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# Eulers Totient Function (Number Theoretic Functions), Right Angle Triangle and Their Applications

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**Abstract:** In this paper, we introduce some new theorem and results(section II,III and VI) on Euler's Totient Function , Right angle triangle and their applications

**Keywords:** Euler's totient function, right angle triangle, Golden ratio, Hardy – Ramanujan number, student's pencil compass and etc.

## I. INTRODUCTION(PRELIMINARY)

1) *Definition 1.1* : for  $m \geq 1$  ,  $\varphi(m)$  denote the number of positive integers not exceeding  $m$  that are relatively prime to  $m$ . Euler's totient function is also known as Euler's phi function.

If  $m = p_1 \cdot p_2$  where  $p_1, p_2$  are the primes ( $p_2 > p_1$ ) , Let divisors of  $m$  are  $1, p_1, p_2$  and  $p_1 \cdot p_2$  .

Then ,  $\varphi(m) = (\text{fourth divisors of } m - \text{second divisors of } m) - (\text{Third divisor of } m - \text{first divisor of } m)$ .

$$\begin{aligned} \text{Or, } \varphi(m) &= (p_1 \cdot p_2 - p_1) - (p_2 - 1) \\ &= p_1(p_2 - 1) - (p_2 - 1) \\ &= (p_1 - 1)(p_2 - 1) \end{aligned}$$

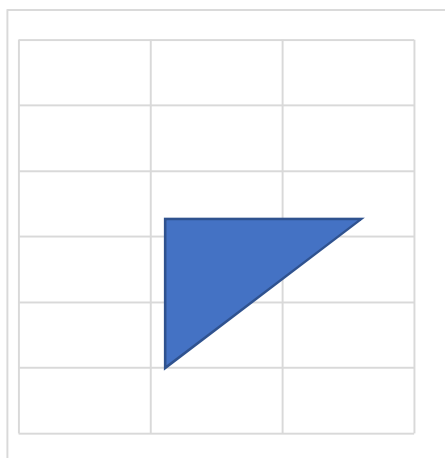
$$\varphi(m) = (p_1 - 1)(p_2 - 1)$$

If  $E [HCF(1, p_1), LCM(1, p_1)]$ ,

$F [HCF(p_1, p_2), LCM(p_1, p_2)]$ , and

$G [HCF(p_2, p_1 \cdot p_2), LCM(p_2, p_1 \cdot p_2)]$

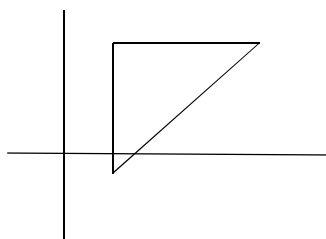
Are the vertices of triangle EFG



F

G

E



Then triangle EFG is inverted right angle triangle

$$\text{And } \varphi(m) = |A - B|$$

Where A,B are the Altitude and Base of triangle EFG

and (i) If  $A > B$

$$\text{then, } A = p_1 \cdot p_2 - p_1 \text{ and } B = p_2 - 1$$

(ii) If  $B > A$

$$\text{then, } B = p_1 \cdot p_2 - p_1 \text{ and } A = p_2 - 1$$

$$\varphi(m) = |A - B|$$

Squaring the both side,

$$(\varphi(m))^2 = A^2 + B^2 - 2 \cdot A \cdot B$$

Since,  $A^2 + B^2 = H^2$  where H is the hypotenuse

$$\text{Then, } (\varphi(m))^2 = H^2 - 2 \cdot A \cdot B$$

divide both side of above equation by 4,

$$\left(\frac{\varphi(m)}{2}\right)^2 = \left(\frac{H}{2}\right)^2 - \frac{1}{2} \cdot A \cdot B$$

$$\text{Or, } T = \left(\frac{H}{2}\right)^2 - \left(\frac{\varphi(m)}{2}\right)^2$$

Where T is the area of triangle EFG =  $\frac{1}{2} \cdot A \cdot B$ .

$H^n - (\varphi(m))^n$  is perfectly divisible by  $2^n, n \geq 1$ .

If  $m = p_1 \cdot p_2 \dots p_z, Z \geq 2$  then total pair of primes =  $\frac{z(z-1)}{2}$  Or  $(Z - 1)^{th}$  triangular number

Therefore, total number of inverted right angle triangle whose area is  $\left(\frac{H}{2}\right)^2 - \left(\frac{\varphi(s)}{2}\right)^2$  is  $(z-1)^{th}$  Triangular number.

Theorem and Results

## II. INEQUALITY RELATION

1) *Theorem 2.1:* If  $m = p_1 \cdot p_2 \dots p_z$ ,

Where  $p_1, p_2, \dots, p_z$  are the primes

$$(p_1 < p_2 < \dots < p_z), Z \geq 3.$$

Let divisors of m are  $d_1, d_2, \dots, d_{\tau(m)}$ , Consider m have  $\tau^{(m)}c_4$  [  $\tau^{(m)}c_4 = \sum_{i=1}^{\tau(m)} i \{ \tau(m) - 2 - i \}^{th}$  triangular number ], where  $n = \tau(m) - 3$  ] groups, each group consists of four elements (at least one element of each group should be distinct). let groups be  $(a', b', c', d')$ ,  $(a'', b'', c'', d'')$ ,  $\dots$   $(a^{''...e \text{ times}}, b^{''...e \text{ times}}, c^{''...e \text{ times}}, d^{''...e \text{ times}})$  where  $e = \tau^{(m)}c_4$

And every four elements of each group belongs to the divisors of m that is  $d_1, d_2, \dots, d_{\tau(m)}$ . Let  $E_1[\text{HCF}(a', b'), \text{LCM}(a', b')]$ ,

$F_1[\text{HCF}(b', c'), \text{LCM}(b', c')]$ ,  $G_1[\text{HCF}(c', d'), \text{LCM}(c', d')]$ ,  $E_2[\text{HCF}(a'', b''), \text{LCM}(a'', b'')]$ ,

$F_2[\text{HCF}(b'', c''), \text{LCM}(b'', c'')], G_2[\text{HCF}(c'', d''), \text{LCM}(c'', d'')], \dots, E_e[\text{HCF}(a''^{\dots e \text{ times}}, b''^{\dots e \text{ times}}), \text{LCM}(a''^{\dots e \text{ times}}, b''^{\dots e \text{ times}})],$

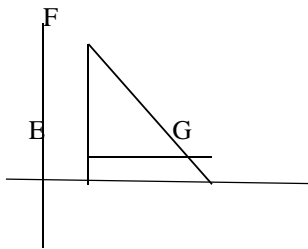
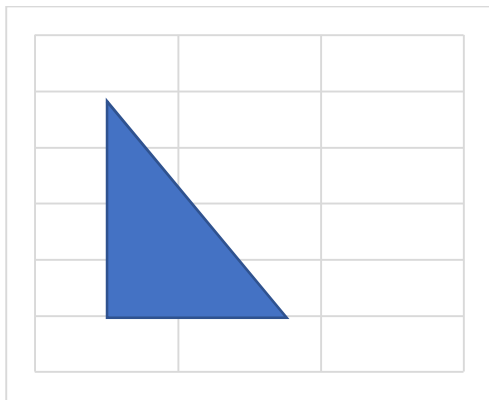
$F_e[\text{HCF}(b''^{\dots e \text{ times}}, c''^{\dots e \text{ times}}), \text{LCM}(b''^{\dots e \text{ times}}, c''^{\dots e \text{ times}})], G_e[\text{HCF}(c''^{\dots e \text{ times}}, d''^{\dots e \text{ times}}), \text{LCM}(c''^{\dots e \text{ times}}, d''^{\dots e \text{ times}})]$  are the vertices (or points) of  $E_1F_1G_1, E_2F_2G_2, \dots, E_eF_eG_e$  and

$W_1 =$  Total number of groups for which  $E_\Psi, F_\Psi, G_\Psi$  form Collinear line.

Where  $\Psi$  belongs to  $[1, e]$

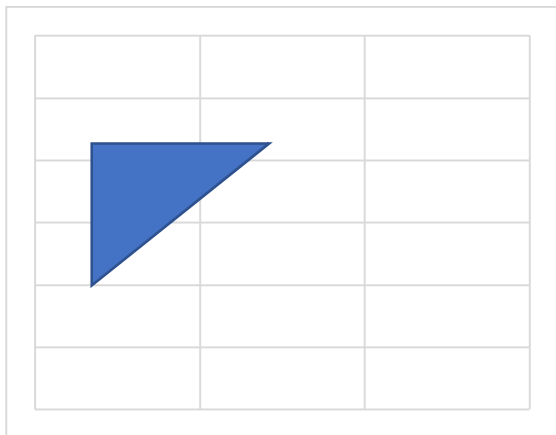
$W_2 =$  Total number of groups for which  $E_\Psi, F_\Psi, G_\Psi$  form triangles (other than right angle triangle).

$W_3 =$  Total number of groups for which  $E_\Psi, F_\Psi, G_\Psi$  form non-inverted right angle triangles



$W_4 =$  Total number of groups for which  $E_\Psi, F_\Psi, G_\Psi$  form inverted right angle triangles whose area is not equal to  $\left(\frac{H}{2}\right)^2 - \left(\frac{\varphi(S)}{2}\right)^2$  and

$W_5 =$  Total number of groups for which  $E_\Psi, F_\Psi, G_\Psi$  form inverted right angle triangles whose area is equal to  $\left(\frac{H}{2}\right)^2 - \left(\frac{\varphi(S)}{2}\right)^2$  where  $S \neq m$ .



If  $S = \text{product of second and third elements of each group } (b \cdot c, b'' \cdot c'', \dots, b^{e \text{ times}} \cdot c^{e \text{ times}})$ , then there exist at least  $Z-1 + {}^{Z-1}C_2$  inverted right angle triangles whose area is  $\left(\frac{H}{2}\right)^2 - \left(\frac{\varphi(S)}{2}\right)^2$ .

That is  $w_3 \geq Z-1 + {}^{Z-1}C_2$  or,  $\sum_{i=1}^n i [\{\tau(m)-2-i\}^{\text{th}} \text{ triangular number}] - \sum_{j=1}^4 w_j \geq Z-1 + {}^{Z-1}C_2$ , where  $n = \tau(m) - 3$ .

Where total number of groups =  $W_1 + W_2 + W_3 + W_4 + W_5$ .

**Proof :** If  $m = P_1 \cdot P_2 \dots P_z, Z \geq 3$ .

Total pair of primes =  $Z-1 + {}^{Z-1}C_2$  and number of groups of the form  $(1, P', P'', P' \cdot P'')$  is  $Z-1 + {}^{Z-1}C_2$

Where  $P'$  and  $P''$  are one of  $P_1, P_2, \dots, P_z (P' < P'')$ . If groups are of the form  $(1, P', P'', P' \cdot P'')$  then  $E, F, G$  form inverted right angle triangles, and every  $Z-1 + {}^{Z-1}C_2$  triangles follow the relation  $T = \left(\frac{H}{2}\right)^2 - \left(\frac{\varphi(S)}{2}\right)^2$ . Where  $S = p \cdot p''$ . By considering the groups of four elements (elements belongs to the divisors of  $m$ ), there exist some other groups also which are not of the form  $(1, P', P'', P' \cdot P'')$  but follow the relation  $T = \left(\frac{H}{2}\right)^2 - \left(\frac{\varphi(S)}{2}\right)^2$ . If  $m = P_1 \cdot P_2 \cdot P_3$ , then total number of groups =  $\tau(m)C_4 = {}^8C_4 = 70$ .

Out of these 70 groups there exist definitely  $Z-1 + {}^{Z-1}C_2$ , that is 3 groups for which the vertices form inverted right angle triangle whose area is  $\left(\frac{H}{2}\right)^2 - \left(\frac{\varphi(S)}{2}\right)^2$ .

Above 3 groups are of the form  $(1, P', P'', P' \cdot P'')$  where  $P'$  and  $P''$  are one of  $P_1, P_2, P_3 (P' < P'')$ . But for some  $m$  there exist some other groups which are not of the form  $(1, P', P'', P' \cdot P'')$  but follow the relation  $T = \left(\frac{H}{2}\right)^2 - \left(\frac{\varphi(S)}{2}\right)^2$ .

If  $m = 2 \cdot P_2 \cdot P_3$ , then the vertices  $(E, F, G)$  for the group  $(2, 2 \cdot p_2, 2 \cdot p_3, 2 \cdot p_2 \cdot p_3)$  form inverted right angle triangle whose area is  $\left(\frac{H}{2}\right)^2 - \left(\frac{\varphi(S)}{2}\right)^2$ .

So, if  $m = 2 \cdot P_2 \cdot P_3$ , then there exist 4 inverted right angle triangle whose area is  $\left(\frac{H}{2}\right)^2 - \left(\frac{\varphi(S)}{2}\right)^2$ .

If  $m = 3 \cdot P_2 \cdot P_3$ , then the vertices  $(E, F, G)$  for the group  $(3, 3 \cdot P_2, 3 \cdot P_3, 3 \cdot P_2 \cdot P_3)$  form inverted right angle triangle but area is not equal to  $\left(\frac{H}{2}\right)^2 - \left(\frac{\varphi(S)}{2}\right)^2$ .

So, If  $m = 3 \cdot P_2 \cdot P_3$  then there exist 3 inverted right angle triangle whose area is  $\left(\frac{H}{2}\right)^2 - \left(\frac{\varphi(S)}{2}\right)^2$ .

Hence, there exist at least  $Z-1 + {}^{Z-1}C_2$  inverted right angle triangle whose area is  $\left(\frac{H}{2}\right)^2 - \left(\frac{\varphi(S)}{2}\right)^2$ .

**NOTE :** (a)  $a < b < c < d, a'' < b'' < c'' < d'', \dots, a^{e \text{ times}} < b^{e \text{ times}} < c^{e \text{ times}} < d^{e \text{ times}}$

(b) let  $m = 2 \cdot 3 \cdot 5$  then, groups are  $(1, 2, 3, 5), (1, 2, 3, 6), (1, 2, 3, 10), (1, 2, 3, 15), (1, 2, 3, 30), (1, 2, 5, 6), (1, 2, 5, 10), (1, 2, 5, 15), (1, 2, 5, 30), (1, 2, 6, 10), (1, 2, 6, 15), (1, 2, 6, 30), (1, 2, 10, 15), (1, 2, 10, 30), (1, 2, 15, 30), (1, 3, 5, 6), (1, 3, 5, 10), (1, 3, 5, 15), \dots, (6, 10, 15, 30)$ . If  $m = 2 \cdot 3 \cdot 5$ , then total number of groups = 70.

(c)  $d_1 = a = 1$ .

Theorem 2.1 in another form :

A lot consists of groups for which  $E_\psi, F_\psi, G_\psi$  form inverted right angle triangle, then probability ( $q_1$ ) of getting the inverted right angle triangle whose area is  $\left(\frac{H}{2}\right)^2 - \left(\frac{\varphi(S)}{2}\right)^2$

is greater than or equal to  $\frac{Z-1 + {}^{Z-1}C_2}{w_4 + w_5}$

Or,  $q_1 \geq \frac{z^{-1} + z^{-1}c_2}{w_4 + w_5}$  and probability ( $q_2$ ) of getting the inverted right angle triangle whose area is not equal to

$\left(\frac{H}{2}\right)^2 - \left(\frac{\varphi(S)}{2}\right)^2$  is less than or equal to  $\frac{w_4 + w_5 - z^{-1}c_2 - z + 1}{w_4 + w_5}$

Or,  $q_2 \leq \frac{w_4 + w_5 - z^{-1}c_2 - z + 1}{w_4 + w_5}$ .

$q_1$  can also be written as ,

$$q_1 \geq \frac{z - 1 + z^{-1}c_2}{\sum_{i=1}^n i \{[\tau(m) - 2 - i]\text{th triangular number}\} - \sum_{j=1}^3 w_j}$$

**Note :** If  $m = p_1 \cdot p_2 \cdot p_3$  then ,

$q_1 \geq 0.1364$  and  $q_2 \leq 0.8637$ .

### III. RESULTS RELATED TO HARDY-RAMANUJAN NUMBER , AREA OF RECTANGLE AND TRAPEZIUM AND GOLDEN RATIO .

1) *Result 3.1:* If  $m = 3.5.7$ , out of the 70 groups if we take 27<sup>th</sup> and 41<sup>th</sup> groups that are (1,7,15,35) and (3,5,15,35) and

if  $E_{27}[HCF(1,7), LCM(1,7)], F_{27}[HCF(7,15), LCM(7,15)] , G_{27}[HCF(15,35), LCM(15,35)]$

and  $E_{41}[HCF(3,5), LCM(3,5)],$

$F_{41}[HCF(5,15), LCM(5,15)],$

$G_{41}[HCF(15,35), LCM(15,35)]$  are the vertices of triangles  $E_{27}, F_{27}, G_{27}$  and  $E_{41}, F_{41}, G_{41}$  , then the average of  $\left(\frac{H_{27}}{2}\right)^2 - \left(\frac{\varphi(S_{27})}{2}\right)^2$

And  $\left(\frac{H_{41}}{2}\right)^2 - \left(\frac{\varphi(S_{41})}{2}\right)^2$  is Hardy – Ramanujan number

Where  $E_{27}, F_{27}, G_{27}$  is inverted right angle triangle ,  $E_{41}, F_{41}, G_{41}$  is non-inverted right angle triangle.  $H_{27}, H_{41}$  are the length of hypotenuse of triangles  $E_{27}, F_{27}, G_{27}$  and  $E_{41}, F_{41}, G_{41}$

$S_{27}=7 \times 15$  ,  $S_{41}=5 \times 15$  and area of both triangle is not equal to  $\left(\frac{H}{2}\right)^2 - \left(\frac{\varphi(S)}{2}\right)^2$ .

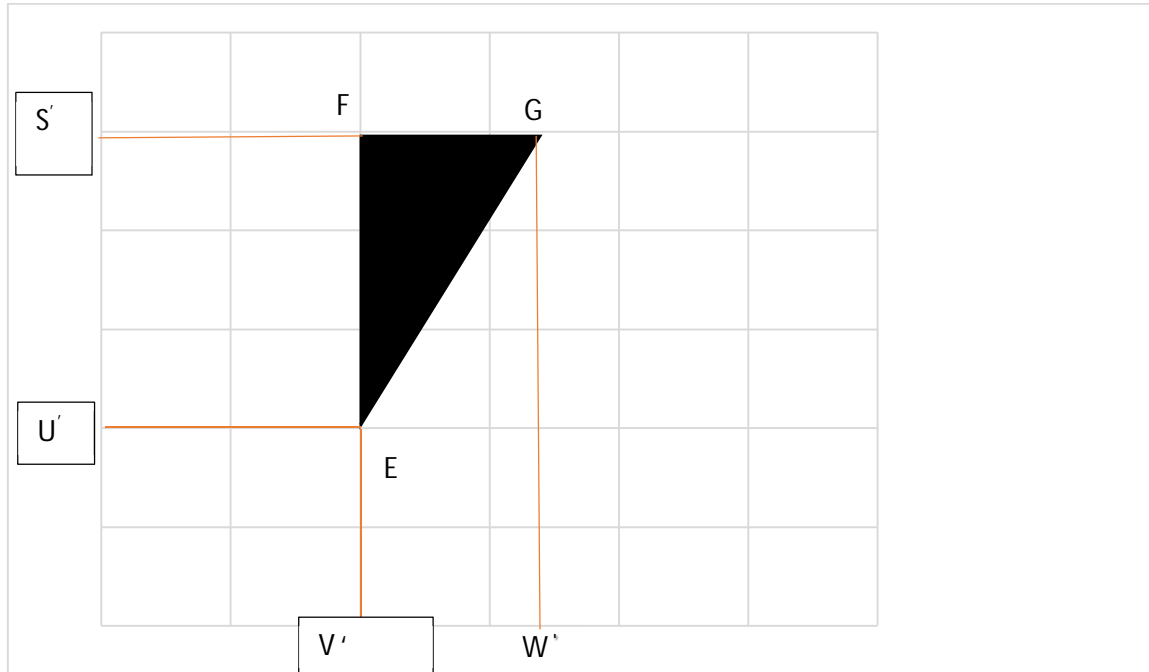
$$\frac{\left(\frac{H_{27}}{2}\right)^2 - \left(\frac{\varphi(S_{27})}{2}\right)^2 + \left(\frac{H_{41}}{2}\right)^2 - \left(\frac{\varphi(S_{41})}{2}\right)^2}{2} = 1729 .$$

