{OX}$  (Ratio of corresponding sides of similar triangle is equal)

$\angle YOR = \angle XOP$

$\therefore \triangle OPR \sim \triangle OXY$

$\angle RPO = \angle YXO$  ..... (11) (Corresponding angle of similar triangle)

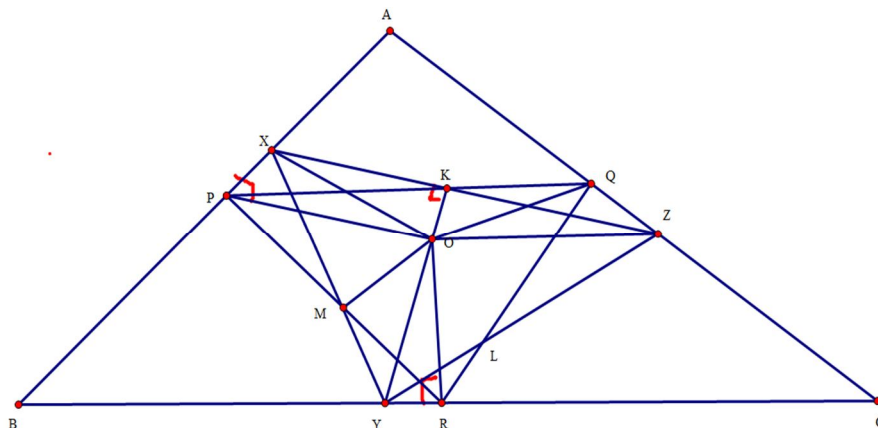
$\angle OPK = \angle OXK$  ..... (12) (Angle subtended on same chord of cyclic quadrilateral)



Adding (11) and (12), we get:

$$\angle RPK = \angle YXK \dots \dots \dots \text{(III)}$$

In the next step, we will join OM, OQ and OZ.



$\angle OQZ = 90^\circ$  (The perpendicular drawn from center to the chord bisects the chord)

$\angle YKZ = 90^\circ$  (Given)

Since,  $\angle OKZ = \angle OQZ = 90^\circ$

O, K, Q and Z are cyclic points.

$\angle ZOQ = \angle ZKQ \dots \dots \dots \text{(13)}$  (Angle subtended on same chord of cyclic quadrilateral)

$\angle PKX = \angle ZKQ \dots \dots \dots \text{(14)}$  (Being vertically opposite angle)

From (13) and (14), we get:

$$\angle ZOQ = \angle PKZ \dots \dots \dots \text{(15)}$$

$\angle YOR = \angle PKX \dots \dots \dots \text{(16)}$  (Already proved)

From (15) and (16), we get:

$$\angle YOR = \angle ZOQ \dots \dots \dots \text{(17)}$$

$\angle ORY = \angle OZQ = 90^\circ$  (Given)

$$\therefore \triangle ORY \sim \triangle OQZ$$

Again,

$$\frac{OZ}{OY} = \frac{OQ}{OR} \text{ (Ratio of corresponding sides of similar triangle is equal)}$$

Adding  $\angle ROZ$  on both sides, we get:

$$\angle YOZ = \angle ROQ$$

$$\text{So, } \triangle YOZ \sim \triangle ROQ$$

$$\therefore \angle OYZ = \angle ORQ \dots \dots \dots \text{(18)}$$
 (Corresponding sides of similar triangle)

$$\therefore \triangle POR \sim \triangle XYO$$

$$\angle PRO = \angle XYO \dots \dots \dots \text{(19)}$$

On adding (18) and (19), we get:

$$\angle XYZ = \angle PRQ \dots \dots \dots \text{(IV)}$$

From (III) and (IV), we have

$$\therefore \triangle XZY \sim \triangle PQR$$

$$\therefore \triangle YZX \sim \triangle ABC$$

Hence, for an acute angled triangle, the inscribed triangle is inversely similar to the given triangle when one altitude of an inscribed triangle passes through the circumcenter of given triangle and the perpendicular foot of altitude lies on the joining of two mid-points of the given triangle.



### III. CONCLUSION

When the vertex of inscribed triangle includes the two mid points of sides of given triangle, then it is not possible for an inscribed triangle to be inversely similar to the given triangle without including the remaining mid points of a sides of given triangle. When the vertex of inscribed triangle includes only one midpoint of the sides of given triangle, then the inscribed triangle will be inversely similar to given triangle if and only if both inscribed triangle and given triangle are right angled triangle. For right angled triangle, inscribed triangle and given triangle are inversely similar if and only if the vertex of right angle of the inscribed triangle lies on the mid-point of the hypotenuse of given triangle. In case of acute angled triangle, inscribed triangle and given triangle are inversely similar if and only if one of the altitude of the inscribed triangle passes through the circumcenter of given triangle.

### IV. FOLLOW-UP RESEARCH

- 1) For obtuse angled triangle, is the necessary for the altitude of inscribed triangle to pass through the circumcenter of given center to be inversely similar to the given triangle?
- 2) Under what conditions of obtuse angled triangle, inscribed triangle will be inversely similar to the given triangle?

### REFERENCES

- [1] He, G., 2015. San Jiao Xing De Liu Xin Ji Qi Ying Yong. Haerbin: Ha er bin gong. ye da xue chu ban she.
- [2] Holshouser, A. and Reiter, H., 2014. Classifying Similar Triangles Inscribed In A Given Triangle. [online] Webpages.uncc.edu. Available at: [Accessed 14 June 2020]
- [3] (not detailed). Retrieved from <https://www.math.ust.hk/~mamu/courses/2023/W7.pdf>



10.22214/IJRASET



45.98



IMPACT FACTOR:  
7.129



IMPACT FACTOR:  
7.429



# INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Call : 08813907089  (24\*7 Support on Whatsapp)