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Connectedness Among the Pixels at the Product Digital Topology

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Abstract: The aim of this paper is to analyze at the concepts connectedness of the separated interior circles at the pixels at the product digital topology with the axioms C1, C2, C3 in the cartesian complex and also at the structure of the elements at the collections of the pixels and interior circles in that region.
Keywords: Cut point, classical axioms of the topological space, pixels, incidence, path, opponent, interior, closure and frontier, locally finite space.

I. INTRODUCTION

Digital topology is to study at the topological properties of digital image arrays. These properties on cathode ray tubes are virtually important in a wide range of diverse applications, including computer graphics, computer tomography, pattern analysis and robotic design. A topological framework contains many pixels or 2-cell. A digital picture can be stored at them. These framework settings are in some of the devices for the focus purpose. In this case one can specify at the pixels on the simple closed curves which states that a simple closed curve separates at the plane into two connected sets. When a pixel is extended to be such a set of pixels possess connectivity and is called a region.

II. PRELIMINARIES

1) *Definition 2.1[2]:* A point x in X is called a cut point (respectively endpoint) if X-{x} has two (one) components. (In the literature our cut-point is usually called a strong cut-point, but here it turns out that these two notions coincide.) The parts of X-{x} are its components if there are two, and X-{x}, φ if there is only one.
2) *Definition 2.2[5]:* A nonempty set S is called a locally finite (LF) space if to each element e of S certain subsets of S are assigned as neighborhoods of e and some of them are finite.
3) *Definition 2.3 [5]:* Axiom 1. For each space element e of the space S there are certain subsets containing e, which are neighborhoods of e. The intersection of two neighborhoods of e is again a neighborhood of e. Since the space is locally finite, there exists the smallest neighborhood of e that is the intersection of all neighborhoods of e. Thus, each neighborhood of e contains its smallest neighborhood. We shall denote the smallest neighborhood of e by SN(e).
4) *Definition 2.4[5]:* Axiom 2. There are space elements, which have in their SN more than one element.
5) *Definition 2.5[5]:* If b $\in$ SN(a) or a $\in$ SN(b), then the elements a and b are called incident to each other.
6) *Definition 2.6[4]:* A path is a sequence (p_i / $0 \leq i \leq n$), and p_i is adjacent to p_{i+1}. In another way Let T be a subset of the space S. In another way [4] a sequence (a_1, a_2, . . ., a_k), a_i, $\in$ T, i = 1, 2, . . ., k; in which each two subsequent elements are incident to each other, is called an incidence path in T from a_1 to a_k.
7) *Definition 2.7 [4]:* A set of pixels is said to be connected if there is a path between any two pixels.
8) *Remark 2.8[5]:* In another way A subset T of the space S is connected iff for any two elements of T there exists an incidence path containing these two elements, which completely lies in T
9) *Definition 2.9 [5]:* The topological boundary, also called the frontier, of a non-empty subset T of the space S is the set of all elements e of S, such that each neighborhood of e contains elements of both T and its complement S−T. It is denoted by the frontier of T $\subseteq$ S by Fr (T, S).
10) *Definition 2.10[5]:* A subset O $\subset$ S is called open in S if it contains no elements of its frontier Fr (O, S). A subset C $\subset$ S is called closed in S if it contains all elements of Fr (C, S).
11) *Definition 2.11[5]:* The neighbourhood relation N is a binary relation in the set of the elements of the space S. The ordered pair (a, b) is in N iff a $\in$ SN(b). We also write aNb for (a, b) in N.
12) *Definition 2.12 [5]:* A pair (a, b) of elements of the frontier Fr (T, S) of a subset T $\subset$ S are opponents of each other, if a belongs to SN(b), b belongs to SN(a), one of them belongs to T and the other one to its complement S − T.
13) *Definition 2.13[5]:* The smallest open subset of the ALF space S that contains the element a $\in$ S is called the smallest open neighborhood of a in S and is denoted by SON (a, S). It is denoted by SON (a, S) = SN(a)

14) *Definition 2.14 [5]:* The topology of a space S is defined if a collection of subsets of S is declared to be the collection of open subsets satisfying the following axioms:
Axiom C1. The entire set S and the empty subset φare open.
Axiom C2. The union of any number of open subsets is open.
Axiom C3. The intersection of a finite number of open subsets is open.

