iJRASET
International Journal For Research in
Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH
IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 11 Issue: II Month of publication: February 2023

DOI: https://doi.org/10.22214/ijraset.2023.49122

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.538
Volume 11 Issue II Feb 2023- Available at www.ijraset.com

Diophantine Triples Involving Octagonal Pyramidal Numbers

C. Saranya[1], R. Janani[2]
[1]Assistant Professor, [2]PG student, PG & Research Department of Mathematics, Cauvery College for Women, (Autonomous) Trichy-18

Abstract: In this work, we strive for three particular polynomials with integer coefficients that may be expanded by non-zero values to the position where the product of any two numbers is a perfect square.
Keywords: Diophantine triples, octagonal pyramidal number, triples, perfect square, pyramidal number.

Notation

$p_n^8 = \frac{n(n+1)}{6}\ [6n-3]$, octagonal pyramidal number of rank n

I. INTRODUCTION

In number theory, a Diophantine equation is a polynomial equation with two or more unknowns that solely considers or searches for integer solutions [1-4]. The term "Diophantine" relates to the Greek mathematician Diophantus of Alexandria who, in the third century, pioneered the introduction of symbolism to variable-based mathematics and studied related problems. Numerous mathematicians have investigated the issue of the occurrence of Dio triples and quadruples with the property D(n) for any integer n as well as for any linear polynomial in n [5-8]. In this case, [9-16,18 &19] provides a full study of the many challenges on Diophantine triples. Triple sequences with a half companion sequences were examined in [19].

Our search for Diophantine triples utilising octagonal pyramidal numbers was prompted by these findings. This study tries to build Dio-Triples that satisfy the necessary property when the product of any two of the triple's members plus a non-zero integer or a polynomial with integer coefficients is added. The Diophantine triples from octagonal pyramidal numbers of various ranks are also presented in each of the three parts along with the relevant features.

II. BASIC DEFINITION

A set of three distinct polynomials with integer coefficients $(a_1, a_2, a_3)$is said to be **Diophantine triple** with property $D(n)$ if $a_i * a_j + n$ is a perfect square for all $1 \leq i < j \leq 3$, where n may be non zero or polynomial with integer coefficients.

III. METHOD OF ANALYSIS

A. Section-A

Construction of the Diophantine triples involving octagonal pyramidal number of rank n and $n-1$

Let $a = 6p_n^8$ and $b = 6p_{n-1}^8$ be octagonal pyramidal numbers of rank n and $n-1$ respectively. Then,

$ab + (-3n^4 - 24n^3 + 43n^2) = (6n^3 - 6n^2 - 4n)^2$

Hence , $ab + (-3n^4 - 24n^3 + 43n^2) = \alpha^2$ (say) (1)

$bc + (-3n^4 - 24n^3 + 43n^2) = \beta^2$ (2)

$ca + (-3n^4 - 24n^3 + 43n^2) = \gamma^2$ (3)

Solving (2) & (3)

$(b-a)(3n^4 + 24n^3 - 43n^2) = (a\beta^2 - b\gamma^2)$ (4)

Put, $\beta = x + by$ and $\gamma = x + ay$

Substituting β, γ in (4)

$x^2(a-b)-(a-b)aby^2=-(3n^4+24n^3-43n^2)(a-b)$

$x^2=aby^2-5n^5-4n^4+5n^3+20n^2-8n+1$

Put y = 1,

$x^2=36n^6-72n^5-12n^4+48n^3+16n^2$

$x=6n^3-56-4n$

Now, $\beta=x+by$

$\beta=12n^3-21n^2+5n$

From equation (2)

bc $-3n^4-24n^3+43n^2=(12n^3-21n^2+5n)^2$

c $=24n^3-24n^2-2n$

c $=4(6p_n^8)$ $-36n^2+14n$

Therefore, the triples

$\{a,b,c\}=\{6p_n^8,6p_{n-1}^8,4(6p_n^8)$ $-36n^2+14n\}$ is a Diophantine triples with the property $D(-3n^4-24n^3+43n^2)$

Some numerical examples are given below in the following table.

