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Estimation of Parameters of Lomax Distribution by Using Maximum Product Spacings Method

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Abstract: The objective of this paper is to estimate the parameters of Lomax distribution with shape parameter(α) and scale parameter(λ). In this heuristic algorithm, the Maximum Product Spacings Method is used to Estimates the parameters of Lomax Distribution. We also computed Average Estimate (AE), Variance (VAR), Mean Absolute Deviation (MAD), Mean Square Error (MSE), Relative Error (RE) and Relative Absolute Bias (RAB) for both the parameters under grouped sample based on 1000 simulations to assess the performance of the estimators. And also compare Maximum Product Spacings Method with Least Squares Regression Method and Median Rank Regression Method.
Keywords: Lomax Parameters, Grouped sample, Maximum Product Spacings method, simulation.

I. INTRODUCTION

Torabi (2008) proposed a general method of Minimum Spacings Distance Estimators and a related 53 method of hypothesis testing based on Spacings. Consider different parameter estimation methods in Generalized Half Log Logistic distribution based on Complete and Censored Data by Torabi and Bagheri (2010). Ramamohan et al. (2011) studied using Minimum Spaceing Square Distance Estimation Method from an optimally constructed grouped sample for Estimation of Scale parameter (σ) when Shape parameter (β) is known in Log Logistic Distribution. We know that the Maximum Likelihood Method of estimation and the Moments Method of estimation are most general methods of estimation. Although Maximum Likelihood Estimation method is advantageous in the good judgment of its efficiency and has good theoretical properties, there is confirmation that it does not execute well, in particular in the case of small samples. The method of moments is simply applicable and often gives precise forms for estimators of unknown parameters. There are several cases where the method of moments does not give explicit estimators (e.g., for the parameters of the Gompertz and Weibull distributions). This spacings-based estimation process provides an alternative to the fixed parametric estimation methods like the Method of Moments, Minimum Two, Maximum Likelihood (ML), and so on. Cheng and Amin (1979, 1983) studied the estimation method that generalizes the initiative contained in the Maximum Spacings Estimator (MSE) and separately discussed by Ranneby (1984) and enjoys similar compensations. Cheng and Amin (1983) communicate that in such situations as a threeparameter Lognormal Gamma, Weibull distribution where the ML method breaks down due to unboundedness of the likelihood, the Maximum Spacings Estimation (MSPE) method produces reliable and asymptotically resourceful estimators. In some situations like mixtures of normals where the MLE is known to turn out inconsistent estimators, the MPS estimators are consistent (see Ranneby, 1984). Kaushik Ghosh (2001) considered a general estimation method using spacings it is shown that the Maximum Spacing Estimator is asymptotically most efficient within the subclass of spacings based estimators. Ehab Mohamed Almetwally et al (2019) calculated the Maximum Product Spacings and Bayesian Method for Parameter Estimation for Generalized Power Weibull Distribution under Censoring Scheme. Yongzhao Shao (2001) deliberated the Consistency of the Maximum Product of Spacings Method and Estimation of a unimodal distribution. Recently many of authors studied some of the different estimation procedures like Vijaya lakshmi, Raja Sekharam and Anjaneyulu (2018) studied Estimation of Scale (λ) and Location (μ) of two-parameter Rayleigh distribution by using Median Ranks estimated method. Vijaya lakshmi, Raja Sekharam and Anjaneyulu (2019) studied Estimation of Scale (θ) and Shape (α) parameters of Power Function Distribution by Least Squares Method using Optimally Constructed Grouped data. Vijaya lakshmi and Anjaneyulu (2019) studied estimation of Location (μ) and Scale (λ) for two-parameter Half Logistic Pareto Distribution (HLPD) by Least Square Regression Method. Vijaya lakshmi and Anjaneyulu (2019) studied Estimation of Location (μ) and Scale (λ) for Two-Parameter Half Logistic Pareto Distribution (HLPD) by Median Rank Regression Method.