15) *Theorem KC (k-dimensional cell) 2.15 [5]:* The dimension of a cell c = (a_1, a_2, . . ., a_n) of an n-dimensional Cartesian complex S is equal to the number of its components a_i, i = 1, 2, . . . n; which are open in their axes.
16) *Definition 2.16[1]:* Let T be a subset in the space S with the axioms C1, C2, C3 then the interior, int (T, S)=□□{O: O is open, O□□□} (i.e.,) the union of all open sets contained in T.
17) *Definition 2.17[1]:* Let T be a subset in the space S with the axioms C1, C2, C3 then the closure, cl (T, S) =□{C:Cis closedand C□□} (i.e.,) the intersection of all closed sets containing T.
18) *Definition 2.18[1]:* Let Tbeanysubsetofaspace S with the axioms C1, C2, C3. Thenext (T,S) = S – { int(T,S) □□Fr (T,S)} where int (T,S),ext (T,S)and Fr (T,S) aredisjoint and also Fr (T,S)isaclosed set.
19) *Theorem 2.19[1]:* Let S be a space with the axioms C1, C2, C3 and let Tbeasubset of S. Then int (T, S) is anopen.
20) *Corollary 2.20[1]:* Let Tbeanysubsetofaspace S with the axioms C1, C2, C3. ThenS=int(T, S) □Fr (T, S) □ext (T, S) where int (T, S),ext (T, S)and Fr (T, S) aredisjoint and also Fr (T, S)isaclosed set.
21) *Remark 2.21[1]:* Let P_1 be the one of the pixels in the Cartesian complex. Now the projections are Π_1 and Π_2 are onto . For if x ϵ X such that (x, y) ϵ X×Y with the axioms C1, C2, C3 for which $\Pi_1(x,y) = x$ and y ϵY such that $(x,y)\epsilon X\times Y$with the axioms C1, C2, C3 for which $\Pi_2(x,y) = y$ if one of X or Y is empty then X×Y is also empty and there is no need to consider the function Π_1 andΠ_2. As Π_1 and Π_2 are surjective we say the Π_1 and Π_2 are projections of X×Y onto its factors respectively.

III. SEPARATION OF INTERIOR CIRCLES AT THE PIXELS

Let $\wp$ be the collection of pixels at the product digital topology with the axioms C1, C2, C3 in the Cartesian complex and the set of all frontier points. Thus it is called rectangular region.

Now we take at the collection of all interior circular region ℓ at a pixel in the product digital topology with the axioms C1, C2, C3 as a plane in the Cartesian complex.

The element of the structure $\wp$ and ℓ are P_1, P_2, P_3,.........and C_1, C_2, C_3,............at the product digital topology with the axioms C1, C2, C3 in the Cartesian complex where P_1be a one of the pixel at $\wp$ and C_1 be the collection of interior circulars c_1, c_2, c_3, ...at a pixel P_1.

1) *Definition 3.1:* Let c_1 and c_2 be interior circulars at C_1 in a pixel P_1 in the Cartesian complex with the axioms C1, C2, C3 are separated at C_1 in a pixel P_1 if and only if $c_1 \bigcap cl(c_2,P_1) = cl(c_1,P_1) \bigcap c_2 = \varphi$.
2) *Theorem 3.2:* If P_1 and P_2 are separated pixels of the product digital topology with the axioms C1, C2, C3 in the Cartesian complex and C_1□ P_1 and C_2□ P_2□□□then C_1 and C_2 are also separated.

Proof : Now P_1□$\bigcap$ cl(P_2, $\wp$)=φ□□and cl(P_1, $\wp$)$\bigcap$ P_2 =φ ....(1). Also C_1□ $P_1 \Rightarrow$ cl(C_1,P_1)□□ cl(P_1, $\wp$) and C_2□ $P_2 \Rightarrow$ cl(C_2,P_2)□□ cl(P_2, $\wp$) ....(2). It follows from (1) and (2) that $C_1 \bigcap$ cl(C_2,P_2)=φ□and cl(C_1,P_1)$\bigcap C_2$ =φ. Hence C_1 and C_2 are separated.

3) *Theorem 3.3:* Two closed (open) interior circles C_1 and C_2 at the pixelsP_1 and P_2at the product digital topology with the axioms C1, C2, C3 in the Cartesian complex are separated ifandonly if they are disjoint.

Proof : Since any two separated interior circles are disjoint. To prove that two disjoint closed (open) interior circles are separated. If C_1 and C_2 are both disjoint and closed, then $C_1 \bigcap C_2$=φ , cl(C_1,P_1) = C_1 and cl(C_2,P_2) = C_2. So that cl(C_1,P_1)$\bigcap C_2$ =φ and $C_1 \bigcap$ cl(C_2,P_2)=φ. To show that C_1 and C_2 are separated. If C_1 and C_2 are both disjoint and open, then P_1-C_1 and P_2-C_2 are both closed interior circles so that P_1- cl(C_1,P_1) = P_1-C_1 and P_2- cl(C_2,P_2) = P_2-C_2. Also $C_1 \bigcap C_2$=φ $\Rightarrow$ C_1□□P_2-C_2 and C_2□□P_1-$C_1 \Rightarrow$ cl(C_1,P_1)□cl(P_2-C_2,P_2)=P_2-C_2 and cl(C_2,P_2)□cl(P_1-C_1,P_1)= P_1-$C_1 \Rightarrow$ cl(C_1,P_1) $\bigcap C_2$=φ and cl(C_2,P_2) $\bigcap C_1$=φ $\Rightarrow$ C_1 and C_2 are separated.