Table 1

S.No	n	(a,b,c)	$property$
1	0	(0,0,0)	0
2	1	(6,0,-2)	16
3	2	(54,6,92)	-68
4	3	(180,54,462)	-504
5	4	(420,180,1144)	-1616

B. Section -B

Construction of the Diophantine triples involving octagonal pyramidal number of rank n and $n-2$

Let $a=6p_n^8$ and $b=6p_{n-2}^8$ be octogonal pyramidal numbers of rank n and $n-2$ respectively.

Now,

$a=6p_n^8$ and $b=6p_{n-2}^8$

$ab+(6n^3+21n^2-90n+64)=(6n^3-15n^2+0n+8)^2$

$ab+(6n^3+21n^2-90n+64)=\alpha^2$ (say) (5)

Equation (5) is a perfect square.

$bc+(6n^3+21n^2-90n+64)=\beta^2$ (6)

$ca+(6n^3+21n^2-90n+64)=\gamma^2$ (7)

Solving (6) & (7)

(b-c) $(6n^3+21n^2-90n+64)=a\beta^2-b\gamma^2$ (8)

Put, $\beta=x+by$ and $\gamma=x+ay$,

Substituting β,γ in (8)

$x^2(a-b)-ab(a-b)y^2=(6n^3+21n^2+90n+64)(a-b)$

$x^2=aby^2+6n^3+21n^2+90n+64$

Put, y=1,

$x^2=36n^6-180n^5+225n^4+96n^3-240n^2+64$

$x = 6n^3 - 15n^2 + 8$

Now, , $\beta = x + by$

$\beta = 12n^3 - 48n^2 + 57n - 22$

From (6)

bc+ $6n^3 + 21n^2 + 90n + 64$ =$(12n^3 - 48n^2 + 57n - 22)^2$

$\Rightarrow$ c =$24n^3 - 60n^2 + 54n - 14$

$\Rightarrow$ c =4$(6p_n^8) - 72n^2 - 66n - 14$

Therefore, the triples

$\{a, b, c\} = \{6p_n^8$, $6p_{n-2}^8, 4(6p_n^8) - 72n^2 - 66n - 14\}$ is a Diophantine triples with the property $D(6n^3 + 21n^2 - 90n + 64)$.

Some numerical examples are given below in the following table.

TABLE 2

S.NO	n	(a, b, c)	property
1	0	(0, $-30, -14$)	64
2	1	(6,0,4)	1
3	2	(54,0,46)	16
4	3	(180,6,256)	145
5	4	(420,54,778)	424

C. Section -C

Construction of the Diophantine triples involving octagonal pyramidal number of rank n and $n - 3$

Let $a = 6p_n^8$ and $b = 6p_{n-3}^8$ be octogonal pyramidal numbers of rank n and $n - 3$ respectively.

Now,

$a = 6p_n^8$ and $b = 6p_{n-3}^8$

$$ab + (9n^4 - 126n^2 + 441) = (6n^3 - 24n^2 + 9n + 21)^2$$

So that

$ab + (9n^4 - 126n^2 + 441) = \alpha^2$ (say) (9)

Equation (9) is a perfect square.

$bc + (9n^4 - 126n^2 + 441) = \beta^2$ (10)

$ca + (9n^4 - 126n^2 + 441) = \gamma^2$ (11)

Solving (10) & (11)

(b—a) $(9n^4 - 126n^2 + 441) = a\beta^2 - b\gamma^2$ (12)

Put, $\beta = x + by$ and $\gamma = x + ay$,

Substituting β & γ in (12)

$(a - b)\, x^2 - ab(a - b)y^2 = -(5n^5 - n^4 - n^3 - 232n^2 + 140n - 64)(a - b)$

Put, y=1,

$x^2 = 36n^6 - 288n^5 + 684n^4 - 180n^3 - 927n^2 + 378n + 441$

$x = 6n^3 - 24n^2 + 9n + 21$

Now , $\beta = x + by$

$\beta = 12n^3 - 75n^2 + 150n - 105$

From,(10)

bc$+(9n^4-126n^2+441)=(12n^3-75n^2+150n-105)^2$

c $=24n^3-96n^2+156n-84$

$c=4(6p_n^8)-180n^2-108n-84$

Therefore, the triples, $\{a,b,c\}=\{6p_n^8, 6p_{n-3}^8, 4(6p_n^8)-180n^2-108n-84\}$ is a Diophantine triples with the property $D(9n^4-126n^2+441)$. Some numerical examples are given below in the following table.

Table 3

S.NO	n	(a,b,c)	$property$
1	0	(0,−126,−84)	441
2	1	(6,−30,0)	324
3	2	(54,0,36)	81
4	3	(110,0,168)	36
5	4	(420,6,540)	729

IV. CONCLUSION

We have shown the octagonal pyramidal number Diophantine triples. In conclusion, given various numbers with their corresponding attributes, one may search for triples or quadruples.

REFERENCES

[1] Beardon, A. F. and Deshpande, M. N. “Diophantine triples”, The Mathematical Gazette, vol.86, pp. 258-260, 2002.

[2] Bugeaud, Y. Dujella, A. and Mignotte, M. “On the family of Diophantine triples” {K – 1, K + 1, 16K^3 $-$ 4K}, Glasgow Math. J. Vol. 49, pp.333 - 344, 2007.

[3] Carmichael, R. D. “Theory of numbers and Diophantine analysis”, Dover Publications.

[4] Deshpande, M. N. “Families of Diophantine triples”, Bulletin of the Marathwada Mathematical Society, Vol. 4, pp. 19 - 21. 2003.

[5] Fujita, Y. “The extendability of Diphantine pairs {k –1, k +1} ”, Journal of Number.

[6] Gopalan, M.A. and Srividhya, G. Two Special Diophantine Triples, Diophantus Journal of Mathematics, Vol, 1 (2012), (accepted).

[7] Gopalan, M.A. and Pandichelvi, V. “On the extendability of the Diophantine triple involving Jacobsthal numbers (, 3,2 3) J 2n–1 J 2n+1 – J 2n + J 2n–1 + J 2n+1 – ”, International Journal of Mathematics & Applications, 2(1), 1-3, 2009. 31.

[8] Hua, L.K. “Introduction to the Theory of Numbers”, Springer-Verlag, Berlin-New york, 1982.

[9] Janaki, G. and Vidhya, S. “Construction of the Diophantine triple involving stella octangular number, Journal of Mathematics and Informatics, vol.10, Special issue, 89-93, Dec 2017.

[10] Janaki, G. and Saranya, C. “Construction of The Diophantine Triple involving Pentatope Number”, International Journal for Research in Applied Science & Engineering Technology, vol.6, Issue III, March 2018.

[11] Janaki, G. and Saranya, C. “Special Dio 3-tuples for pentatope number”, Journal of Mathematics and Informatics, vol.11, Special issue, 119-123, Dec 2017.

[12] “Some Non-Extendable Diophantine Triples Involving Centered Square Number”, International journal of scientific research in mathematical and statistical , Volume 6, Issue 6, Pg. No. 109-107, December 2019.

[13] Saranya, C and Achya, B, “Special Diophantine triples involving square pyramidal number”, Indian journal of advanced mathematics, volume 1,issue 2,October 2021.

[14] Saranya, C and Achya, B., “Diophantine triples Involving Square Pyramidal Numbers”, Advances and Applications in mathematical sciences , Volume 21,Issue 3, pg. No.1541-1547, January 2022.

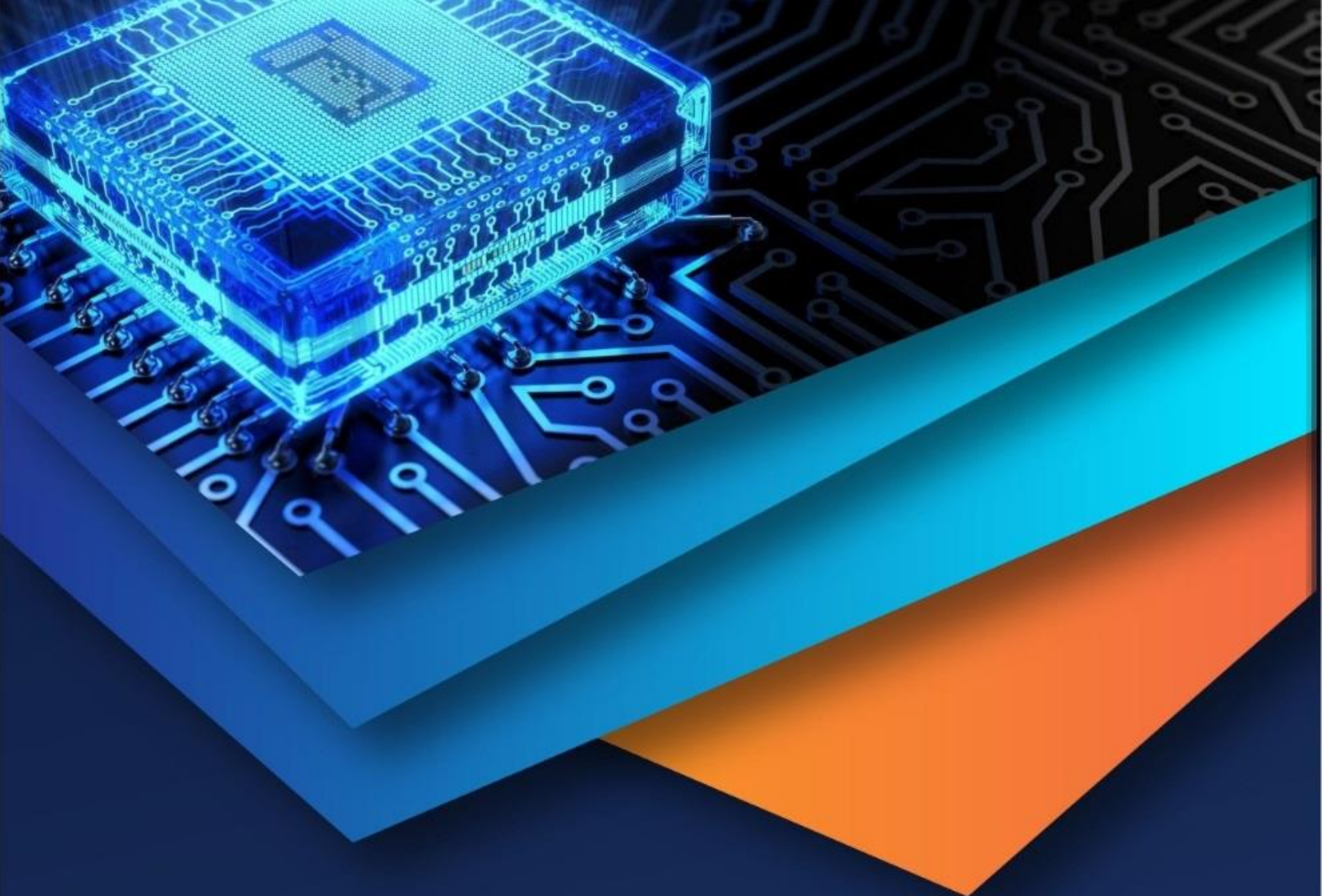
[15] Saranya, C and Manjula, K., “Construction of Diophantine triples involving hexagonal Saranya, C and Janaki, G., pyramidal numbers”, International journal science research in mathematics and statistical sciences, volume 9, issue 4, 2022.

[16] Saranya, C and Achya, B, “Special Dio 3-Tuples Involving Square Pyramidal Numbers”, International Journal for Research in Applied Science & Engineering Technology, vol.10, Issue III, March 2022.

[17] Saranya, C and Janaki, G., “Half companion sequences of special dio 3-tuples involving centered square number”, International journal for recent technology and engineering, volume 8, issue 3, pages 3843-3845, 2019.

[18] Vidhya S, Janaki G., “Special Dio 3-tuples for Pronic number-I”, International Journal for Research in Applied Science and Engineering Technology, 5(XI), 159-162, 2017.

[19] Janaki G, Vidhya S., “Special Dio 3-tuples for Pronic number-II”, International Journal of Advanced Science and Research, 2(6), 8-12, 2017.



10.22214/IJRASET

45.98

IMPACT FACTOR: 7.129

IMPACT FACTOR: 7.429

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY