iJRASET
International Journal For Research in
Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 11 **Issue:** I **Month of publication:** January 2023

DOI: https://doi.org/10.22214/ijraset.2023.48577

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.538
Volume 11 Issue I Jan 2023- Available at www.ijraset.com

Estimation Methods of the Parameters in Fuzzy Pareto Distribution

Pullooru Bhavana[1], Dr. D. Kalpana Priya[2]
[1, 2]Department of Mathematics, VIT University, Vellore, Tamil Nadu, India

Abstract: In this paper considered the estimation of the parameters in the Fuzzy Pareto Distribution of two parameters. Now we used the Method of moments, Method of Maximum likelihood, and Method of least squares. From this it seemed to establish the consistent parameters when the sample size is increased. This paper aims at sample size increased when the parameters are consistent.
Keywords: Fuzzy Pareto Distribution, Method of Moments, MLE, Ridge Regression, Consistent.

I. INTRODUCTION

The pareto Distribution was first of all invented for model for distribution of incomes. Now a days it is one of statistical distribution where the having great future. It also used as a model for the distribution of city population within a given area [1].

The pareto distribution is a power law probability distribution. It is used in a model of social, scientific, geophysical and actuarial science. It mostly applied in area of economics, trade, business, social science and meteorology of some real appliances [2] [3] [4]. The pareto distribution is a heavy tailed distribution. The shape parameter is sufficiently large means the mean,all variance and other moments are finite. Now generalized by adding a scale parameter and x takes from in interval [τ,∞). The pareto distribution with shape parameter τ and scale parameter ς.

The cdf is $F(x) = 1-(\frac{\varsigma}{x})^{\tau}$, τ ≥0, ς >0 and 0<τ≤x

And the pdf is $(x) = \frac{\tau\, \varsigma^{\tau}}{x^{\tau+1}}$, x € [τ,∞) (see Pareto, 1965) [5].,

A. *Fuzzy Pareto Distribution*

In real data analysis the data is not provide exact values but it provides in imprecise information. So we necessarily to take fuzzy concepts in statistical inference to deal of the lack of precision of data. Now a days, it extremely used in all areas and many papers on generalization classical statistical methods to analysis of fuzzy data have been published.

Some. Maximum likelihood estimation of exponential model using type-II fuzzy censored data is considered by Khoolenjani and Shahsanaie [6], The inferential estimation in the Weibull and exponential distribution on fuzzy data by Pak and others [7],[8[, Makhdoom et., al., explained the Bayesian estimation of parameter of exponential distribution under type-II censoring from fuzzy data [9], and estimating the parameters of lomax distribution from imprecise information studied by Abbas Pak [10].

Now in our knowledge generate the two parameters in pareto distribution on fuzzy data by using the different methods.

The main aim of this paper is to estimate the two parameters in fuzzy pareto distribution. In section 2. for this we take the data on triangular fuzzy number and apply om different α-cut values to estimate the consistent parameters and explain about different methods i.e., method of moments, MLE, method of least squares and ridge regression. In section 3, we provide a comparison of all estimation procedures developed in this paper by goodness of fits i.e., Mean Square Error and Total Deviation often used to get the accuracy of result. In section 4, we take a real data with 1000 random samples with different α-cuts on different samples size. When the sample size increased then the we reach the consistent of parameters. Lastly, in section 5, conclusion and recommendation and also future improvements are provided.

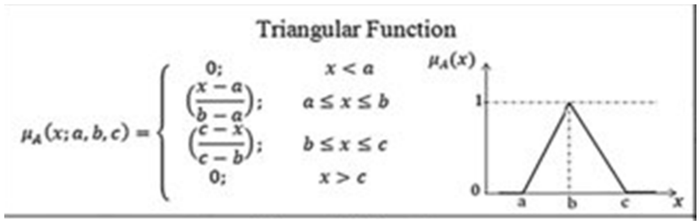
.

II. METHOD OF ANALYSIS

A. *α-cut values on Fuzzy Pareto Distribution*

The estimating of two parameters in fuzzy pareto distribution, can be defined by using the triangular fuzzy number (a1,a2,a3) this representation is interpreted as membership function.

ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.538
Volume 11 Issue I Jan 2023- Available at www.ijraset.com



now get crisp interval by α-cut operation, interval A_α shall be obtained as follows ∀ α € [0,1].