2) *Result 3.2:* If  $m=P_1 \cdot P_2$  ( $P_1 < P_2$ ), divisors of  $m$  are 1,  $P_1, P_2, P_1 \cdot P_2$ .

Let E [ $HCF(1, P_1), LCM(1, P_1)$ ],

F [ $HCF(P_1, P_2), LCM(P_1, P_2)$ ], and

G [ $HCF(P_2, P_1 \cdot P_2), LCM(P_2, P_1 \cdot P_2)$ ] are the vertices of triangle EFG



Let Area of Rectangle  $S'U'EF = R$  and Area of trapezium

$V'W'GE = K$  then,

(a)  $\varphi(m) = R - \sqrt{H^2 - R^2}$

(b)  $\varphi(m) = 2R - \sqrt{H^2 + 4(K - R)}$

where H is the length of hypotenuse of right angle triangle EFG .

Proof :  $EG = \sqrt{(p_2 - 1)^2 + (p_1 \cdot p_2 - p_1)^2}$

$$= (p_2 - 1) \cdot \sqrt{1 + p_1^2}$$

$$EG = H = (p_2 - 1) \cdot \sqrt{1 + p_1^2}$$

Area of Rectangle =  $S'U'EF = R = p_1(p_2 - 1)$

Area of Trapezium =  $V'W'GE =$

$$K = \frac{1}{2} p_1(p_2 - 1)(p_2 + 1)$$

Put  $R = p_1(p_2 - 1)$  and  $H = (p_2 - 1) \cdot \sqrt{1 + p_1^2}$  in (a) that is  $R - \sqrt{H^2 - R^2}$ , We get

$$p_1(p_2 - 1) - \sqrt{(p_2 - 1)^2(1 + p_1^2) - p_1^2(p_2 - 1)^2}$$

$$= p_1(p_2 - 1) - (p_2 - 1)$$

$$= (p_2 - 1)(p_1 - 1)$$

$$= (p_1 - 1)(p_2 - 1)$$

$$= \varphi(m)$$

Hence (a) part is proved.

Now, put  $R = p_1(p_2 - 1)$ ,  $H = (p_2 - 1) \sqrt{1 + p_1^2}$

And  $K = \frac{1}{2} p_1(p_2 - 1)(p_2 + 1)$  in (b) that is

$2R - \sqrt{H^2 + 4(K - R)}$ , we get

$$= 2p_1(p_2 - 1) - [(p_2 - 1)^2(1 + p_1^2) + 2\{p_1(p_2 - 1)(p_2 + 1) - 2p_1(p_2 - 1)\}]^{1/2}$$

$$\begin{aligned}
 &= 2p_1(p_2-1) - [(p_2-1)^2(1+p_1^2)+2 p_1(p_2-1)(p_2+1-2)]^{1/2} \\
 &= 2p_1(p_2-1) - [(p_2-1)^2(1+p_1^2)+2 p_1(p_2-1)^2]^{1/2} \\
 &= 2p_1(p_2-1) - [(p_2-1)^2(1+p_1^2+2p_1)]^{1/2} \\
 &= 2p_1(p_2-1) - (p_2-1)(p_1+1) \\
 &= (p_2-1)[2p_1-p_1-1] \\
 &= (p_2-1)(p_1-1) \\
 &= (p_1-1)(p_2-1) \\
 &= \varphi(m)
 \end{aligned}$$

Hence (b) part is proved.