In this chapter, we discuss about the estimation procedure for the unknown parameters for Lomax distribution. The idea behind the Maximum Product Spacings parameter estimation is to determine the parameters for the given sample data. We present MPS of the unknown scale and shape parameters of Lomax distribution using Newton-Raphson iterative procedure.

We also compute Average Estimate (AE), Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB) for both the parameters under sample based on 10,000 simulations to assess the performance of the estimators. Finally, the proposed estimation method be examined with applied on real and generalized data sets.

II. ESTIMATION OF PARAMETERS OF LOMAX DISTRIBUTION MAXIMUM PRODUCT SPACINGS(MPS) METHOD

Let $x_1, x_2,..., x_n$ be a random sample of size n from LD(α, λ), its probability density function (pdf) and Cumulative Distribution Function(CDF) are given by

$$f(x) = \frac{\alpha}{\lambda}\left[1+\frac{x}{\lambda}\right]^{-(\alpha+1)};\ x \geq 0,\ \alpha > 0,\ \lambda > 0 \quad \ldots(1.2.1)$$

α = shape parameter
λ = scale parameter

$$\text{CDF} \quad F(X) = 1-\left(1+\frac{x}{\lambda}\right)^{-\alpha} \quad \ldots(1.2.2)$$

A random variable X~LD(α, λ) has quantile function and the p^{th} quantile x_p of Lomax distribution is

$$x_p = \lambda\left[(1-P)^{\frac{-1}{\alpha}}-1\right] \quad \ldots(1.2.3)$$

Let U~U(0,1) then equation(4.2.3) can be used to simulate a random sample of size'n' from the Lomax distribution as follows

$$x_p = \lambda\left[(1-u_i)^{\frac{-1}{\alpha}}-1\right]\ i=1,2,\ldots,n. \quad \ldots(1.2.4)$$

Let $(\alpha, \lambda) = F_{LD}(X_i/\alpha, \lambda) - F_{LD}(X_{i-1}/\alpha, \lambda)$ for i=1 to n, be the uniform spacings of a random sample from the Lomax distribution, Where

$F_{LD}(X_0/\alpha, \lambda)=0$

$F_{LD}(X_{n+1}/\alpha, \lambda)=1$and

$$D_i = \begin{cases} D_1 = F_{LD}(X_1) \\ D_I = F_{LD}(\mathrm{Xi}) - F_{LD}(\mathrm{Xi}-1) = F_{LD}(\mathrm{X}(2{:}\,\mathrm{m}));\ \mathrm{i}=2,\ldots,\mathrm{m}. \\ D_m = 1 - F_{LD}(\mathrm{Xm}) \end{cases}$$

Clearly $\sum_{i=1}^{n+1} D_i(\alpha,\lambda) = 1$

The MPS estimates, $\hat{\alpha}_{MPS}$ and $\hat{\lambda}_{MPS}$ are obtained by maximizing the Geometric mean of spacings,

$$G(\alpha, \lambda)=\left[\prod_{i=1}^{n+1} D_i(\alpha,\lambda)\right]^{\frac{1}{n+1}} \quad \ldots(1.2.5)$$

With respect to α , λ or equivalently by maximizing the logarithm of the Geometric mean of sample spacings:

$$(\alpha, \lambda)=\frac{1}{n+1}\sum_{i=1}^{n+1} \log D_i(\alpha,\lambda) \quad \ldots(1.2.6)$$

The estimates $\hat{\alpha}_{MPS}$ and $\hat{\lambda}_{MPS}$ of the parameters α, λ can be obtained by solving the following non-linear equations

$$(\alpha, \lambda)=\frac{1}{n+1}\sum_{i=1}^{n+1} \log\left[\left(1+\frac{x_{i-1}}{\lambda}\right)^{-\alpha}-\left(1+\frac{x_i}{\lambda}\right)^{-\alpha}\right] \quad \ldots(1.2.7)$$