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4) *Definition 3.4:* A pixelP_1at the product digital topology with the axioms C1, C2, C3 in the Cartesian complex is disconnected if and only if there are disjoint non empty interior circular regions c_1 and c_2 at C_1 in P_1such that P_1= c_1□ c_2.
5) *Remark 3.5:* In another way of disconnected if and only if a pixel P_1 is the union of two non empty separated interior circular regions c_1 and c_2 at C_1 in P_1 with the axioms C1, C2, C3.
6) *Definition 3.6:* A pixel P_1 at the product digital topology with the axioms C1, C2, C3 in the Cartesian complex is said to be connected if it cannot be expressed as union of two non empty separated interior circular regions c_1 and c_2 at C_1 in P_1 with the axioms C1, C2, C3. In another way, two non empty separated interior circular regions c_1 and c_2 at C_1 in P_1 with the axioms C1, C2, C3is said to be connected if and only if it is not disconnected.
7) *Theorem 3.7:* A pixel P_1 is connected at thenon empty separated interior circular regions c_1 and c_2 at C_1 in P_1 with the axioms C1, C2, C3 that are both open and closed in a pixel P_1 at the product digital topology with the axioms C1, C2, C3 in the Cartesian complex.

Proof : If c_1 is non empty proper circular region of P_1 which is both open and closed at a pixel P_1 at the product digital topology with the axioms C1, C2, C3 in the Cartesian complex then $u=c_1$ and $v=P_1-c_1$. Consistute a separation of P_1 at the product digital topology with the axioms C1, C2, C3 in the Cartesian complex for they are open disjoint and non-empty whose union is P_1.
Conversely if u and v are separation of a pixel P_1 at the product digital topology with the axioms C1, C2, C3 in the Cartesian complex then $u=P_1-v$ that implies u is closed. Therefore v is open. Similarly $v=P_1-u$ that implies v is closed therefore u is open. Thus u and v are both open and closed at a pixel P_1 at the product digital topology with the axioms C1, C2, C3 in the Cartesian complex.

8) *Theorem 3.8:* A pixel P_1 at the product digital topology with the axioms C1, C2, C3 in the Cartesian complexis disconnected if and only if there exists a non-empty proper interior circles of P_1 which is both open and closed at the product digital topology with the axioms C1, C2, C3 in the Cartesian complex.

Proof: Let c_1 be a non-empty proper interior circles of P_1 which is both open and closed. To show that P_1 is disconnected. Let $c_2=P_1-c_1$. Then c_2 is non-empty since c_1 is a proper interior circle of a pixel P_1 at the product digital topology with the axioms C1, C2, C3 in the Cartesian complex. Moreover, c_1□ $c_2=P_1$ and $c_1 \bigcap c_2=\varphi$. Since c_1 is both closed and open interior circle, c_2 is also both closed and open interior circle. Hence $cl(c_1 ,P_1)=c_1$ and $cl(c_2,P_2)=c_2$. It follows that $cl(c_1 ,P_1) \bigcap c_2=\varphi$ and $c_1 \bigcap cl(c_2,P_2)=\varphi$. Ths P_1 has been expressed as a nion of two separated interior circles and so P_1 is disconnected at the product digital topology with the axioms C1, C2, C3 in the Cartesian complex.
Conversely, let P_1be disconnected. Then there exist non-empty interior circles c_1 and c_2 of a pixel P_1 at the product digital topology with the axioms C1, C2, C3 in the Cartesian complex such that $c_1 \bigcap cl(c_2,P_1)=\varphi$, $cl(c_1 ,P_1) \bigcap c_2=\varphi$ and c_1□ $c_2=P_1$. Since c_1□ $cl(c_1,P_1)$, $cl(c_1 ,P_1) \bigcap c_2=\varphi \Rightarrow c_1 \bigcap c_2=\varphi$. Hence $c_1=P_1-c_2$. Since c_2 is non-empty interior circle, and c_2□ P_1- $c_2=P_1$, it follows that $c_2=P_1-c_1$ is a proper interior circle of P_1. Now c_1□ $cl(c_2,P_1)=P_1$.
Also $c_1 \bigcap cl(c_2,P_1)=\varphi \Rightarrow c_1=P_1- cl(c_2,P_1)$ and similarly $cl(c_1 ,P_1) \bigcap c_2=\varphi \Rightarrow c_2=P_1- cl(c_1,P_1)$. Since $cl(c_2,P_1)$ and $cl(c_1 ,P_1)$ are closed interior circles, it follows that c_1 and c_2 are open interior circles. Since $c_1=P_1-c_2$, c_1 is also closed interior circle. Thus c_1 is non-empty proper sub interior circle of a pixel P_1 at the product digital topology with the axioms C1, C2, C3 in the Cartesian complex which is both open and closed interior circles.