3) *Result 3.3:* If  $m=p_1.p_2$  where  $p_1, p_2$  are the primes ( $p_1 < p_2$ ), divisors of  $m$  are  $1, p_1, p_2, p_1.p_2$ . let  $E [HCF(1, p_1), LCM(1, p_1)]$ ,  $F [HCF(p_1, p_2), LCM(p_1, p_2)]$  and  $G [HCF(p_2, p_1.p_2), LCM(p_2, p_1.p_2)]$  are the vertices of triangle EFG, then roots of the quadratic equation

$$2. (\varphi(p_1))^2 x^2 - 2(\varphi(p_1))^2 x^1 - p_1 = 0 \quad \text{are } \frac{\sqrt{H^2-4T} \pm H}{2\sqrt{H^2-4T}}$$

Proof :-  $2. (\varphi(p_1))^2 x^2 - 2(\varphi(p_1))^2 x^1 - p_1 = 0$

$$x = \frac{(p_1-1) \pm \sqrt{1+p_1^2}}{2(p_1-1)}$$

Since ,  $H = (p_2 - 1) \sqrt{1 + p_1^2}$  and  $T = \frac{1}{2} p_1 (p_2 - 1)^2$

Where T is the area of triangle EFG

$$\frac{\sqrt{H^2 - 4T} \pm H}{2\sqrt{H^2 - 4T}} = \frac{(p_1 - 1) \pm \sqrt{1 + p_1^2}}{2(p_1 - 1)}$$

Hence, Proved.

4) *Result 3.4:*

*If  $m = p_1.p_2$  where  $p_1$  is either 2 or 3 and*

*$p_1 < p_2$ , Divisors of  $m$  are  $1, p_1, p_2, p_1.p_2$ .*

*Let  $E[HCF(1, p_1), LCM(1, p_1)]$ ,  $F[HCF(p_1, p_2), LCM(p_1, p_2)]$*

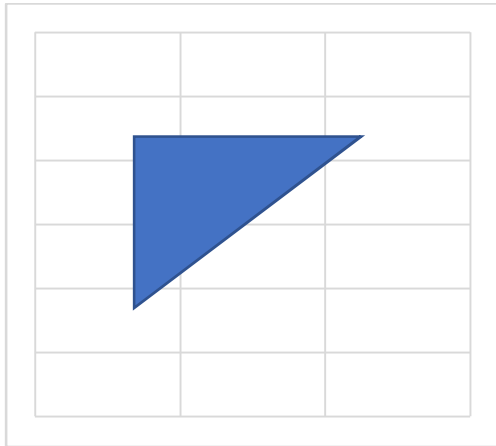
*And  $G[HCF(p_2, p_1.p_2), LCM(p_2, p_1.p_2)]$  are the vertices of triangle EFG , then*

$$2. \lim_{n \rightarrow \infty} \left( \frac{U_{n+1}}{U_n} \right) - 1 = \sqrt{\frac{4T + (\varphi(m))^2}{\varphi(p_2) \cdot \varphi(m)}} = \frac{H}{\varphi(p_2) \cdot \sqrt{\varphi(p_1)}}$$

*where T is the area of triangle EFG.*

**Proof :** *If  $m = p_1.p_2$ , then  $\varphi(m) = (p_1 - 1) \cdot (p_2 - 1)$ ,*





Vertices of above triangle EFG are  $E(1,p_1)$ ,  $F(1,p_1.p_2)$  and  $G(p_2,p_1.p_2)$ .

$$T = \frac{1}{2} \cdot EF \cdot FG$$

$$\text{Or, } T = \frac{1}{2} \cdot p_1 \cdot (p_2 - 1)^2$$

Put  $T$  and  $\varphi(m)$  in R. H. S,

$$\sqrt{\frac{2p_1(p_2 - 1)^2 + (p_1 - 1)^2(p_2 - 1)^2}{(p_1 - 1)(p_2 - 1)^2}} = \sqrt{\frac{p_1^2 + 1}{p_1 - 1}}$$

(i) If  $p_1 = 2$ , then  $\sqrt{\frac{p_1^2 + 1}{p_1 - 1}} = \sqrt{5}$

(ii) If  $p_1 = 3$ , then  $\sqrt{\frac{p_1^2 + 1}{p_1 - 1}} = \sqrt{5}$

Since, for  $n \geq 1$   $\lim_{n \rightarrow \infty} \left( \frac{U_{n+1}}{U_n} \right) = \frac{1 + \sqrt{5}}{2}$

Or,  $2 \cdot \lim_{n \rightarrow \infty} \left( \frac{U_{n+1}}{U_n} \right) - 1 = \sqrt{5}$

Hence,  $2 \cdot \lim_{n \rightarrow \infty} \left( \frac{U_{n+1}}{U_n} \right) - 1 = \sqrt{\frac{4T + (\varphi(m))^2}{\varphi(p_2) \cdot \varphi(m)}} = \frac{H}{\varphi(p_2) \cdot \sqrt{\varphi(p_1)}}$ .

