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Design of Rocket Engine Nozzle Using Method of Characteristics

Dr. R. S. Tarnacha[1], Mehak Virmani[2]
[1]*Sr. Consultant, TATA Technologies Ltd., Amity Institute of Technology*
[2]*Student at Amity Institute of Technology, Amity University Uttar Pradesh*

Abstract: This report focuses on the design of a 2D theoretical layout of a rocket engine nozzle. Convergent and divergent are to be designed separately with the focus on divergent part for a minimum length nozzle. The divergent part is to be designed using method of characteristics with left and right running characteristics using the known (sonic) values at the throat of the convergent-divergent nozzle. These characteristics would be pressure, density, velocity and temperature. A computer code has been designed to calculate the values of these characteristics and the end characteristics at the exit.
CFD analysis has also been carried out on the designed nozzle using commercial software ANSYS.

I. INTRODUCTION

A rocket engine nozzle typically consists of a dual bell-shaped design, also referred to as a convergent-divergent nozzle, as depicted in Figure 1. The convergent portion of the nozzle converges the flow, reducing pressure while increasing velocity from the combustion chamber to sonic conditions at the throat. Once the flow reaches sonic conditions, further velocity increase becomes unattainable, leading to a state known as choked flow. To enhance flow velocity, the nozzle incorporates a divergent section, where the flow expands, resulting in decreased pressure and increased velocity. This paper will primarily concentrate on the design aspects of the divergent section of the rocket engine nozzle.

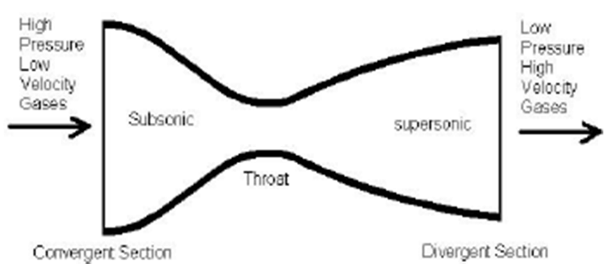

Figure 1: Dual bell shaped nozzle

The divergent segment of a nozzle can be divided into two sections: the expansion section and the straightening section. As the names imply, the flow gradually expands within the expansion section and initiates straightening within the straightening section. This configuration is illustrated in Figure 2.

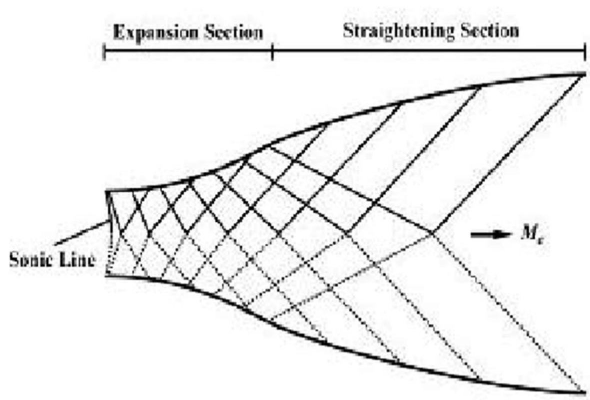

Figure 2: Gradual expansion nozzle(divergent section)

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Gradual expansion nozzles, due to their considerable size and weight, are primarily utilized in wind tunnels. To mitigate these drawbacks, a novel design concept known as the minimum length nozzle has emerged. These nozzles maintain the same area ratio (exit area to throat area) as gradual expansion nozzles, ensuring equivalent flow expansion. However, the principal difference lies in their length, as implied by the name. A minimum length nozzle eliminates the expansion section, allowing the flow to reach maximum expansion at a single point. This sharp corner at the nozzle's entry initiates initial expansion, followed by further straightening in the straightening section. At this point, the contour of the nozzle has maximum wall angle $\theta_{w,max}$. Due to this sharp corner at the throat, Prandtl-Meyer expansion takes place which is generally used to describe the expansion occurring when the flow turns supersonic.