From $\frac{a(\alpha)-a}{b-a} = \alpha$ and $\frac{c-c(\alpha)}{c-b} = \alpha$

we get, $a^{(\alpha)} = a+(b-a)\alpha$ and $c^{(\alpha)} = c-(c-a)\alpha$ thus $A_\alpha = [a^{(\alpha)}, c^{(\alpha)}] = [a+(b-a)\alpha,\ c-(c-b)\alpha]$.
(see George J.Klir Bo Yuan)[10].
The pareto distribution exhibits the α-cut values and calculate on imprecise data to compare the effect on leptin, [12]. Now we calculate the two parameters in fuzzy pareto distribution by using the following methods.

B. *Method of Moments [MOM]*

By the kth moment of pareto distribution

$$E(x) = \int_{\alpha}^{\infty} x f(x)\,dx$$

We estimate τ by equating mean $\bar{x}$, $\hat{\tau} = \frac{\bar{x}}{\bar{x}-\hat{\varsigma}}$, where $\hat{\varsigma}$ is some estimate of ς.

The estimation ς from the samples, i.e., the probability all n samples are greater than x is $\left(\frac{\varsigma}{x}\right)^{n\tau}$

Therefore, the probability that the lowest sample value is greater than x. Thus probability distribution of lowest sample value is $G(x) = 1 - \left(\frac{\varsigma}{x}\right)^{n\tau}$

The corresponding density function is $g(x) = 1 - \left(\frac{\varsigma}{x}\right)^{n\tau}$

And expected value or lowest sample observation is $\int g(x) = \int_{\varsigma}^{\infty} \frac{\varsigma^{n\tau}(n\tau)}{x^{n\tau}}\,dx$

$$= \frac{n\tau\varsigma}{\tau-1}$$

Equating with lowest sample mean x_0, $\hat{\varsigma} = \frac{(n\tau-1)\,x_0}{n\tau}$

The procedure we obtain see Quandts(1964)[13]

Therefore, we obtain the method of moments estimates as

$\hat{\varsigma} = \frac{(n\hat{\tau}-1)x_0}{n\tau}$ and $\hat{\tau} = \frac{n\bar{x}-x_0}{n(\bar{x}-x_0)}$ where x_0 is the minimum value and $\bar{x}$ is the mean. (see Akpan)[14]

Now we obtain the estimates of the two parameters in the fuzzy pareto distribution by using α-cut,

the scale parameter as $\tilde{\varsigma}(x) = [\hat{\varsigma}_l(x), \hat{\varsigma}_r(x)]$ where $\tilde{\varsigma}_l(x) = \frac{(n\hat{\tau}_l(x)-1)x_{0(l)}}{n\hat{\tau}_l(x)}$ and $\tilde{\varsigma}_r(x) = \frac{(n\hat{\tau}_r(x)-1)x_{0(r)}}{n\hat{\tau}_r(x)}$

And the shape parameter as $\tilde{\tau}(x) = [\tilde{\tau}_l(x), \tilde{\tau}_r(x)]$ where $\hat{\tau}_l(x) = \frac{n\,\tilde{\bar{x}}_l - x_{0(l)}}{n(\tilde{\bar{x}}_l - x_{0(l)})}$ and $\hat{\tau}_r(x) = \frac{n\,\tilde{\bar{x}}_r - x_{0(r)}}{n(\tilde{\bar{x}}_r - x_{0(r)})}$

C. *Method of Maximum Likelihood (MLE)*

Let the random variables $x_1, x_2, x_3, \ldots\ldots\ldots\ldots x_n$ obey the pareto distribution, the likelihood is denoted by

L=L(x,ς,τ) for sample is $L = L(\bar{x}, \bar{\varsigma}, \bar{\tau}) = \frac{\varsigma^{b\tau}\,\tau^{n}}{(\Pi x_i)^{\tau+1}}$

By taking natural log,

$\text{Ln } L = \ln(\varsigma^{b\tau}\,\tau^{n}) - \sum \ln (x_i)^{\tau+1}$

$\quad = n\tau \ln \varsigma + n \ln \tau - (\tau+1) \ln x_i$

To obtain estimate, we differentiate wrt to ς and τ and equate it to zero, we get,

$$\frac{\partial}{\partial\tau}(lnL) = nln\varsigma + \frac{n}{\tau} - \sum ln\, x_i = 0$$

$$\frac{n}{\tau} = \sum ln\, x_i - nln\varsigma$$

Thus, $\tilde{\tau} = \frac{n}{\sum(lnx_i - ln\varsigma)} = \frac{n}{\sum ln(\frac{x_i}{\varsigma})}$

Again we partial differentiate wrt ς but it is not bounded.