#### IV. ANGLES OF TRIANGLE EFG IN TERMS OF EULERS PHI FUNCTION

1) *Theorem 4.1:* If  $m=p_1.p_2$  where  $p_1, p_2$  are the primes ( $p_1 < p_2$ ), divisors of  $m$  are  $1, p_1, p_2, p_1.p_2$ . let  $E$  [ $HCF(1, p_1)$ ,  $LCM(1, p_1)$ ],  $F$  [ $HCF(p_1, p_2)$ ,  $LCM(p_1, p_2)$ ] and  $G$  [ $HCF(p_2, p_1.p_2)$ ,  $LCM(p_2, p_1.p_2)$ ] are the vertices of triangle EFG and angle  $E = \alpha$  and angle  $G = \beta$ , then the angles of triangle EFG ( in terms of euler's phi function ) are  $\alpha = \frac{1}{2} \cot^{-1} \left[ \frac{\varphi(m) \sqrt{2H^2 - (\varphi(m))^2}}{H^2 - (\varphi(m))^2} \right]$  and

$$\beta = \frac{1}{2} \cot^{-1} \left[ \frac{\varphi(m) \sqrt{2H^2 - (\varphi(m))^2}}{(\varphi(m))^2 - H^2} \right]$$

**Proof :** Put  $\varphi(m) = (p_1 - 1)(p_2 - 1)$  and

$$H = (p_2 - 1) \sqrt{1 + p_1^2} \text{ in}$$

$$\alpha = \frac{1}{2} \cot^{-1} \left[ \frac{\varphi(m) \sqrt{2H^2 - (\varphi(m))^2}}{H^2 - (\varphi(m))^2} \right],$$

$$= \frac{1}{2} \cot^{-1} \left[ \frac{(p_1 - 1)(p_2 - 1) \{ 2(p_2 - 1)^2 (1 + p_1^2) - (p_1 - 1)^2 (p_2 - 1)^2 \}^{1/2}}{(p_2 - 1)^2 (1 + p_1^2) - (p_1 - 1)^2 (p_2 - 1)^2} \right]$$

$$= \frac{1}{2} \cot^{-1} \left[ \frac{(p_1 - 1) \{ 2(1 + p_1^2) - (p_1 - 1)^2 \}^{1/2}}{\{ (1 + p_1^2) - (p_1 - 1)^2 \}} \right]$$

$$= \frac{1}{2} \cot^{-1} \left[ \frac{(p_1 - 1) \{ 2 + 2p_1^2 - p_1^2 - 1 + 2p_1 \}^{1/2}}{1 + p_1^2 - p_1^2 - 1 + 2p_1} \right]$$

$$= \frac{1}{2} \cot^{-1} \left[ \frac{(p_1 - 1) \{ p_1^2 + 2p_1 + 1 \}^{1/2}}{2p_1} \right]$$

$$= \frac{1}{2} \cot^{-1} \left[ \frac{(p_1 - 1)(p_1 + 1)}{2p_1} \right]$$

$$= \frac{1}{2} \cot^{-1} \left[ \frac{p_1^2 - 1}{2p_1} \right] = \frac{1}{2} \tan^{-1} \left[ \frac{2p_1}{p_1^2 - 1} \right] \text{-----} (*)$$

Since,  $\tan \alpha = \frac{GF}{EF}$

Or,  $\tan \alpha = \frac{p_2 - 1}{p_1(p_2 - 1)} = \frac{1}{p_1}$

From (\*) equation,

$$\frac{1}{2} \tan^{-1} \left[ \frac{2p_1}{p_1^2 - 1} \right] = \alpha$$

Or,  $\tan(2\alpha) = \frac{2p_1}{p_1^2 - 1}$

Or,  $\frac{2 \tan \alpha}{1 - \tan^2 \alpha} = \frac{2p_1}{p_1^2 - 1}$

Put  $\alpha = \frac{1}{p_1}$ ,

$$\frac{2 \times \frac{1}{p_1}}{1 - \frac{1}{p_1^2}} = \frac{2p_1}{p_1^2 - 1}$$

L.H.S = R.H.S

Hence, Proved.

Since,  $\alpha + \beta = \frac{\pi}{2}$

$$\frac{1}{2} \tan^{-1} \left[ \frac{2p_1}{p_1^2 - 1} \right] + \beta = \frac{\pi}{2}$$

$$\beta = \frac{\pi}{2} - \frac{1}{2} \tan^{-1} \left[ \frac{2p_1}{p_1^2 - 1} \right]$$

$$\beta = \frac{1}{2} \left[ \pi - \tan^{-1} \left\{ \frac{2p_1}{p_1^2 - 1} \right\} \right]$$

$$\text{Or, } \beta = \frac{1}{2} \left[ \pi - \cot^{-1} \left\{ \frac{p_1^2 - 1}{2p_1} \right\} \right]$$

$$\beta = \frac{1}{2} \left[ \cot^{-1} \left\{ \frac{1 - p_1^2}{2p_1} \right\} \right]$$

$$\text{Hence, } \beta = \frac{1}{2} \cot^{-1} \left[ \frac{\varphi(m) \sqrt{2H^2 - (\varphi(m))^2}}{(\varphi(m))^2 - H^2} \right]$$

### V. OBSERVATIONS

(5.1) If  $m = 2.3.5$ , then  $w_4 = 18$ ,  $w_5 = 4$ , and  $w_4 + w_5 = 22$ .

