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Diophantine Triples Involving Octagonal Pyramidal Numbers

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***Abstract:** In this work, we strive for three particular polynomials with integer coefficients that may be expanded by non-zero values to the position where the product of any two numbers is a perfect square.*
***Keywords:** Diophantine triples, octagonal pyramidal number, triples, perfect square, pyramidal number.*

Notation
$p_n^8 = \frac{n(n+1)}{6}\ [6n-3]$, octagonal pyramidal number of rank n

I. INTRODUCTION

In number theory, a Diophantine equation is a polynomial equation with two or more unknowns that solely considers or searches for integer solutions [1-4]. The term "Diophantine" relates to the Greek mathematician Diophantus of Alexandria who, in the third century, pioneered the introduction of symbolism to variable-based mathematics and studied related problems. Numerous mathematicians have investigated the issue of the occurrence of Dio triples and quadruples with the property D(n) for any integer n as well as for any linear polynomial in n [5-8]. In this case, [9-16,18 &19] provides a full study of the many challenges on Diophantine triples. Triple sequences with a half companion sequences were examined in [19].
Our search for Diophantine triples utilising octagonal pyramidal numbers was prompted by these findings. This study tries to build Dio-Triples that satisfy the necessary property when the product of any two of the triple's members plus a non-zero integer or a polynomial with integer coefficients is added. The Diophantine triples from octagonal pyramidal numbers of various ranks are also presented in each of the three parts along with the relevant features.

II. BASIC DEFINITION

A set of three distinct polynomials with integer coefficients $(a_1, a_2, a_3)$is said to be **Diophantine triple** with property $D(n)$ if $a_i * a_j + n$ is a perfect square for all $1 \le i < j \le 3$, where n may be non zero or polynomial with integer coefficients.

III. METHOD OF ANALYSIS

A. Section-A

Construction of the Diophantine triples involving octagonal pyramidal number of rank n and $n-1$

Let $a = 6p_n^8$ and $b = 6p_{n-1}^8$ be octagonal pyramidal numbers of rank n and $n-1$ respectively. Then,

$ab + (-3n^4 - 24n^3 + 43n^2) = (6n^3 - 6n^2 - 4n)^2$

Hence , $ab + (-3n^4 - 24n^3 + 43n^2) = \alpha^2$ (say) (1)

$bc + (-3n^4 - 24n^3 + 43n^2) = \beta^2$ (2)

$ca + (-3n^4 - 24n^3 + 43n^2) = \gamma^2$ (3)

Solving (2) & (3)

$(b-a)(3n^4 + 24n^3 - 43n^2) = (a\beta^2 - b\gamma^2)$ (4)

Put, $\beta = x + by$ and $\gamma = x + ay$

Substituting β, γ in (4)

$x^2(a-b)-(a-b)aby^2=-(3n^4+24n^3-43n^2)(a-b)$

$x^2=aby^2-5n^5-4n^4+5n^3+20n^2-8n+1$

Put y = 1,

$x^2=36n^6-72n^5-12n^4+48n^3+16n^2$

$x=6n^3-56-4n$

Now, $\beta=x+by$

$\beta=12n^3-21n^2+5n$

From equation (2)

bc $-3n^4-24n^3+43n^2=(12n^3-21n^2+5n)^2$

c $=24n^3-24n^2-2n$

c $=4(6p_n^8)$ $-36n^2+14n$

Therefore, the triples

$\{a,b,c\}=\{6p_n^8,6p_{n-1}^8,4(6p_n^8)$ $-36n^2+14n\}$ is a Diophantine triples with the property $D(-3n^4-24n^3+43n^2)$

Some numerical examples are given below in the following table.

Table 1

S.No	n	(a,b,c)	$property$
1	0	(0,0,0)	0
2	1	(6,0,-2)	16
3	2	(54,6,92)	-68
4	3	(180,54,462)	-504
5	4	(420,180,1144)	-1616

B. Section -B

Construction of the Diophantine triples involving octagonal pyramidal number of rank n and $n-2$

Let $a=6p_n^8$ and $b=6p_{n-2}^8$ be octogonal pyramidal numbers of rank n and $n-2$ respectively.

Now,

$a=6p_n^8$ and $b=6p_{n-2}^8$

$ab+(6n^3+21n^2-90n+64)=(6n^3-15n^2+0n+8)^2$

$ab+(6n^3+21n^2-90n+64)=\alpha^2$ (say) (5)

Equation (5) is a perfect square.

$bc+(6n^3+21n^2-90n+64)=\beta^2$ (6)

$ca+(6n^3+21n^2-90n+64)=\gamma^2$ (7)

Solving (6) & (7)

(b-c) $(6n^3+21n^2-90n+64)=a\beta^2-b\gamma^2$ (8)

Put, $\beta=x+by$ and $\gamma=x+ay$,

Substituting β,γ in (8)

$x^2(a-b)-ab(a-b)y^2=(6n^3+21n^2+90n+64)(a-b)$

$x^2=aby^2+6n^3+21n^2+90n+64$

Put, y=1,

$x^2=36n^6-180n^5+225n^4+96n^3-240n^2+64$

$x = 6n^3 - 15n^2 + 8$

Now, , $\beta = x + by$

$\beta = 12n^3 - 48n^2 + 57n - 22$

From (6)

bc+ $6n^3 + 21n^2 + 90n + 64$ =$(12n^3 - 48n^2 + 57n - 22)^2$

$\Rightarrow$ c =$24n^3 - 60n^2 + 54n - 14$

$\Rightarrow$ c =4$(6p_n^8) - 72n^2 - 66n - 14$

Therefore, the triples

$\{a, b, c\} = \{6p_n^8$, $6p_{n-2}^8, 4(6p_n^8) - 72n^2 - 66n - 14\}$ is a Diophantine triples with the property $D(6n^3 + 21n^2 - 90n + 64)$.

Some numerical examples are given below in the following table.

TABLE 2

S.NO	n	(a, b, c)	property
1	0	(0, $-30, -14$)	64
2	1	(6,0,4)	1
3	2	(54,0,46)	16
4	3	(180,6,256)	145
5	4	(420,54,778)	424

C. Section -C

Construction of the Diophantine triples involving octagonal pyramidal number of rank n and $n - 3$

Let $a = 6p_n^8$ and $b = 6p_{n-3}^8$ be octogonal pyramidal numbers of rank n and $n - 3$ respectively.

Now,

$a = 6p_n^8$ and $b = 6p_{n-3}^8$

$$ab + (9n^4 - 126n^2 + 441) = (6n^3 - 24n^2 + 9n + 21)^2$$

So that

$ab + (9n^4 - 126n^2 + 441) = \alpha^2$ (say) (9)

Equation (9) is a perfect square.

$bc + (9n^4 - 126n^2 + 441) = \beta^2$ (10)

$ca + (9n^4 - 126n^2 + 441) = \gamma^2$ (11)

Solving (10) & (11)

(b—a) $(9n^4 - 126n^2 + 441) = a\beta^2 - b\gamma^2$ (12)

Put, $\beta = x + by$ and $\gamma = x + ay$,

Substituting β & γ in (12)

$(a - b)\, x^2 - ab(a - b)y^2 = -(5n^5 - n^4 - n^3 - 232n^2 + 140n - 64)(a - b)$

Put, y=1,

$x^2 = 36n^6 - 288n^5 + 684n^4 - 180n^3 - 927n^2 + 378n + 441$

$x = 6n^3 - 24n^2 + 9n + 21$

Now , $\beta = x + by$

$\beta = 12n^3 - 75n^2 + 150n - 105$

From,(10)

bc$+(9n^4-126n^2+441)=(12n^3-75n^2+150n-105)^2$

c $=24n^3-96n^2+156n-84$

c =4$(6p_n^8)-180n^2-108n-84$

Therefore, the triples, $\{a,b,c\}=\{6p_n^8,6p_{n-3}^8,4(6p_n^8)-180n^2-108n-84\}$ is a Diophantine triples with the property $D(9n^4-126n^2+441)$. Some numerical examples are given below in the following table.

Table 3

S.NO	n	(a,b,c)	$property$
1	0	(0,$-126,-84$)	441
2	1	(6,-30,0)	324
3	2	(54,0,36)	81
4	3	(110,0,168)	36
5	4	(420,6,540)	729

IV. CONCLUSION

We have shown the octagonal pyramidal number Diophantine triples. In conclusion, given various numbers with their corresponding attributes, one may search for triples or quadruples.

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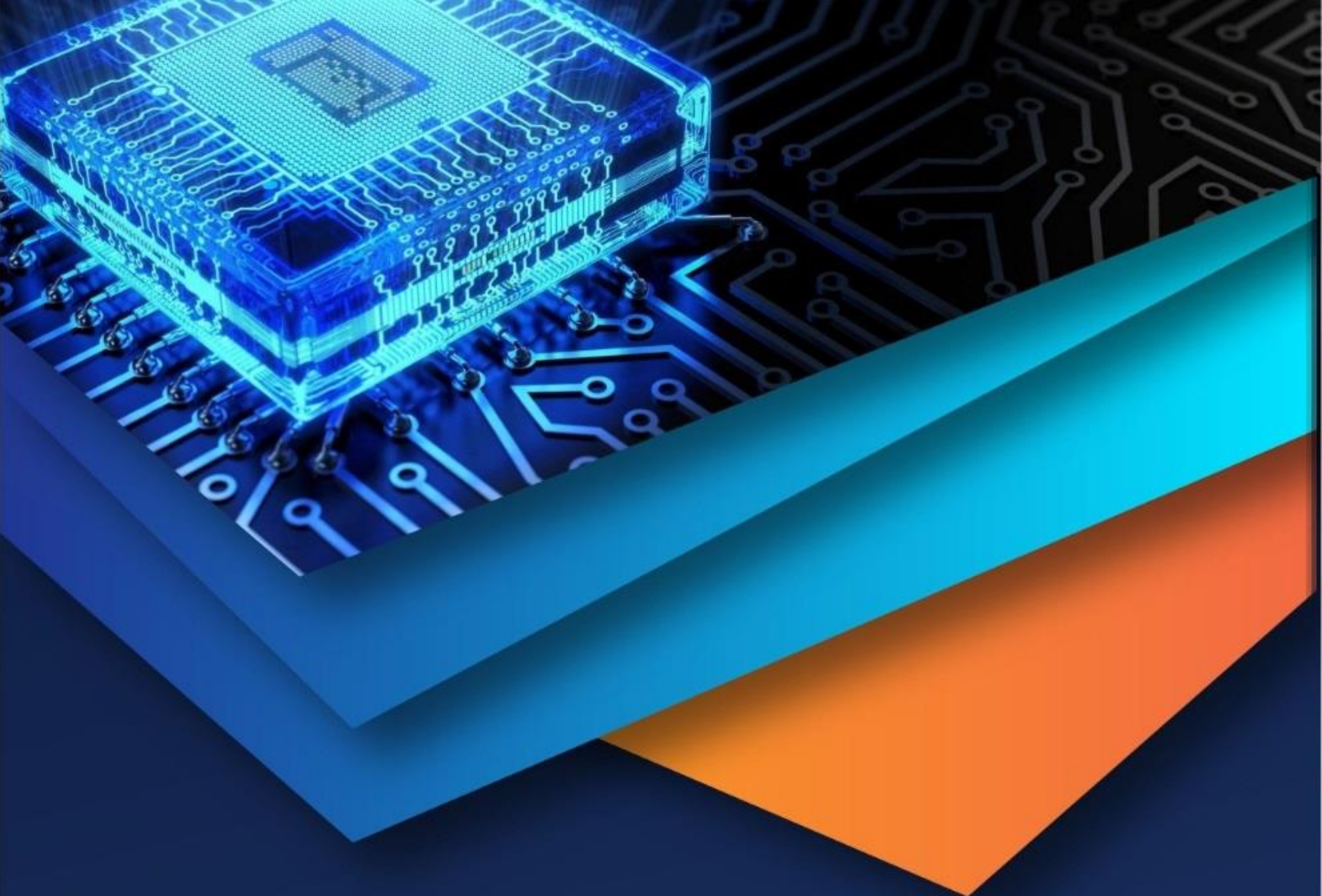

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