Since ς is lower bound of x, we may maximize L subject to constraint $\hat{\varsigma} \leq \min x_i$ (See [14] and [13])

Now we take the parameters by α-cuts in fuzzy pareto distribution, the scale parameter as $\tilde{\varsigma}(x) = [\tilde{\varsigma}_l(x), \tilde{\varsigma}_r(x)]$

where $\tilde{\varsigma}_l(x) = min\overline{x_{i(l)}}$ and $\tilde{\varsigma}_r(x) = min\overline{x_{i(r)}}$,

and the shape parameter as $\tilde{\tau}(x) = [\tilde{\tau}_l(x), \tilde{\tau}_r(x)]$ where

$\tilde{\tau}_l(x) = \frac{n}{\sum ln\frac{\tilde{x}_{i\,(l)}}{\tilde{\varsigma}_l}}$ and $\tilde{\tau}_r(x) = \frac{n}{\sum ln\frac{\tilde{x}_{i\,(r)}}{\tilde{\varsigma}_r}}$. (see [12])

D. Least Square Method (LSE)

The cdf of pareto distribution $F(x) = 1 - (\frac{\varsigma}{x})^{\tau}$,

$$1 - F(x) = (\frac{\varsigma}{x})^{\tau}$$

Taking log on both sides, we get

$$Ln\,(1-F(x)) = \tau \ln\varsigma - \tau \ln x$$

The equation can be written in the form of $Y_i = A + BX_i$, where $Y_i = \ln(1-F(x))$, $A = \tau \ln\varsigma$, $B = \tau$ and $X_i = \ln x_i$

The method of least squares estimates is given by

$B = \frac{\sum lnx_i \sum ln(1-F(x)) - n\sum lnx_i ln(1-F(x))}{(\sum lnx_i)^2 - n\sum lnx_i{}^2}$ and $A = \frac{1}{n}\sum ln\big(1 - F(x)\big) - \frac{1}{n}B\sum lnx_i$

Here F(x) is obtained by using the median rank method also known as $F(x) = \frac{i-0.3}{N+0.4}$ (small to large assign The ranks).

Once obtained A and B the values of $\hat{\tau}$ and $\hat{\varsigma}$ can easily obtained.(see[18]).

Now we take the parameters by α-cuts in fuzzy pareto distribution, the scale parameter as $\tilde{\varsigma}(x) = [\tilde{\varsigma}_l(x), \tilde{\varsigma}_r(x)]$

where $\tilde{\varsigma}_l(x) = \tilde{A}_l$ and $\tilde{B}_l$ and $\tilde{\varsigma}_r(x) = \tilde{A}_r$ and $\tilde{B}_r$

and the shape parameter as $\tilde{\tau}(x) = [\tilde{\tau}_l(x), \tilde{\tau}_r(x)]$ where $\tilde{\tau}_l(x) = \tilde{A}_l$ and $\tilde{B}_l$ and $\tilde{\tau}_r(x) = \tilde{A}_r$ and $\tilde{B}_r$. (see[12])

III. GOODNESS OF FIT TESTS

The uses of the method of the MSE and TD as the goodness of fit test referred by Al-Fawazan[17].

A. Mean Square Error (MSE)

The MSE can be calculated using the formula below,

$MSE = \frac{\sum[\tilde{F}(x_i) - F(x_i)]^2}{N}$ where $\tilde{F}(x)$ is the value of the cumulative distribution of two parameters in fuzzy pareto Distribution using estimated parameters and F(x) is empirical cumulative distribution function (see15).

$$F(x) = 1 - (\frac{\varsigma}{x})^{\tau}$$

B. Total Deviation (TD)

The total deviation can be calculated for each method as

$TD = \left|\frac{\hat{\tau} - \tilde{\tau}}{\tilde{\tau}}\right| + \left|\frac{\hat{\varsigma} - \tilde{\varsigma}}{\tilde{\varsigma}}\right|$ where $\hat{\tau}$, $\hat{\varsigma}$ are estimated parameters and $\tilde{\tau}$, $\tilde{\varsigma}$ are the known parameters.

Here the best fitting method estimating efficient parameter by selected as minimum total absolute deviation.

IV. RESULTS AND DISCUSSIONS

A stimulated data from R package is used for evaluating the performance of the proposed estimation method of the Fuzzy pareto distribution. In this stimulation 1000 random numbers with pair of (1,1), (1,2), (3,2) follows Fuzzy pareto distribution of shape and scale (τ,ς). We generated different α-cuts by taking of 0.2, 0.5, 0.8 and Also generated different samples of sizes 20, 50, 100. Now we have to compare the values of the results.

Table 1 Estimation of two parameters in Fuzzy Pareto Distribution when α=0.2

Sample size	Method	True value				Estimated Value				MSE		TD	
		Shape τ		Scale ς		Shape $\hat{\tau}$		Scale $\hat{\varsigma}$		Left	Right	Left	Right
n = 20	MME	1	1.100674	1	1.059405	1.100674	1.100674	1.059405	1.059405	0.00057	0.000297	0	0
		1	1.026073	2	2.044494	1.025102	1.027043	1.968297	2.120699	0.00018	0.000729	-0.03821569	0.03821863
		3	3.28938	2	1.985503	3.198538	3.380221	1.905883	2.065143	0.0100452	0.00083769	-0.06771742	0.06772719
	MLE	1	0.840856	1	1.10982	0.8139065	0.8669906	1.02982	1.18982	0.00442	0.01456825	-0.10413331	0.10316531
		1	0.718887	2	2.149225	0.7081906	0.7294083	2.069225	2.229225	0.0096528	0.01467114	-0.05210128	0.05185883
		3	3.395812	2	2.016149	3.290835	3.500406	1.936149	2.096149	0.0065716	0.002312	-0.07059327	0.07048049
	LSE	1	0.684609	1	0.889243	0.6746142	0.694218	0.8405306	0.9379107	0.004625	0.006848	-0.06937776	0.06876632
		1	0.574142	2	1.606164	0.5704749	0.5777044	1.5622261	1.649883	0.0111843	0.01108699	-0.03374244	0.03342471
		3	2.935311	2	1.947634	2.871457	2.998733	1.87424	2.021192	0.0098092	0.00026612	-0.05943741	0.05937445
n = 50	MME	1	1.093941	1	1.014162	1.086666	1.101216	0.9355072	1.092833	0.0030403	0.00028778	-0.08420671	0.08422269
		1	1.012522	2	1.963107	1.012022	1.013022	1.884668	2.041547	0.001437	0.00002081	-0.04045038	0.04045088
		3	3.823519	2	2.045391	3.713661	3.933375	1.965503	2.125284	0.006492	0.00159	-0.06778974	0.06779166
	MLE	1	0.826512	1	1.033048	0.7983855	0.8537496	0.953048	1.113048	0.0019918	0.00194082	-0.11147192	0.1103947
		1	0.897372	2	2.002665	0.8816463	0.9128158	1.92265	2.082665	0.000644	0.000466	-0.05747845	0.05715679
		3	3.890774	2	2.056146	3.772395	4.008842	1.976146	2.136146	0.005516	0.001984	-0.06933331	0.06925338
	LSE	1	0.740781	1	0.917604	0.7276524	0.7533642	0.8624853	0.9725698	0.0038262	0.00068987	-0.07779135	0.07688717
		1	0.601435	2	1.188459	0.5985982	0.6041705	1.1549016	1.2218705	0.020515	0.00211	-0.03295203	0.03266233
		3	3.398647	2	1.990344	3.308467	3.4885	1.913235	2.067552	0.011225	0.000348	-0.06527563	0.06522916
n = 100	MME	1	1.374676	1	0.991325	1.34488	1.404473	0.9121898	1.070496	0.016608	0.005371	-0.10150245	0.10153969
		1	1.151355	2	1.965729	1.145302	1.157408	1.886942	2.044528	0.004561	0.000847	-0.04533758	0.04534368
		3	3.22748	2	1.996395	3.138046	3.315399	1.91655	2.076242	0.007685	0.00213	-0.06770475	0.06723635
	MLE	1	1.090722	1	1.00596	1.04225	1.138389	0.92596	1.08596	0.002819	0.001014	-0.12396631	0.12322827
		1	1.221109	2	2.000479	1.195561	1.246319	1.920479	2.080479	0.004488	0.00176	-0.06091239	0.06063559
		3	3.215765	2	2.008843	3.116926	3.31293	1.928843	2.088843	0.005604	0.003262	-0.07055968	0.07003912
	LSE	1	1.296832	1	1.171718	1.259135	1.333043	1.100056	1.242798	0.003977	0.011795	-0.0902283	0.08858572
		1	0.991196	2	1.669206	0.9814216	1.000729	1.6145822	1.72378	0.015351	0.007485	-0.04258578	0.04231212
		3	3.18185	2	2.007765	3.102995	3.257935	1.93156	2.083872	0.005098	0.003022	-0.06273789	0.06181852