$$\frac{dH(\alpha,\lambda)}{d\alpha} = \frac{1}{n+1}\sum_{i=1}^{n+1}\frac{1}{D_i(\alpha,\lambda)}\left[\left(1+\frac{x_i}{\lambda}\right)^{-\alpha}\log\left(1+\frac{x_i}{\lambda}\right)-\left(1+\frac{x_{i-1}}{\lambda}\right)^{-\alpha}\log\left(1+\frac{x_{i-1}}{\lambda}\right)^{-\alpha}\right]=0$$
$$\ldots(1.2.8)$$

$$\frac{dH(\alpha,\lambda)}{d\lambda} = \frac{1}{n+1}\sum_{i=1}^{n+1}\frac{1}{D_i(\alpha,\lambda)}\left[\frac{\alpha}{\lambda^2}(x_{i-1})\left(1+\frac{x_{i-1}}{\lambda}\right)^{-\alpha-1}-\frac{\alpha}{\lambda^2}(x_i)\left(1+\frac{x_i}{\lambda}\right)^{-\alpha-1}\right]=0 \quad \ldots(4.2.9)$$

Let us take,

$\Delta_1 = \left(1+\frac{x_i}{\lambda}\right)^{-\alpha} log\left(1+\frac{x_i}{\lambda}\right)$

$\Delta_2 = \left(1+\frac{x_{i-1}}{\lambda}\right)^{-\alpha} log\left(1+\frac{x_{i-1}}{\lambda}\right)^{-\alpha}$

$\Delta_3 = \frac{\alpha}{\lambda^2}(x_{i-1})\left(1+\frac{x_{i-1}}{\lambda}\right)^{-\alpha-1}$

$\Delta_4 = \frac{\alpha}{\lambda^2}(x_i)\left(1+\frac{x_i}{\lambda}\right)^{-\alpha-1}$

$\Delta_5 = (x_{i-1})\left(1+\frac{x_{i-1}}{\lambda}\right)^{-\alpha-1} - (x_i)\left(1+\frac{x_i}{\lambda}\right)^{-\alpha-1}$

$\Delta_6 = -\left(1+\frac{x_{i-1}}{\lambda}\right)^{-\alpha} log\left(1+\frac{x_{i-1}}{\lambda}\right)$

The reduced form of equations (4.2.8) and (4.2.9) are becomes

$$\frac{dH(\alpha,\lambda)}{d\alpha} = \frac{1}{n+1}\sum_{i=1}^{n+1}\frac{1}{D_i(\alpha,\lambda)}[\Delta_1 - \Delta_2] = 0 \quad \ldots(1.2.10)$$

$$\frac{dH(\alpha,\lambda)}{d\lambda} = \frac{1}{n+1}\sum_{i=1}^{n+1}\frac{1}{D_i(\alpha,\lambda)}[\Delta_3 - \Delta_4] = 0 \quad \ldots(1.2.11)$$

$$\frac{dH(\alpha,\lambda)}{d\alpha d\lambda} = \frac{1}{n+1}\sum_{i=1}^{n+1}\frac{1}{D_i(\alpha,\lambda)}\left[\Delta_1 log\left(1+\frac{x_{i-1}}{\lambda}\right) + \Delta_4 log\left(1+\frac{x_i}{\lambda}\right) + \frac{\alpha}{\lambda^2}\Delta_5[\Delta_6 + \Delta_1] + \frac{1}{\lambda^2}\Delta_5\right] \quad \ldots(1.2.12)$$

By solving (4.2.10) and (4.2.11) we get the estimated values of $\hat{\alpha}_{MPS}$ and $\hat{\lambda}_{MPS}$.

But the equations have to be solved numerically using nonlinear optimization techniques. Note that if $x_{i+k} = x_{i+k-1} = \ldots = x_i$.. We get $D_{i+k-1}(\alpha,\lambda) = \cdots = D_i(\alpha,\lambda) = 0$. Therefore, the MPS estimators are sensitive to closely spaced observations, especially ties. When the ties are due to multiple observations, $D_i(\alpha,\lambda)$ should be replaced by the corresponding likelihood $f_{LD}(x;,\lambda)$.Since $x_i = x_{i-1}$. For the Exponentiated Exponential Gompertz (EEG) distribution, the MPS estimators are asymptotically normally distributed (see Cheng et al (1983)) with joint bi-variate normal distribution given by

$$(\hat{\alpha}_{MPS}, \hat{\lambda}_{MPS}) \sim [(\alpha,\lambda), I^{-1}(\alpha,\lambda)] \text{ for } n \to \infty \quad \ldots(1.2.13)$$

where $I(\alpha,\lambda)$ is Fisher information matrix

$$I(\alpha,\lambda) = -\begin{bmatrix} I_{11}(\alpha,\lambda) & I_{12}(\alpha,\lambda) \\ I_{21}(\alpha,\lambda) & I_{22}(\alpha,\lambda) \end{bmatrix} \quad \ldots(1.2.14)$$

$$I_{11}(\alpha,\lambda) = \frac{dH(\alpha,\lambda)}{d\alpha}$$

$$I_{22}(\alpha,\lambda) = \frac{dH(\alpha,\lambda)}{d\lambda}$$

$$\mathbf{I_{12}(\alpha,\lambda) = I_{21}(\alpha,\lambda) = \frac{dH(\alpha,\lambda)}{d\alpha d\lambda}} \quad \ldots(4.2.15)$$

III. SIMULATION STUDY

In this section, we develop a simulation study. The major goal of these simulations is to calculate the efficiency of the Maximum Product Spacings estimation method for the parameters of the Lomax distribution. The subsequent procedure is adopted as follows:

1) *Step 1:* Set the sample size 'n' and the vector of parameter values $\Psi = (\alpha, \boldsymbol{\lambda})$.
2) *Step 2:* Using the values obtained in step (1), compute $\hat{\boldsymbol{\alpha}}_{MPS}$ ***and*** $\hat{\boldsymbol{\lambda}}_{MPS}$ through Maximum Product Spacings.
3) *Step3:* Repeat steps (2) and (3) N times
4) *Step4:* Using $\hat{\boldsymbol{\Psi}}$ of Ψ, compute the Average Estimate (AE), Variance (VAR), Standard Deviation (SD), Mean Square Error (MSE) and Relative Absolute Bias (RAB). If Ψ_{lm} is Maximum Product Spacings estimate method of Ψ_m, m=1, 2 where $\Psi_m$is a general notation that can be replaced by $\Psi_1 = \alpha$, $\Psi_2 = \boldsymbol{\lambda}$ based on sample l, (l =1,2,…,r), then the Average Estimate (AE), Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB) are given respectively by

Average Estimate $(\hat{\psi}_m) = \frac{\sum_{i=1}^{r} \hat{\psi}_{lm}}{r}$

Variance$(\hat{\psi}_m) = \frac{\sum_{i=1}^{r}(\hat{\psi}_{lm} - \overline{\hat{\psi}_{lm}})^2}{r}$

Standard Deviation$(\hat{\psi}_m) = \sqrt{\frac{\sum_{i=1}^{r}(\hat{\psi}_{lm} - \overline{\hat{\psi}_{lm}})^2}{r}}$

Mean Absolute Deviation $= \frac{\sum_{i=1}^{r}(|\hat{\psi}_{lm} - \overline{\hat{\psi}_{lm}}|)}{r}$

Mean Square Error $(\hat{\psi}_m) = \frac{\sum_{i=1}^{r}(\hat{\psi}_{lm} - \psi_m)^2}{r}$

Relative Absolute Bias$(\hat{\psi}_m) = \frac{\sum_{i=1}^{r}|(\hat{\psi}_{lm} - \psi_m)|}{r\psi_m}$

The results are computed using the software R (R Core Development Team). The seed used to generate the random values. The chosen values to perform this procedure are N = 10,000, and n = (3,5,15,20,,…, 200). For different population parameter values.

A. *Simulated Data Sets*

We evaluated the performance of the Least Square Regression (LR) method for estimating the Shape(α) and Scale (λ) parameters of Lomax distribution by using Newton-Raphson simulation. The process is repeated 10,000 times for different sample sizes *n*= 3(50)200.

Average Estimate (AE), Variance (VAR), Standard Deviation(SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB) of Least Squares method for the estimators of location and scale parameters of the Lomax distribution are evaluated.

The simulated results are presented in Table-4.1 for the values of the parameters Shape (α)=1 and Scale(λ)=1.

Table-1.1
Maximum product Spacings Method- Lomax Distribution

Sample size	Parameter	AE	VAR	SD	MAD	MSE	RAB
3	α	-1.2887	0.73748	0.8588	0.4401	5.9758	2.2887
	λ	0.35326	0.43256	0.6577	0.3252	0.8508	0.7884
5	α	-1.0597	0.03201	0.1789	0.1198	4.2742	2.0597
	λ	0.14462	0.02312	0.1521	0.1027	0.7548	0.8569
15	α	-1.0057	0.00048	0.0219	0.0153	4.0231	2.0057
	λ	0.03108	0.00043	0.0207	0.0163	0.9392	0.9689
20	α	-1.0032	0.00017	0.013	0.0091	4.0129	2.0032
	λ	0.02125	0.00018	0.0134	0.0106	0.9581	0.9788
25	α	-1.0021	0.00008	0.0089	0.0062	4.0083	2.0021
	λ	0.01593	0.00009	0.0095	0.0076	0.9685	0.9841
50	α	-1.0005	0.00001	0.0032	0.0019	4.0021	2.0005
	λ	0.00653	0.00001	0.0032	0.0028	0.987	0.9935
100	α	-1.0001	0	0	0.0006	4.0005	2.0001
	λ	0.00275	0	0	0.0011	0.9945	0.9973
200	α	-1	0	0	0.0002	4.0001	2
	λ	0.0012	0	0	0.0004	0.9976	0.9988

We evaluated the performance of the Least Square Regression (LR) method for estimating the Shape(α)and Scale (λ) parameters of Lomax distribution by using Newton-Raphson simulation the process is repeated 10,000 times for different sample sizes *n*=3(50)200.

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Average Estimate (AE), Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB) of Least Squares method for the estimators of location and scale parameters of the Lomax distribution are evaluated.

The simulated results are presented in Table-4.2 for the values of the parameters Shape (α)=3 and Scale(λ) =1.5.

Table-1.2

Maximum Product Spacings Method- Lomax Distribution

Sample size	Parameter	AE	VAR	SD	MAD	MSE	RAB
3	α	-1.2832	0.81474	0.90263	0.43747	19.1605	4.28319
	λ	0.47188	0.68624	0.8284	0.43298	1.74328	1.1971
5	α	-1.0625	0.03102	0.17612	0.11845	16.5353	4.06254
	λ	0.21596	0.05215	0.22836	0.15266	1.7009	1.28714
15	α	-1.0058	0.00046	0.02145	0.01501	16.0466	4.00576
	λ	0.05163	0.00126	0.0355	0.02757	2.09903	1.44837
20	α	-1.0031	0.00017	0.01304	0.00917	16.0249	4.00309
	λ	0.03604	0.00054	0.02324	0.01814	2.14372	1.46396
25	α	-1.0021	0.00008	0.00894	0.00624	16.0172	4.00213
	λ	0.02705	0.00028	0.01673	0.01317	2.16987	1.47295
50	α	-1.0005	0.00001	0.00316	0.00185	16.0039	4.00049
	λ	0.01129	0.00004	0.00632	0.00497	2.2163	1.48871
100	α	-1.0001	0	0	0.00056	16.001	4.00012
	λ	0.0048	0.00001	0.00316	0.00194	2.23563	1.4952
200	α	-1	0	0	0.00018	16.0003	4.00003
	λ	0.00208	0	0	0.00077	2.24376	1.49792

We evaluated the performance of the Least Square Regression (LR) method for estimating the Shape(α)and Scale (λ) parameters of Lomax distribution by using Newton-Raphson simulation. The process is repeated 10,000 times for different sample sizes *n*=3(50)200.

Average Estimate (AE), Variance (VAR), Standard Deviation(SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB) of Least Squares method for the estimators of location and scale parameters of the Lomax distribution are evaluated.

The simulated results are presented in Table-4.3 for the values of the parameters Shape(α)=1.5 and Scale (λ)=3.

Table-1.3

Maximum Product Spacings Method- Lomax Distribution

Sample size	Parameter	AE	VAR	SD	MAD	MSE	RAB
3	α	-1.2926	1.42072	1.1919	0.447	9.2194	2.7926
	λ	1.00787	4.13341	2.0331	0.9239	8.102	2.3618
5	α	-1.0623	0.03136	0.1771	0.1192	6.5968	2.5623
	λ	0.42674	0.19282	0.4391	0.302	6.8145	2.577
15	α	-1.006	0.00051	0.0226	0.0156	6.2806	2.506
	λ	0.09866	0.00457	0.0676	0.0526	8.4224	2.9013
20	α	-1.0033	0.00017	0.013	0.009	6.2668	2.5033
	λ	0.0673	0.00182	0.0427	0.0339	8.6025	2.9327
25	α	-1.0019	0.00008	0.0089	0.0061	6.2597	2.5019
	λ	0.05029	0.00094	0.0307	0.0243	8.7018	2.9497
50	α	-1.0005	0.00001	0.0032	0.0019	6.2527	2.5005
	λ	0.02075	0.00013	0.0114	0.009	8.876	2.9793
100	α	-1.0001	0	0	0.0006	6.2507	2.5001
	λ	0.00879	0.00002	0.0045	0.0035	8.9474	2.9912
200	α	-1	0	0	0.0002	6.2502	2.5
	λ	0.00384	0	0	0.0014	8.977	2.9962

IV. OBSERVATIONS FOR THE SIMULATION RESULT

1) When we increase the sample size the AE value of shape parameter (α) increased, scale parameter (λ) decreased.
2) In all the tables when we increase the sample size VAR, SD, MAD and MSE are decreased.
3) In some of the tables when we increase the sample size RAB of scale parameter (λ) increased.

V. COMPARISON OF MAXIMUM PRODUCT SPACINGS METHOD WITH LEAST SQURES AND MEDIAN RANK REGRESSION METHOD ESTIMATORS FOR DIFFERENT VALUES OF SHAPE PARAMETER (α) = 2 AND SCALE PARAMETER (λ) = 1.5

		MRR		LSR		MPS	
Sample Size	Para meters	AE	VAR	AE	VAR	AE	VAR
3	α	0.13355	0.000007	0.126487	0.000006	-1.31249	4.61748
	λ	0.758815	0.006755	0.796278	0.005506	0.4928	3.696
5	α	0.1423	0.000011	0.1348	0.000009	-1.0596	0.03107
	λ	0.730828	0.002009	0.758069	0.001774	0.21675	0.05124
15	α	0.154193	0.000009	0.148818	0.000008	-1.00568	0.00048
	λ	0.613205	0.002408	0.628311	0.002271	0.04998	0.0012
20	α	0.153081	0.000013	0.148839	0.000012	-1.00305	0.00017
	λ	0.631949	0.004402	0.644091	0.00426	0.03471	0.0005
25	α	0.157278	0.000013	0.15316	0.000011	-1.00204	0.00008
	λ	0.587024	0.004466	0.597643	0.004363	0.02589	0.00026
50	α	0.162812	0.000005	0.16004	0.000005	-1.00051	0.00001
	λ	0.754326	0.00547	0.760134	0.005269	0.01086	0.00003
100	α	0.168612	0.000002	0.166655	0.000002	-1.00014	0
	λ	1.101465	0.000189	1.101546	0.0002	0.00461	0.00001
200	α	0.168919	0.000008	0.167749	0.000008	-1.00003	0
	λ	1.180636	0.002108	1.181462	0.002148	0.00199	0

- *Observations*
- When comparing MPS with MRR and LSR , in MPS the AE is decreasing for the both shape and scale parameters when the sample size is increasing.
- In MRR and in LSR the AE is increasing for the both shape and scale parameters when the sample size is increasing.
- When comparing MPS with MRR and LSR , in MPS the VAR is decreasing for the both shape and scale parameters when the sample size is increasing.

		MRR		LSR		MPS	
Sample Size	Para meters	SD	MAD	SD	MAD	SD	MAD
3	α	0.000002	0.002415	0	0.002115	2.14883	0.47254
	λ	0.000046	0.05694	0.00003	0.051332	1.9225	0.45431
5	α	0.000001	0.002799	0	0.002561	0.17627	0.11911
	λ	0.000004	0.026562	0.000003	0.025024	0.22636	0.15487
15	α	0.000003	0.002421	0	0.002259	0.02191	0.01525
	λ	0.000006	0.032053	0.000005	0.031124	0.03464	0.02691
20	α	0.000001	0.003376	0	0.003143	0.01304	0.00908
	λ	0.000019	0.03914	0.000018	0.038492	0.02236	0.01745
25	α	0.000001	0.002873	0	0.002738	0.00894	0.00619
	λ	0.00002	0.048829	0.000019	0.048335	0.01612	0.01269
50	α	0.000001	0.001752	0	0.001644	0.00316	0.00189
	λ	0.00003	0.05703	0.000028	0.056071	0.00548	0.00474
100	α	0.000001	0.001121	0	0.001073	0	0.00057
	λ	0.000001	0.010745	0	0.01104	0.00316	0.00186
200	α	0.000002	0.002439	0	0.002369	0	0.00018
	λ	0.000004	0.033769	0.000005	0.034022	0	0.00074

- *Observations*

➢ When comparing MPS with MRR and LSR , in MPS the SD and MAD are decreasing for the both shape and scale parameters when the sample size is increasing.

➢ In MRR and LSR methods SD is decreasing and MAD is increasing when the sample size is increasing.

		MRR		LSR		MPS	
Sample Size	Para meters	MSE	RAB	MSE	RAB	MSE	RAB
3	α	0.026353	0.001682	0.021976	0.002258	15.5901	3.31249
	λ	0.064925	0.013274	0.047009	0.012459	4.71046	1.20432
5	α	0.034908	0.003806	0.029543	0.003144	9.39225	3.0596
	λ	0.074463	0.012622	0.060305	0.011841	1.69796	1.2853
15	α	0.050104	0.005136	0.045207	0.004927	9.03457	3.00568
	λ	0.152019	0.00877	0.140424	0.008606	2.10374	1.45002
20	α	0.050016	0.006878	0.046032	0.006068	9.01847	3.00305
	λ	0.139863	0.025271	0.130931	0.024954	2.14758	1.46529
25	α	0.051838	0.012577	0.048083	0.012698	9.01232	3.00204
	λ	0.175016	0.011681	0.166253	0.011475	2.17326	1.47411
50	α	0.0391	0.022202	0.037441	0.022058	9.00308	3.00051
	λ	0.065825	0.000604	0.062804	0.000734	2.21757	1.48914
100	α	0.023341	9.644795	0.022829	9.70607	9.00083	3.00014
	λ	0.010484	0.101466	0.010511	0.101546	2.2362	1.49539
200	α	0.020424	5.488071	0.020117	5.466375	9.0002	3.00003
	λ	0.034738	0.180636	0.035076	0.181462	2.24403	1.49801

- *Observations*

➢ In MPS the MSE is decreasing and RAB is decreasing for shape parameter and increasing for scale parameter when the sample size is increasing.

➢ In MRR and LSR, the RAB is increasing when sample size is increasing.

VI. CONCLUSIONS

A. MPS estimator is the best one for estimating the parameters of the Lomax distribution in comparison with MRR and LSR.

B. The MPS estimator has the smallest MSE, Variance (VAR), Standard deviation (SD), Mean absolute deviation (MAD), Mean Square Error (MSE), Relative Absolute Bias(RAB) for both parameters, proving to be the slightly most efficient method compared to MRR and LSR for estimating the unknown parameters.

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