9) *Theorem 3.9:* A pixel of $\wp$ at the product digital topology with the axioms C1, C2, C3 in the Cartesian complexis connected if and only if every non-empty proper interior circles of pixels has a non-empty frontier.

Proof: Let every non-empty proper interior circle of pixels have a non-empty frontier. To show that a pixel $P_1$is connected. Suppose, if possible, P_1 is disconnected. Then there exist non-empty disjoint interior circles c_1 and $c_2$both open and closed interior circles at a pixel P_1 such that P_1= c_1□ c_2. Therefore $c_1=int (c_1,P_1)=cl(c_1,P_1)$. But $Fr(c_1,P_1)= cl(c_1,P_1)-int(c_1,P_1)$. Hence $Fr(c_1,P_1)= cl(c_1,P_1)-int(c_1,P_1)=\varphi$, which iscontrary to our hypothesis. Hence P_1 must be connected.
Conversely, let P_1 be connected and suppose, if possible, there exists a non-empty proper interior circle c_1 of P_1 such that $Fr(c_1,P_1) =\varphi$. Now $cl(c_1,P_1)= int(c_1,P_1)$□□$Fr(c_1,P_1)=c_1$□ $Fr(cl(c_1,P_1))$. Hence $cl(c_1,P_1)=int (c_1,P_1)=c_1$ showing that c_1 is both open and closed interior circle in P_1 and therefore P_1 is disconnected. But this is a contradiction. Hence every non-empty proper interior circles of P_1 must have a non-empty frontier.

10) *Theorem 3.10:* Let P_1 be a pixel at the product digital topology with the axioms C1, C2, C3 in the Cartesian complex and let c_1 be separated interior circle of P_1 such that c_1□□C_1□ C_2 where C_1 and C_2 are separated interior circles. Then c_1□□C_1 or c_1□□C_2, that is, c_1 cannot intersect C_1 and C_2.

Proof: Since C_1 and C_2 are separated interior circles , $C_1 \cap cl(C_2,P_1)=\varphi$, $cl(C_1,P_1) \cap C_2=\varphi$. Let c_1□□C_1□ $C_2 \Rightarrow c_1 \cap$ (C_1□ C_2)= ($c_1 \cap C_1$)□($c_1 \cap C_2$).........(1).

Claim that at least one of the interior circle $c_1 \cap C_1$ and $c_1 \cap C_2$ is empty. If possible, suppose none of these interior circles is empty, that is, suppose that $c_1 \cap C_1 \neq \varphi$ and $c_1 \cap C_2 \neq \varphi$. Then $(c_1 \cap C_1) \cap (c_1 \cap C_2)$□□ $(c_1 \cap C_1) \cap (cl(c_1,P_1) \cap cl(C_2,P_1))=(c_1 \cap (cl(c_1,P_1)) \cap (C_1 \cap cl(C_2,P_1))= (c_1 \cap (cl(c_1,P_1)) \cap \varphi=\varphi$. Clearly $(c_1 \cap C_1) \cap (c_1 \cap C_2) =\varphi$. Since $c_1 \cap C_1$ and $c_1 \cap C_2$ are separated interior circles. Thus c_1 has been expressed at the union of two non-empty separated interior circles and consequently c_1 is disconnected . But this is a contraction. Hence at one of the interior circles $c_1 \cap C_1$ and $c_1 \cap C_2$ is empty. If $c_1 \cap C_1 =\varphi$......(1) gives $c_1= c_1 \cap C_2$ which implies that c_1□□C_2 . Similarly if $c_1 \cap C_2 =\varphi$, then c_1□□C_1. Hence either c_1□□C_1 or c_1□□C_2.

11) *Corollary 3.11:* If c_1 is a connected interior circle of a pixel P_1 such that c_1□□C_1□ C_2 where C_1 , C_2 are disjoint open (closed) interior circle of a pixel P_1, then C_1 and C_2 are separated interior circles.
12) *Remark 3.12:* Let c_1 be a connected interior circle of a pixel P_1 such that c_1□ c_1□ $cl(c_1,P_1)$. Then c_1 is connected. In particular $cl(c_1,P_1)$ is connected.

IV. CONCLUSIONS

Further work, at the connectedness among the pixels and interior circles will be extended upto the concepts compactness at the product digital topology with the axioms C1, C2, C3 in the cartesian complex.

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