Table 2 Estimation of two parameters in Fuzzy Pareto Distribution when α=0.5

Sample size	Method	True value				Estimated Value				MLE		TD	
		Shape τ		Scale ς		Shape $\tilde{\tau}$		Scale $\tilde{\varsigma}$		Left	Right	Left	Right
n = 20	MME	1	1.10067	1	1.059405	1.096138	1.10521	1.011477	1.107349	0.0004257	0.001384993	-0.0493616	0.049376703
		1	1.02607	2	2.044494	1.025466	1.026679	1.99687	2.092121	0.0000464	0.000406902	-0.02388536	0.023885852
		3	3.28938	2	1.985503	3.232603	3.346155	1.935738	2.035276	0.0056753	0.000326455	-0.04232488	0.042328298
	MLE	1	0.84086	1	1.10982	0.824114	0.85728	1.05982	1.15982	0.0058069	0.01222953	-0.06496259	0.064585776
		1	0.71889	2	2.149225	0.712223	0.725483	2.099225	2.199225	0.0104372	0.013587369	-0.03253395	0.03243936
		3	3.39581	2	2.016149	3.330247	3.461225	1.966149	2.066149	0.0032894	0.001018344	-0.04410736	0.044062599
	LSE	1	0.68461	1	0.889243	0.67841	0.690657	0.8588048	0.919664	0.004622	0.006074165	-0.04328269	0.043044438
		1	0.57414	2	1.606164	0.571863	0.57638	1.5787293	1.633513	0.0110238	0.010974727	-0.02105052	0.020926513
		3	2.93531	2	1.947634	2.895455	2.974998	1.901743	1.993589	0.0055955	#########	-0.03714055	0.037115839
n = 50	MME	1	1.09394	1	1.014162	1.089394	1.098488	0.9650004	1.063329	0.001495	0.000102	-0.05263163	0.052636952
		1	1.01252	2	1.963107	1.012209	1.012834	1.914083	2.012132	0.000815	0.0000011	-0.02528179	0.025281309
		3	3.82352	2	2.045391	3.754858	3.892178	1.99546	2.095324	0.003481	0.001084	-0.04236901	0.042369465
	MLE	1	0.82651	1	1.033048	0.809044	0.843633	0.983048	1.083048	0.00229	0.001438	-0.06953566	0.069114566
		1	0.89737	2	2.002665	0.887578	0.907057	1.952665	2.052665	0.000606	0.000345	-0.03588128	0.03575879
		3	3.89077	2	2.056146	3.816824	3.9646	2.006146	2.106146	0.003101	0.00394	-0.04332384	0.04329197
	LSE	1	0.74078	1	0.917604	0.732644	0.748706	0.883175	0.951974	0.003272	0.000506	-0.04850553	0.048152805
		1	0.60143	2	1.188459	0.599674	0.603156	1.1675035	1.209358	0.018786	0.00234	-0.02055958	0.020446609
		3	3.39865	2	1.990344	3.342325	3.454842	1.942139	2.038588	0.003243	0.000187	-0.04079132	0.040773547
n = 100	MME	1	1.37468	1	0.991325	1.356053	1.393299	0.9418609	1.040802	0.013206	0.006356	-0.06344396	0.063457373
		1	1.15136	2	1.965729	1.147572	1.155138	1.916486	2.014977	0.00343	0.001142	-0.02833645	0.028338995
		3	3.22748	2	1.996395	3.1713	3.282146	1.946489	2.046297	0.003703	0.000704	-0.04240483	0.041933728
	MLE	1	1.09072	1	1.00596	1.060565	1.120637	0.95596	1.05596	0.001559	0.000565	-0.07735242	0.077130553
		1	1.22111	2	2.000479	1.205183	1.236904	1.950479	2.050479	0.003625	0.001947	-0.03803626	0.037928977
		3	3.21577	2	2.008843	3.153778	3.276278	1.958843	2.058843	0.00234	0.001339	-0.04416592	0.043707555
	LSE	1	1.29683	1	1.171718	1.273356	1.319531	1.126898	1.216104	0.00506	0.010021	-0.0563541	0.055384552
		1	0.9912	2	1.669206	0.985116	0.997182	1.6350723	1.70332	0.013566	0.008672	-0.02658281	0.026476024
		3	3.18185	2	2.007765	3.132197	3.22903	1.960091	2.055286	0.002087	0.001229	-0.03934988	0.038496458