If  $m = 3.5.7$ , then  $w_4 = 19$ ,  $w_5 = 3$ , and  $w_4 + w_5 = 22$

So,  $w_4 + w_5$  is constant but  $w_4, w_5$  are not constant.

If  $m = p_1.p_2.p_3$ , for different  $m$   $w_1, w_2, w_3$  and  $w_4 + w_5$  are constants.

$w_1 = 10, w_2 = 36, w_3 = 2$  and  $w_4 + w_5 = 22$ .

$$w_2 > w_4 + w_5 > w_1 > w_3.$$

Total number of right angle triangles =  $w_3 + w_4 + w_5$

$$= \varphi(\sum_{k=1}^5 w_k) = \varphi(8c_4) =$$

$$\varphi[\sum_{i=1}^5 i \{ \tau(m) - 2 - i \} \text{th triangular number}] = 24.$$

$$\text{Or, } w_3 + w_4 + w_5 = \varphi(w_1 + w_2 + w_3 + w_4 + w_5)$$

$$\text{Or, } \sum_{k=1}^5 w_k - \varphi(\sum_{k=1}^5 w_k) = w_1 + w_2$$

$$\text{That is } 70 - 24 = 10 + 36$$

$$\text{From Theorem 2.1 : } w_5 \geq z - 1 + z^{-1}c_2$$

If  $m = p_1.p_2.p_3$ , then,

$$\varphi(\sum_{k=1}^5 w_k) \geq z - 1 + z^{-1}c_2 + w_3 + w_4, \text{ Where } \sum_{k=1}^5 w_k = \text{Total No. of groups.}$$

$$\text{And } \varphi(\sum_{k=1}^5 w_k) - z + 1 - z^{-1}c_2 - w_3 \geq w_4 \geq \frac{z-1+z^{-1}c_2}{q_1} - w_5. \text{ where } q_1 \text{ is the probability of getting the inverted right angle triangle}$$

$$\text{whose area is } \left(\frac{H}{2}\right)^2 - \left(\frac{\varphi(S)}{2}\right)^2$$

### VI. APPLICATION OF EULER'S TOTIENT FUNCTION AND NUMBER THEORETIC FUNCTIONS IN STUDENT'S PENCIL COMPASS

Mathematical instruments are used to understand mathematical constructions and concepts . If we talk about mathematical instruments then there are many mathematical instruments like protractor , ruler , set-square , divider , pencil compass and etc . There are two types of mathematical instruments , one is used by student and other are used by teachers . Mathematical instruments used by teachers are very large as compare to mathematical instruments used by student because they are used in black boards or white boards . The mathematical instruments used by student are smaller because they are usable in their books only . If we talk about the student's pencil compass , then one can form the circles with a radius of about 1 to 15 centimeters .

Here we have applied the concepts of number theory .

If it is assumed that a circle of radius up to 20 centimeter is being formed by student's pencil compass then also the following result will be correct . Normally circles of radius up to 14 or 15 centimeters are formed by the student's pencil compass .

## VII. RESULT

If we draw all the possible circles by student's pencil compass having radius =  $r$  ( $r = 1, 2, 3, 4, \dots$ , maximum possible radius to be made by student's pencil compass), then 6 is the only number whose every positive proper divisors as a radius follow the relation

$$\varphi(2r) + \tau(2r) + \sigma(2r) = \text{greatest integer function of } P$$

and the number 6 (itself) as a radius follow the relation

$$\varphi(2r) + \tau(2r) + \sigma(2r) = \text{lowest integer function of } P.$$

$$\varphi(2r) + \tau(2r) + \sigma(2r) \text{ can also be written as } \varphi(d) + \tau(d) + \sigma(d),$$

where  $P = \pi \times d$ ,  $\pi = 3.1415$  (four digits after the decimal point),  $P$  is circumference(perimeter) of circle and  $d$  is diameter of circle.

NOTE: (a) Draw all the possible circles by student's pencil compass means to make circles of each radius which is made by student's pencil compass.

Where  $\varphi(d)$  is the Euler's phi function which denotes the number of positive integers not exceeding  $d$  that are relatively prime to  $d$ .

$\tau(d)$  denotes the number of positive divisors of  $d$  and  $\sigma(d)$  denotes the sum of these divisors.

In mathematical form: If  $r = 1$ ,  $\varphi(2) + \tau(2) + \sigma(2) = \text{greatest integer function of } 2 \times 3.1415 \times 1 = 6$ ,

If  $r = 2$ ,  $\varphi(4) + \tau(4) + \sigma(4) = \text{greatest integer function of } 2 \times 3.1415 \times 2 = 12$ ,

If  $r = 3$ ,  $\varphi(6) + \tau(6) + \sigma(6) = \text{greatest integer function of } 2 \times 3.1415 \times 3 = 18$ ,

If  $r = 6$ ,  $\varphi(12) + \tau(12) + \sigma(12) = \text{lowest integer function of } 2 \times 3.1415 \times 6 = 38$ ,

Where 1, 2 and 3 are the positive proper divisors of 6.

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