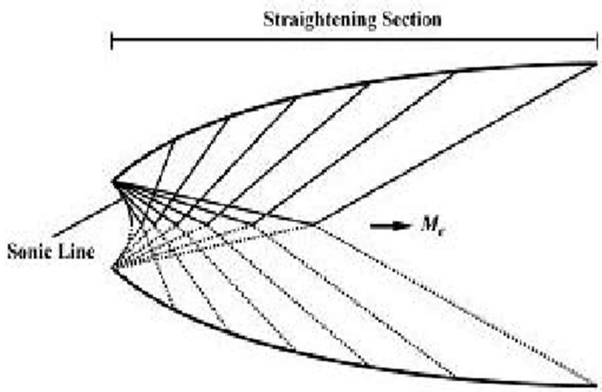

Figure 3: Minimum length nozzle

The core stage of the GSLV Mk - III rocket for a space launch in July 2019 utilized two Vikas engines, designed by LPSC. These engines inspired our design process. Employing the Method of Characteristics, we have designed the divergent section of a minimum length nozzle based on the conditions of the Vikas engine, aiming to minimize weight and size. Figure 4 illustrates this design.

II. GENERAL THEORY

The physical characteristics of a steady, two-dimensional rotational flow under isentropic conditions can be expressed through the nonlinear differential equation of a velocity potential. The method of characteristics offers a mathematical framework for determining solutions to the velocity potential, which must adhere to specified boundary conditions or transform the governing partial differential equations into ordinary ones. Conventionally, a supersonic nozzle is segmented into two distinct parts. The supersonic section operates independently of upstream conditions up to the sonic line, allowing for the separate analysis of the subsonic section. This latter portion is designed to achieve sonic flow at the throat. We propose a specific type of nozzle that ensures a parallel and uniform flow at the exit section. Termed the "minimum length nozzle with centered expansion," it minimizes length compared to other existing nozzle types.

Figure 4: Vikas engine

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A. Method of Characteristics

Characteristics represent the lines within a supersonic flow that are oriented in specific directions along which disturbances, mainly perturbations of pressure waves, propagate. The Method of Characteristics (MOC) serves as a numerical technique suited for solving problems related to two-dimensional compressible flows. Utilizing this approach, various flow properties such as direction and velocity can be computed at different points across a flow field.

In the vicinity immediately following the sonic line at the throat, where the flow diverges from itself, the airflow undergoes expansion, transitioning into supersonic velocities. This expansion occurs gradually across the initial expansion region. In the context of Prandtl-Meyer expansion, the expansion is envisioned to occur via a centered fan originating from a sharp corner. This phenomenon is commonly modelled as a continuous sequence of expansion waves, each altering the airflow's direction to an infinitesimal amount in accordance with the channel wall's contour.

Expansion waves can be considered as the converse of shock compression waves, which decelerate airflow. This phenomenon is dictated by the Prandtl-Meyer function,

$$d\theta = \pm\sqrt{M^2 - 1}\frac{dV}{v}$$

where the alteration in flow angle is denoted by $d\theta$.

Integrating the equation yields the parameter v known as the Prandtl-Meyer angle.

$$v(M) = \sqrt{\frac{\gamma + 1}{\gamma - 1}} tan^{-1} \sqrt{\frac{\gamma - 1}{\gamma + 1}(M^2 - 1)} - tan^{-1}\sqrt{M^2 - 1}$$

Within the Method of Characteristics (MOC) equations, the angle of flow relative to the horizontal axis is represented by θ. The Mach angle μ is formally defined as $\mu = sin^{-1}\left(\frac{1}{M}\right)$. These equations are depicted in the reference figure below.

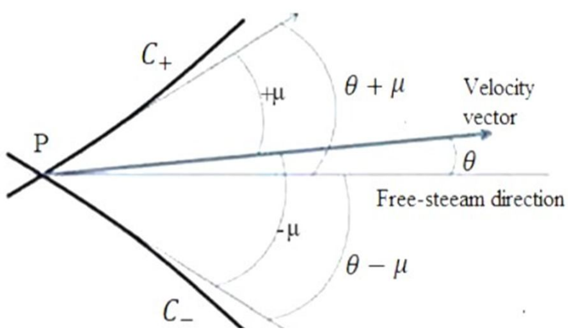

Figure 5: Characteristic lines schematic diagram

$$\left(\frac{dy}{dx}\right) = tan^{-1}(\theta - \alpha)$$
$$\left(\frac{dy}{dx}\right) = tan^{-1}(\theta + \alpha)$$

$$\theta + v(M) = K_- \ (along\ C - characteristic)$$
$$\theta - v(M) = K_+ \ (along\ C + characteristic)$$

K_-, K_+ are constants along their respective characteristic lines.

$$\theta = \frac{1}{2}(K_- + K_+)$$
$$v(M) = \frac{1}{2}(K_- - K_+)$$

B. Calculation Of Flow Parameters At Different Nodal Points For A Nozzle

The divergent part of a convergent-divergent nozzle can be designed according to the required flow parameters. The contour of its outer wall can be designed particularly knowing the location and slope at these points. Here, a glimpse of such calculation has been shown.

The calculations can be done for two different points on a nozzle, namely:

1) *Internal Grid Points:* These are the points inside a grid. By calculating values at these points, we can further calculate values at other points that lie downstream of these points.

Let us assume that two points 1 and 2 lie on the sonic line in the throat region of the nozzle. A third point 3 lies downstream of these two points, at the intersection of characteristics line C- and C+ originating from points 1 and 2 respectively. Since points 1 and 2 lie on the sonic line, we assume that we know the flow parameters at these points.

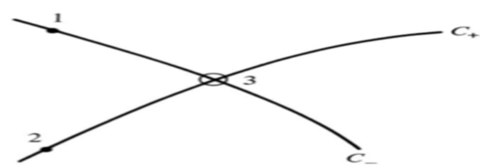

Figure 6: Schematic showing internal grid points

Since point 1 and 3 lie on the same characteristic line C-,

$$(K-)_1 = (K-)_3$$
$$\theta_1 + v(M_1) = \theta_3 + v(M_3)$$

Since point 2 and 3 lie on the same characteristic line C+,

$$(K+)_2 = (K+)_3$$
$$\theta_2 - v(M_2) = \theta_3 - v(M_3)$$
$$\theta_3 = \frac{1}{2}[(K-)_1 + (K+)_2]$$
$$v(M_3) = \frac{1}{2}[(K-)_1 - (K+)_2]$$

Flow parameters as calculated at point 3:

- Static pressure p_{o3} and temperature T_{o3}: These are constant throughout the flow for an inviscid adiabatic flow.
- Total pressure p_3 and temperature T_3: These can be found out from the table of isentropic flow properties. Then M_3 is found out for that value of p_3.
- Prandtl Meyer function $v(M_3)$: This can be found out from the table of Prandtl Meyer function vs Mach number for that value of M_3.
- Velocity V_3: The speed of sound at that temperature T_3 can be found. Further, velocity at point 3 can be found.

$$a_3 = \sqrt{\gamma R T_3}$$
$$V_3 = M_3 a_3$$

2) *Wall Points:* These are the points at the wall of the nozzle. The location of these points gives the contour of the nozzle.

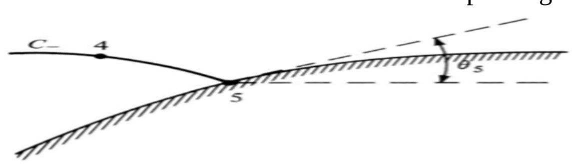

Figure 7: Schematic showing wall points

Let us assume that point 4 lies on a characteristic line C- and we have calculated the flow parameters at this point from the relations discussed earlier. Point 5 lies on the wall, downstream of point 4 at the same characteristics line C-. Since point 5 is a wall point, the slope at this point θ_5 is known. Hence, we can find the Prandtl Meyer function from the relation. All other parameters follow from these relations, as discussed earlier.

$$(K-)_4 = (K-)_5$$
$$\theta_4 + v(M_4) = \theta_5 + v(M_5)$$

III. DESIGN PROCESS AND PERFORMANCE ANALYSIS

A. Algorithm Design for Flow Parameters

Calculate total number of nodes by using n (number of characteristics) as done in [6],

total nodes = $\frac{n(n+2)}{2}$

Calculate the value θ, v, K-, K+ for each node

Calculate x, y co-ordinates of each node

Connect the outermost point located on the wall by using average slope formulae

Average slope between point of expansion fan and first wall point = $(\theta_A+\theta_i)/2$

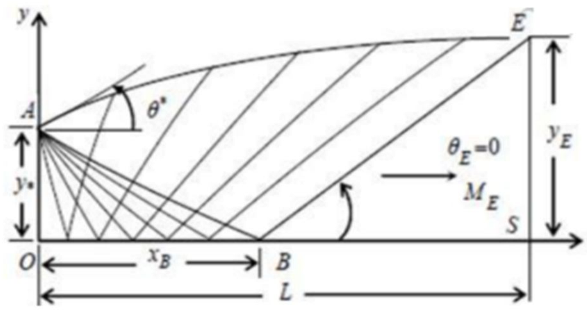

Figure 8: Schematic of minimum length nozzle with expansion fan

For nodes located on first right running characteristics line (C-)

First we divide the wall slope into equal parts as the number of characteristics lines,

$$\theta_{max} = n\,\Delta\theta$$
$$\theta_i = i\,\Delta\theta$$

For a wall point, $\theta_i = \theta_{i-1}$

At each node, $v = \theta$

$$(K\text{-})_i = v + \theta$$
$$(K+)_i = v - \theta$$

For nodes located on other right running characteristics lines

For nodal points located on centerline of nozzle i.e. the central axis

$$\theta_i = 0$$
$$(K\text{-})_i = (K\text{-})_{i-1}$$
$$v = (K\text{-})_i - \theta$$
$$(K+)_i = v - \theta$$

For nodes located between nozzle centerline and nozzle wall points

$$(K+)_i = (K+)_{i-1}$$
$$(K\text{-})_i = (K\text{-})_A$$
$$\theta_i = \frac{(C_+)_i + (C_-)_i}{2}$$
$$v_i = \frac{(C_+)_i - (C_-)_i}{2}$$

For nodal points located on the wall of the nozzle

$$\theta_i = \theta_{i-1}$$
$$(K\text{-})_i = (K\text{-})_{i-1}$$
$$v_i = v_{i-1}$$
$$(K+)_i = (K+)_{i-1}$$

For each node, values of θ, v, C-, C+ have been calculated. Now at these points we can calculate the following values as well:

M_i calculated from Prandtl-Meyer table for the value of v_i

Mach angle $\mu = \sin^{-1}(1/M)$

p and T calculated using isentropic flow tables

Speed of sound $a = \sqrt{\gamma RT}$

Relative velocity $V = a\,M$

Horizontal velocity $u = V\cos\theta$

Vertical velocity $v = V\sin\theta$

Values of average slope and Mach angle:

a) Along *(C+)* characteristic line

$$\theta_{avg} = \frac{1}{2}[\theta_{i-1} + \theta_i]$$

$$\mu_{avg} = \frac{1}{2}[\mu_{i-1} + \mu_i]$$

$$\frac{dy}{dx} = tan(\theta_{avg} + \mu_{avg})$$

b) Along *(C-)* characteristic line

$$\theta_{avg} = \frac{1}{2}[\theta_{i-1} + \theta_i]$$

$$\mu_{avg} = \frac{1}{2}[\mu_{i-1} + \mu_i]$$

$$\frac{dy}{dx} = tan(\theta_{avg} - \mu_{avg})$$

These values of $\frac{dy}{dx}$ will give the respective values for *x*- and *y*- co-ordinates of the nodal point. These values of *(x,y)* give the exact locations of the nodal points.

B. Boundary Conditions for the Design

The engine in consideration for our design is Vikas engine fueled by *UDMH* and uses N_2O_4 as oxidizer. The engine operates for about *130 s* at an altitude of *41 km*. The latest version of the engine is able to produce *800 kN* of vacuum thrust with a maximum chamber pressure of *5.85 MPa* and average chamber pressure of *3.99 MPa*. The engine produces a maximum specific impulse of *274 s* and average specific impulse of *227 s*. The area ratio (exit area to throat area) of the engine is *13.9* which is the basis of our calculation and design.

Equation of combustion:

$$(CH_3)_2N_2H_2 + 2\,N_2O_4 \rightarrow 4\,H_2O + 2\,CO_2 + 3\,N_2$$

Calculation for γ at *41 km* altitude:

C_p for each gas is found and interpolated for 41 km altitude as follows:

$$Cp\ (water) = 2.147\ kJ/kgK$$

$$Cp\ (CO_2) = 1.168\ kJ/kgK$$

$$Cp\ (nitrogen) = 1.122\ kJ/kgK$$

Further, C_v is calculated using the following relation:

$$Cv = Cp - R_0\ /\ MW$$

where R_o is the universal gas constant in $J{\cdot}mol^{-1}{\cdot}K^{-1}$ and MW is the molecular weight of the respective gas.

γ is then calculated using the relation given below where 4, 2 and 3 are the number of moles of gases 1, 2 and 3 respectively as discussed earlier in the chemical equation.

$$\gamma = \frac{4\,Cp_1 + 2\,C_{p_2} + 3\,Cp_3}{4\,Cv_1 + 2\,C_{v_2} + 3\,C_{v_3}} = 1.23$$

Calculation for T_{th}:

$$p_c = 58.5bar$$

$$T_c = 2500\ K$$

$$\frac{p_{th}}{p_c} = \left(\frac{2}{\gamma - 1}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{T_{th}}{T_c} = \left(\frac{p_{th}}{p_c}\right)^{\frac{\gamma-1}{\gamma}}$$

These equations are valid for throat and further values are calculated based on method of characteristics.
A MATLAB code has been designed to calculate the values as specified earlier.
Throat diameter = *0.4816 m*
Throat area = *0.182* m^2
Chamber pressure and temperature are considered to be total (freestream conditions). Further, throat pressure and temperature have been calculated from chamber pressure and temperature using the relations:

$$\frac{p_{th}}{p_0} = \left(1 + \frac{\gamma-1}{2}M^2\right)^{\frac{-\gamma}{\gamma-1}}$$

$$\frac{T_{th}}{T_0} = \left(1 + \frac{\gamma-1}{2}M^2\right)^{-1}$$

As we know, the velocity of flow at the throat reaches sonic conditions. So we take $M = 1$. The above relations reduce to:

$$\frac{p_{th}}{p_0} = \left(1 + \frac{\gamma-1}{2}\right)^{\frac{-\gamma}{\gamma-1}}$$

$$\frac{T_{th}}{T_0} = \left(1 + \frac{\gamma-1}{2}\right)^{-1}$$

Using the equations of average slope and Mach angle along the characteristics C+ and C-, we get the slope dy/dx of the line joining those two characteristic points.

$$\frac{dy}{dx} = tan(\theta_{avg} + \mu_{avg})$$

along characteristic line C+

$$\frac{dy}{dx} = tan(\theta_{avg} - \mu_{avg})$$

along characteristic line C-

The table below shows the co-ordinates *(x,y)* of the nozzle contour obtained from the MATLAB code.

X – Axis	Y -Axis (+ ve)	Y -Axis (- ve)
0	0.2408	-0.2408
0.1509	0.3069	-0.3069
0.4179	0.4199	-0.4199
0.4695	0.441	-0.441
0.5321	0.4657	-0.4657
0.5894	0.4875	-0.4875
0.645	0.5079	-0.5079
0.7002	0.5274	-0.5274
0.7557	0.5463	-0.5463
0.8122	0.5647	-0.5647
0.8701	0.5828	-0.5828
0.9296	0.6006	-0.6006
0.9911	0.6182	-0.6182
1.0549	0.6357	-0.6357
1.1211	0.6529	-0.6529
1.19	0.6699	-0.6699
1.2619	0.6868	-0.6868
1.3369	0.7035	-0.7035
1.4153	0.7199	-0.7199
1.4974	0.736	-0.736
1.5834	0.7518	-0.7518
1.6736	0.7673	-0.7673
1.7682	0.7823	-0.7823
1.8675	0.7969	-0.7969
1.9718	0.8109	-0.8109
2.0814	0.8242	-0.8242
2.1968	0.8369	-0.8369
2.3182	0.8487	-0.8487
2.446	0.8595	-0.8595
2.5806	0.8694	-0.8694
2.7225	0.878	-0.878
2.8721	0.8852	-0.8852
3.03	0.891	-0.891
3.1966	0.895	-0.895
3.3724	0.8972	-0.8972
3.5614	0.8983	-0.8983

The minimum length is plotted below using these points from the contour data.

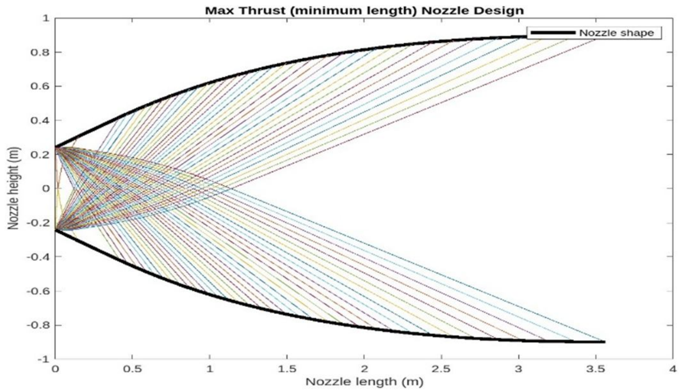

Figure 9: Minimum length nozzle contour and characteristics

The code also gave us the variation of Mach number and pressure ratio with respect to the length of the nozzle. This is plotted as below.

The plot shows us that the Mach number increases gradually and consistently from 1 at the nozzle throat up to 3.79 at the nozzle exit. The pressure ratio P_{exit}/P_c, on the other hand, decreases from 16.48 at the throat to 0.27 at the nozzle exit.

IV. CONTOUR PLOT AND ANSYS ANALYSIS

After knowing the co-ordinates of the contour, the same contour has been plotted in CREO Parametric. This 2D sketch of the nozzle contour has been used in ANSYS for 2D analysis of the flow to find Mach number and to verify it with the numerical approach from the MATLAB code.

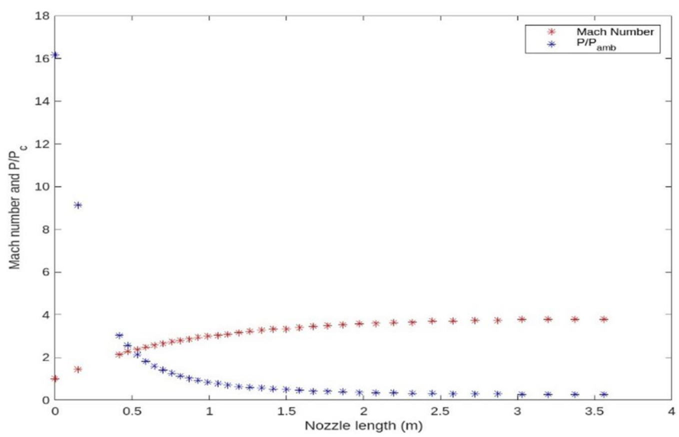

Figure 10: Mach number and P/Pc vs Nozzle length

First, we have a supersonic nozzle with a specified chamber pressure, temperature, force or mass flow rate at specific altitude. This nozzle is designed using the method of characteristics in MATLAB. So, an exit Mach number is calculated by using the isentropic flow relations. Then, based on that design nozzle wall to achieve the specified flow rate at the exit, geometry can be seen. Then for this simulation purpose, we use the two-dimensional geometry and only half of it because it is a symmetric flow. Now, we extend the domain so shock waves can be seen.

The boundary conditions taken for the nozzle flow are shown in the table below:

Chamber pressure	5.85 MPa
Chamber temperature	2500 K
Ambient pressure	101325 Pa
Ambient temperature	300 K

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The results of the simulation are shown below.

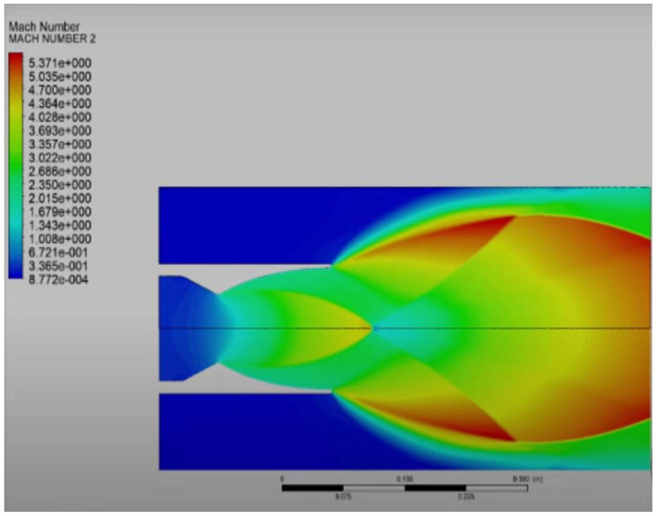

Figure 11: ANSYS simulation result

V. RESULT AND DISCUSSION

We have successfully applied the method of characteristics to generate the contour for a minimum length nozzle. The resulting nozzle points can be imported into any CAD environment for further refinement and manufacturing processes. It is highly recommended to use this method with an iterative design process to achieve the best results according to the input specifications. Additionally, it is crucial to select an optimal number of characteristics; the number should be sufficiently large to produce a smooth, bell-shaped contour using straight lines. The overall effectiveness of the method of characteristics can be enhanced by running multiple iterations of our algorithm with small variations in the input parameters. The nozzle design gave us *Mach 3.79* at the nozzle exit from the MATLAB code at an exit length of *3.56m.* The nozzle design when imported in a simulation software for 2D flow analysis gave us *Mach 3.693* at the nozzle exit, which is close the results obtained from the code as well. This helps us verify that our calculations have been precise and accurate in calculating the Mach no. obtained from the method of characteristics. The Mach no. produced by the flow at the nozzle exit expands further according to the Prandtl-Meyer expansion theory. Since there is a difference between the exit nozzle pressure and the ambient pressure at the higher altitude, this thrust increases further with increasing operating altitude. These results are in accordance with the original design as our design is able to produce thrust and impulse close to the original design of *725 kN.* Since there is not much change in the *y co-ordinates* of the nozzle design after nozzle length of *3*.03m, we can truncate the length of the nozzle to *3.03m* giving us the precise thrust and Mach no.

VI. CONCLUSION

The design of a minimum length supersonic nozzle focuses on minimizing its length by eliminating the expansion section. As the expansion section contracts, the overall length of the nozzle decreases. In design, the supersonic nozzle achieves minimum length due to the minimized expansion section, which contracts to a point at the throat's end.

Previously, it was noted that the nozzle is optimized for the desired exit Mach number. It's important to remember that the flow streamlines diverge from the axis and then converge back towards it in the diverging section. This divergence of streamlines occurs in the expansion section, and the angle of divergence depends on the local Mach number. Hence, the final local and maximum divergence angles crucially determine the exit Mach number.

Supersonic nozzles find applications in diverse fields such as rocket engine propulsion and wind tunnels, where they encounter intricate flow patterns. While this design approximation simplifies these complex flow patterns, the method of characteristics remains the most suitable for supersonic nozzle design. To address real-world conditions accurately, factors like viscous effects, changes in pressure difference concerning back pressure, heat conduction, and others must be considered.

The design presented here serves as a benchmark for comparison with similar nozzle designs under similar conditions. We achieved comparable Mach numbers and thrust while significantly reducing the nozzle's length. Maintaining the nozzle's area ratio ensures similar flow conditions and expansion as the original design.

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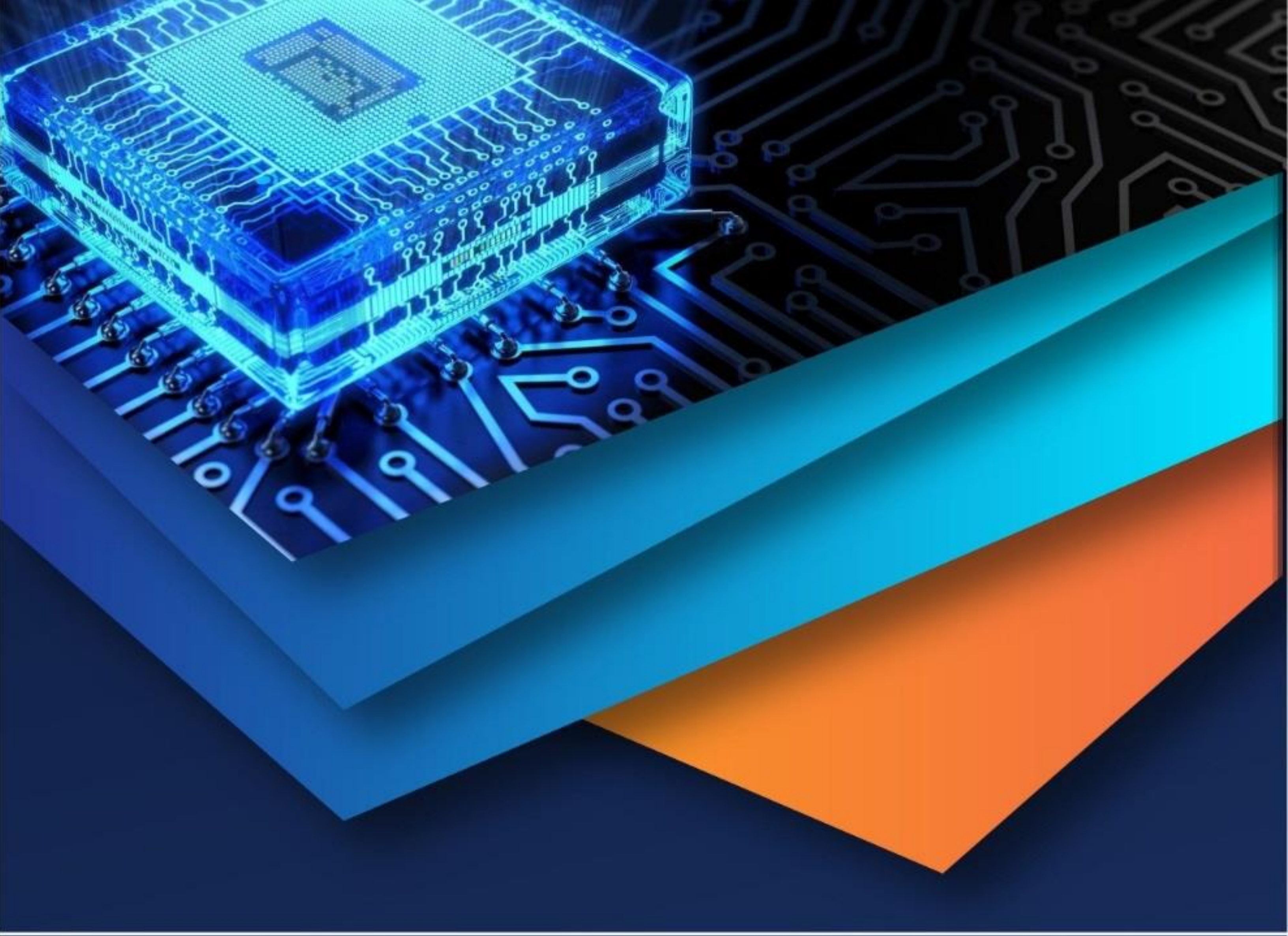

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