Table 3 Estimation of two parameters in Fuzzy Pareto Distribution when α=0.8

Sample size	Method	True value				Estimated Value				MSE		TD	
		Shape τ		Scale ς		Shape $\tilde{\tau}$		Scale $\tilde{\varsigma}$		left	right	Left	Right
n = 20	MME	1	1.100674	1	1.059405	1.09886	1.102488	1.040231	1.0786	0.00028053	0.00068795	-0.01974692	0.019747864
		1	1.026073	2	2.044494	1.02583	1.026315	2.025444	2.0635	0.0000309	0.00017822	-0.009554534	0.009554049
		3	3.28938	2	1.985503	3.266669	3.31209	1.965596	2.0054	0.00272837	0.00067563	-0.016930515	0.016930715
	MLE	1	0.8408559	1	1.10982	0.8341987	0.847463	1.08982	1.1298	0.00747212	0.01005819	-0.025937669	0.02587875
		1	0.7188866	2	2.149225	0.7162294	0.721533	2.129225	2.1692	0.01130048	0.01256349	-0.013001951	0.012986928
		3	3.395812	2	2.016149	3.369603	3.421993	1.996149	2.0361	0.00134916	0.00052441	-0.017637938	0.017629693
	LSE	1	0.6846085	1	0.889243	0.6821478	0.687045	0.87707	0.9014	0.00484423	0.00543855	-0.017283045	0.017245177
		1	0.5741417	2	1.606164	0.573235	0.575042	1.595201	1.6171	0.01093462	0.01091752	-0.008404868	0.008385266
		3	2.935311	2	1.947634	2.919389	2.951205	1.92927	1.966	0.0026902	0.00053696	-0.014853174	0.014848769
n = 50	MME	1	1.093941	1	1.014162	1.092122	1.095759	0.9944961	1.0338	0.000599	0.0000255	-0.021054076	0.021053261
		1	1.012522	2	1.963107	1.012397	1.012647	1.943497	1.9827	0.000378	0.00001509	-0.010112721	0.010112721
		3	3.823519	2	2.045391	3.796054	3.850982	2.025418	2.0654	0.002136	0.000741	-0.016948055	0.016947531
	MLE	1	0.8265124	1	1.033048	0.8195681	0.833401	1.013048	1.053	0.003047	0.001003	-0.027762495	0.027694087
		1	0.8973721	2	2.002665	0.8914678	0.901259	1.982665	2.0227	0.000735	0.000246	-0.016566139	0.014318107
		3	3.890774	2	2.056146	3.861208	3.920318	2.036146	2.0761	0.002311	0.000962	-0.017325938	0.017320283
	LSE	1	0.7407814	1	0.917604	0.7375529	0.743976	0.90384	0.9314	0.003072	0.000374	-0.019358278	0.019301674
		1	0.6014345	2	1.188459	0.6007351	0.602128	1.1800839	1.1968	0.017197	0.002593	-0.008209995	0.008191947
		3	3.398647	2	1.990344	3.3761343	3.42114	1.971057	2.0096	0.002967	0.0002	-0.016314303	0.016311521
n = 100	MME	1	1.374676	1	0.991325	1.367227	1.382125	0.9715372	1.0111	0.010489	0.007786	-0.025379495	0.025381109
		1	1.151355	2	1.965729	1.149842	1.152868	1.946031	1.9854	0.002516	0.001608	-0.011334814	0.011335322
		3	3.22748	2	1.996395	3.204553	3.248892	1.97643	2.0164	0.001315	0.000217	-0.017104211	0.016631799
	MLE	1	1.090722	1	1.00596	1.078739	1.102765	0.98596	1.026	0.000791	0.000423	-0.030867807	0.030922816
		1	1.221109	2	2.000479	1.214755	1.227443	1.980479	2.0205	0.002937	0.002271	-0.015201072	0.015184694
		3	3.215765	2	2.008843	3.190582	3.239581	1.988843	2.0288	0.000611	0.00031	-0.017787087	0.017361993
	LSE	1	1.296832	1	1.171718	1.287401	1.305867	1.1537	1.1894	0.006358	0.008358	-0.022649758	0.022042278
		1	0.9911962	2	1.669206	0.9887759	0.993602	1.6555548	1.6829	0.011935	0.009982	-0.010619997	0.010603007
		3	3.18185	2	2.007765	3.161327	3.20006	1.988634	2.0267	0.000507	0.000258	-0.015978527	0.015159947

Table 1-3 gives the estimates of the parameters of the fuzzy pareto distribution by using different methods under different sample size with α-cut value is 0.2, 0.5 and 0.8 respectively. The (1,1), (1,2) and (3,2) also the estimates of shape and scale parameters in fuzzy pareto distribution were preferred Maximum Likelihood Estimation Method based on the least Means Square Error and the seconded by Least Square Method and the Last by Method of Moment. But in alpha-cut manner the left alpha cut preferred in order as MLE, LSE and MME and in right alpha cut preferred in order as MME, LSE and MLE. The Table also shows that as the sample size increases, the parameter estimates tend to be closer to the original values. So far, estimation methods have demonstrated the properties of consistency.

By using the goodness of fit criteria of MSE and TD we prefered MLE method is the best method and followed by LSE and MME respectively by taking least and smaller values. (by Quandts 1964).

V. CONCLUSION

From the above results we conclude that the Maximum Likelihood Estimation method is more preferrable and suitable method for fitting the two parameter fuzzy pareto distribution. And also we proven that MLE is the most efficient estimator compared with Least Square Method and Method of Moment estimators. It is also conclude that analysis all the methods is the consistent. The alpha cut we used here to calculate the estimate parameters among from the imprecise data adequately.

REFERENCES

[1] Joseph Lee Peterson, A estimating the parameters of a pareto distribution, Introducing a Quantile regressin method.

[2] Amand.M., and Pelgrin.F., (2016), Pareto distribution in International trade: Hard to identify, easy to estimate, https://www.tse-fr-eu/sites/default/files/TSE/documents/sem2016/macro/amand.pdf.

[3] Ghitany.M.E., Gomez-Deniz.E., and Nadarajah.S., (2018), A new generalization of pareto distribution and its application to insurance data, Journal of Risk and Financial Management, 11(10), 1-114.

[4] Nursamsiah.D., Sugianto.N., Jusup.S.M., and Yulianto.B., (2018), Seasonal and Non-seasonal generalized pareto distribution to estimate extreme significant wave height in the Banda Sea, A paper presented at the 3rd International conference on Tropical and Coastal Region Eco Development,2017,IOP Conf. Services, 116 012066.

[5] Pareto.V., (1965), La Courbe da la Repartition de la Richese, originally published in 1896 in Busino.G., Editor Oevves completes the Vilfredo Pareto Geneva: Librairie Drez., pp.1-5.

[6] Khoolenjani.N.B., and Shahsana.E., (2016), Estimating the parameter of Exponential distribution under type-II censoring from fuzzy data. Journal of statistical theory and applications, 15(1.2), 181-195.

[7] Pak.A., Parham.G.H., and Saraj, (2013), Inference for the Weibull distribution based on fuzzy data, Revista colombianica de estadistica, 36(1.2), 339-358.

[8] Pak.A., Parham.G.H., and Saraj, (2014), Inference on the competing Risk Reliability problem for exponential distribution based on fuzzy data, IEEE Transactions on Reliability,63(1.1), 1-10.

[9] Makhdoom.I., Nasisn.P., and Pak.A., (2016), Estimating the parameter of exponential distribution under Type-II censoring from fuzzy data, Journal of Modern applied statistical methods, 15(1.2), 495-509.

[10] Abbas Pak and Mohammad Reza Mahmoudi, (2018), Estimating the parameters of Lomax distribution from imprecise information, Journal of Statistical theory and application, vol 17,no.1,122-135.

[11] George J Klir Bo Yuan, (2008),Fuzzy sets and Fuzzy logic, Prentice Hall of India.

[12] Venkatesh.A., Vetrivel.K., and Manikandan.P., (2017), A mathematical model using fuzzy pareto distribution for the effect of leptin, International Journal of Pune and applied mathematics, Volume 117,No.6,209-215.

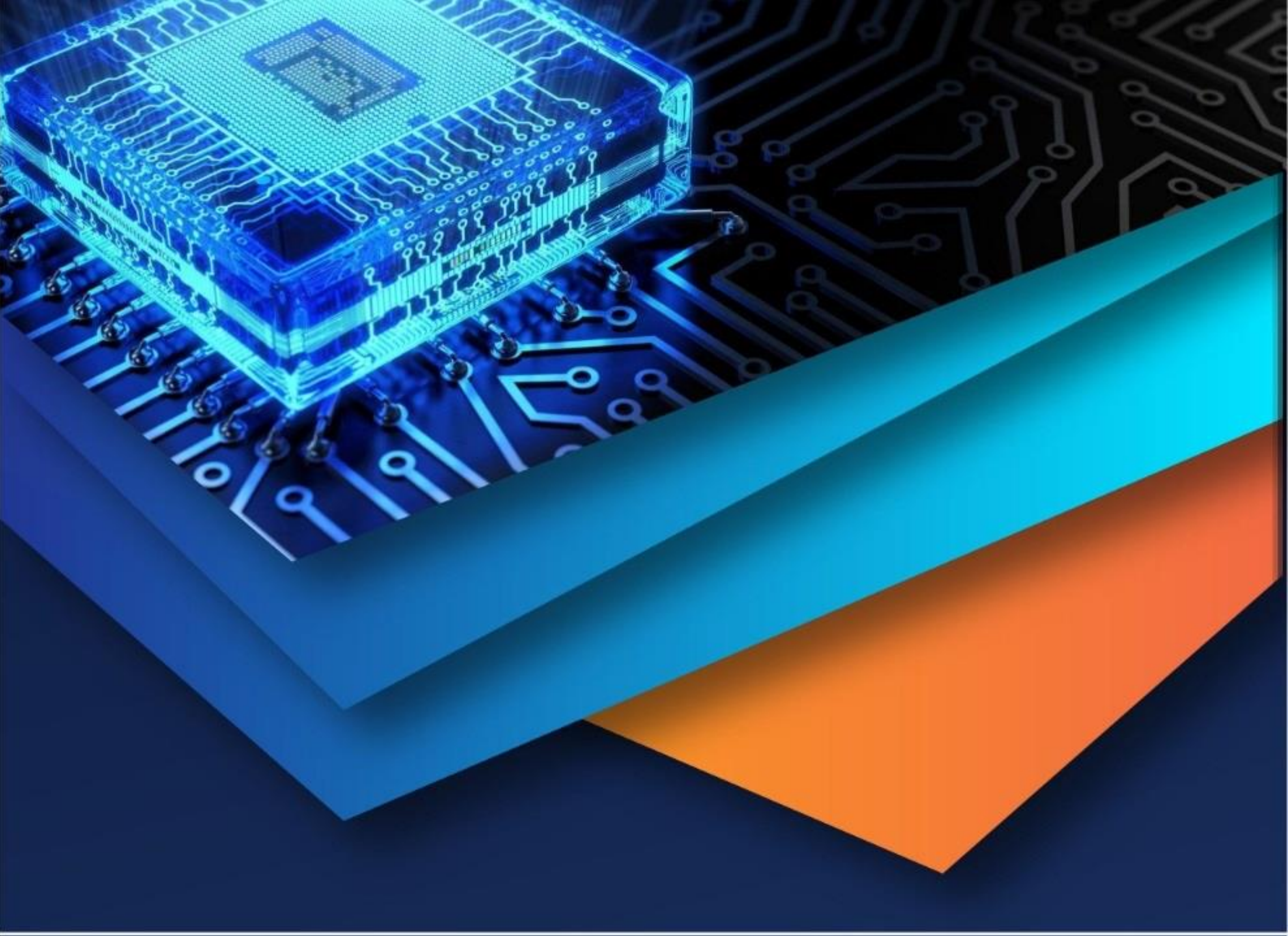
[13] Quandt.R.E., (1966), Old and New methods of estimation in the pareto distribution, Metrika, 10(1),55-82.

[14] Ekpan.M.V., Njoku.O.S., Ekpenyong.E.J., (2018), Efficiency of some estimation methods of the parameters of a two parameter pareto distribution, American Journal of Mathematics and statistics, 8(5),105-110.

[15] Rashid.M.Z., Akhter.A.S., (2011), Estimation accuracy of Exponential distribution parameters, Pakistan Journal of Statistics and operation research, 7(2),217-232.

[16] Ronald.E.W., Raymond.H.M.,(1978), Probability and Statistics for Engineers and Scientists (2nd Ed), Mc Millan Publishing Co., Inc., New.

[17] Al-Fawzan.M., (2000), Methods for estimating the parameters of Weibull distribution, King Abdulaziz city for sciences & technology, Soudi Arabia.



doi crossref

10.22214/IJRASET

INDEX COPERNICUS

45.98

ISRA JIF

IMPACT FACTOR: 7.129

SJIFactor Scientific Journal Impact Factor

TOGETHER WE REACH THE GOAL

IMPACT FACTOR: 7.429

ISI

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY