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Exposure of Positive Integer Solutions to an Equation $l^\alpha + m^\beta = \omega^\gamma$ Comprising Kaprekar Numbers

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Abstract: In this manuscript, the technique of evaluating all possible positive integer solutions to the Kaprekar number and an even perfect number of base 2 based on an equation $l^\alpha + m^\beta = \omega^\gamma$ is enlightened.

Keywords: Diophantine equation, Even perfect number, Kaprekar number.

I. INTRODUCTION

A number sequence is an ordered list of numbers that follow a specific rule or pattern. There are abundant research studies in the literature that concern number sequences, see [1 – 4]. In [6], Saranya. C and Janaki. G presented an approach to finding an infinite number of integer solutions to the generalized Fermat equations using Jarasandha numbers. In [10], Pandichelvi.V and Vanaja.R exhibited that there is no solution for the equation $p^x + (p + 2)^y = z^2$ together with the condition of $x + y$. In [11], the authors analysed an equation $(3u^2 + 5)p + (6u^2 + 11)q = w^2$ with $p + q = 0, 1, 2, 3$. In [12], an exponential Diophantine equation $q^x + p^y = z^2$ where p is a Safe prime, q is a Sophie Germain prime and x, y, z are non-negative integers such that $x + y = 0, 1, 2, 3$. is scrutinized. For a more comprehensive study of numbers, one may refer [5,7 – 9].

In this manuscript, an integer solution to the equation $l^\alpha + m^\beta = \omega^\gamma$ concerning a Kaprekar number and an even perfect number of base 2, a special type of Kaprekar number is exposed.

II. PRELIMINARIES

A. Even Perfect Number

Every number of the form $2^{p-1}(2^p - 1)$ where $2^p - 1$ is a Mersenne prime number for all prime values of p is called an even perfect number. Some even perfect numbers are 6, 28, 496, 8128, 33550336, etc

B. Kaprekar Number

A Kaprekar number is a number whose square is split up into two pieces such that their sum equal to the original number and in which none of the sections has a value of zero.

Take a look at an illustration, 7272 is a Kaprekar number because the square of 7272 is 52881984; divide it into two equal parts 5288 and 1984; and then add the two parts together $5288 + 1984$ to get back to 7272.

Here are a few examples.

$$\begin{aligned} 9^2 &= 81, & 8 + 1 &= 9; \\ 45^2 &= 2025, & 20 + 25 &= 45; \\ 55^2 &= 3025 & 30 + 25 &= 55. \end{aligned}$$

C. Special Kind of Kaprekar Number

An even perfect number is a Kaprekar number in base 2.

Example: 6 is an even perfect number, which is a Kaprekar number in base 2.

It is known that $6_2 = 110$

Then,

$$110^2 = 100100 = 10 + 0100 = 110$$

$$110^2 = 110$$

Hence, 6 is a Kaprekar number in base 2.

III. TECHNIQUE OF ANALYSIS

In this section, the technique for establishing numerous collections of positive integer solutions to the equation $l^\alpha + m^\beta = \omega^\gamma$ where k is a Kaprekar number is analysed in the following theorems.

A. Theorem 3.1

If k is a Kaprekar number, then the set of positive integer solutions of $l^\alpha + m^\beta = \omega^\gamma$ are

$$\begin{aligned} \begin{pmatrix} l \\ m \\ \omega \end{pmatrix} &= \begin{pmatrix} a^{(k+1)k^2t} (a^{(k+1)k^2t} + 1)^{k+1} \\ a^{(k-2)k^2t} (a^{(k+1)k^2t} + 1)^{k-1} \\ a^{(k+1)(k-2)t} (a^{(k+1)k^2t} + 1) \end{pmatrix} \\ \begin{pmatrix} l \\ m \\ \omega \end{pmatrix} &= \begin{pmatrix} a^{(k+2)k^2t} (a^{(k-1)k^2t} + 1)^{k+1} \\ a^{(k-1)k^2t} (a^{(k-1)k^2t} + 1)^{k-1} \\ a^{(k^2-1)t} (a^{(k-1)k^2t} + 1) \end{pmatrix} \\ \begin{pmatrix} l \\ m \\ \omega \end{pmatrix} &= \begin{pmatrix} a^{(k+1)(k^2+1)t} (a^{(k^2-1)t} + 1)^{k+1} \\ a^{(k-1)k^2t} (a^{(k^2-1)t} + 1)^{k-1} \\ a^{(k^2-1)t} (a^{(k^2-1)t} + 1) \end{pmatrix} \\ \begin{pmatrix} l \\ m \\ \omega \end{pmatrix} &= \begin{pmatrix} a^{(k-1)^2kt} (a^{(k-1)^2kt} + 1)^{k-1} \\ a^{(k-1)k^2t} (a^{(k-1)^2kt} + 1)^{k+1} \\ a^{(k-1)^2t} (a^{(k-1)^2kt} + 1) \end{pmatrix} \\ \begin{pmatrix} l \\ m \\ \omega \end{pmatrix} &= \begin{pmatrix} a^{k^3t} (a^{(k+1)k^2t} + 1)^{k-1} \\ a^{(k+1)k^2t} (a^{(k+1)k^2t} + 1)^{k+1} \\ a^{(k^2-1)t} (a^{(k+1)k^2t} + 1) \end{pmatrix} \quad \text{and} \\ \begin{pmatrix} l \\ m \\ \omega \end{pmatrix} &= \begin{pmatrix} a^{(k-1)(k^2+1)t} (a^{(k^2-1)t} + 1)^{k-1} \\ a^{(k+1)k^2t} (a^{(k^2-1)t} + 1)^{k+1} \\ a^{(k^2-1)t} (a^{(k^2-1)t} + 1) \end{pmatrix} \end{aligned}$$

Proof

$$\text{The equation to be analysed is } l^\alpha + m^\beta = \omega^\gamma \tag{3.1}$$

where l, m, ω belongs to Z^+ .

Let us consider the following cases.

1) Case 1

Let $\alpha = k - 1, \beta = k + 1$ and $\gamma = k^2$ where k is a Kaprekar number. Then the above equation becomes,

$$l^{k-1} + m^{k+1} = \omega^{k^2} \tag{3.2}$$

Subcase 1(i):

Assume that $l = a^x (a^x + 1)^{k+1}, m = a^y (a^y + 1)^{k-1}, \omega = a^z (a^z + 1)$

where $x, y, z \in Z^+$ and $n > 0$

Then, the original equation (3.2) can be transformed into the equation

$$a^{(k-1)x}(a^x + 1)^{k^2-1} + a^{(k+1)y}(a^x + 1)^{k^2-1} = a^{k^2z}(a^x + 1)^{k^2}$$

Equating the exponent on either side of the equation provides that

$$(k - 1)x = x + k^2 \tag{3.3}$$

$$(k + 1)y = k^2 z \tag{3.4}$$

It is deeply experiential that (3.3) and (3.4) are fulfilled by the succeeding chances of x, y and z

$$x = (k + 1)k^2t, y = (k - 2)k^2t \text{ and } z = (k + 1)(k - 2)t$$

Subsequently, the values of the variables l, m and ω are invented by

$$l = a^{(k+1)k^2t}(a^{(k+1)k^2t} + 1)^{k+1}$$

$$m = a^{(k-2)k^2t}(a^{(k+1)k^2t} + 1)^{k-1}$$

$$\omega = a^{(k+1)(k-2)t}(a^{(k+1)k^2t} + 1)$$

The following Table 3.1 displays specific numerical solutions for (3.1)

Table 3.1

| k | a | t | l | m | ω |
|-----|-----|-----|--------------------------------------|-------------------------------------|------------------------------|
| 9 | 2 | 1 | $2^{810}(2^{810} + 1)^{10}$ | $2^{567}(2^{810} + 1)^8$ | $2^{70}(2^{810} + 1)$ |
| 45 | 3 | 2 | $3^{186300}(3^{186300} + 1)^{46}$ | $3^{174150}(3^{186300} + 1)^{44}$ | $3^{3956}(3^{186300} + 1)$ |
| 55 | 2 | 2 | $2^{338800}(2^{338800} + 1)^{56}$ | $2^{320650}(2^{338800} + 1)^{54}$ | $2^{5936}(2^{338800} + 1)$ |
| 99 | 2 | 3 | $2^{2940300}(2^{2940300} + 1)^{100}$ | $2^{2852091}(2^{2940300} + 1)^{98}$ | $2^{29100}(2^{2940300} + 1)$ |

The values of each side of (3.2) are illustrated in the following table

Table 3.1(i)

| LHS of (3.2) | RHS of (3.2) |
|---|---|
| $2^{5670}(2^{810} + 1)^{81}$ | $2^{5670}(2^{810} + 1)^{81}$ |
| $3^{8010900}(3^{186300} + 1)^{2025}$ | $3^{8010900}(3^{186300} + 1)^{2025}$ |
| $2^{17956400}(2^{338800} + 1)^{3025}$ | $2^{17956400}(2^{338800} + 1)^{3025}$ |
| $2^{285209100}(2^{2940300} + 1)^{9801}$ | $2^{285209100}(2^{2940300} + 1)^{9801}$ |

Subcase 1(ii):

Replacing the chances of l, m and ω by $l = a^x(a^y + 1)^{k+1}$, $m = a^y(a^y + 1)^{k-1}$ and $\omega = a^z(a^y + 1)$ in (3.2) and employing the analogous approach described in subcase 1(i), the feasible values of x, y and z pointed out by

$$x = (k + 2)k^2t, y = (k - 1)k^2t \text{ and } z = (k^2 - 1)t$$

The corresponding triples (l, m, ω) satisfying the given equation is stated by

$$l = a^{(k+2)k^2t}(a^{(k-1)k^2t} + 1)^{k+1}$$

$$m = a^{(k-1)k^2t}(a^{(k-1)k^2t} + 1)^{k-1}$$

$$\omega = a^{(k^2-1)t}(a^{(k-1)k^2t} + 1)$$

Numerical specimens fulfilling (3.2) for few Kaprekar are shown in the following table 3.2

Table 3.2

| k | a | t | l | m | ω |
|-----|-----|-----|------------------------------------|-----------------------------------|----------------------------|
| 9 | 3 | 2 | $3^{1782}(3^{1296} + 1)^{10}$ | $3^{1296}(3^{1296} + 1)^8$ | $3^{160}(3^{1296} + 1)$ |
| 45 | 2 | 3 | $2^{285525}(2^{267300} + 1)^{46}$ | $2^{267300}(2^{267300} + 1)^{44}$ | $2^{6072}(2^{267300} + 1)$ |
| 55 | 2 | 2 | $2^{344850}(2^{326700} + 1)^{56}$ | $2^{326700}(2^{326700} + 1)^{54}$ | $2^{6048}(2^{326700} + 1)$ |
| 99 | 3 | 1 | $3^{989901}(3^{960498} + 1)^{100}$ | $3^{960498}(3^{960498} + 1)^{98}$ | $3^{9800}(3^{960498} + 1)$ |

The following table displays the values of each side of (3.2)

Table 3.2(i)

| LHS of (3.2) | RHS of (3.2) |
|--|--|
| $3^{12960} (3^{1296} + 1)^{81}$ | $3^{12960} (3^{1296} + 1)^{81}$ |
| $2^{12295800} (2^{267300} + 1)^{2025}$ | $2^{12295800} (2^{267300} + 1)^{2025}$ |
| $2^{18295200} (2^{326700} + 1)^{3025}$ | $2^{18295200} (2^{326700} + 1)^{3025}$ |
| $3^{96049800} (3^{960498} + 1)^{9801}$ | $3^{96049800} (3^{960498} + 1)^{9801}$ |

Subcase 1(iii):

Interchange the values of l, m, ω in (3.2) as mentioned below

$$l = a^x (a^z + 1)^{k+1}, m = a^y (a^z + 1)^{k-1} \text{ and } \omega = a^z (a^z + 1)$$

Following the technique as explained in subcase 1(i), the possible values of x, y and z are evaluated by

$$x = (k + 1)(k^2 + 1)t, y = (k - 1)k^2t \text{ and } z = (k^2 - 1)t$$

Consequently, the assessment of l, m and ω are indicated by

$$l = a^{(k+1)(k^2+1)t} (a^{(k^2-1)t} + 1)^{k+1}$$

$$m = a^{(k-1)k^2t} (a^{(k^2-1)t} + 1)^{k-1}$$

$$\omega = a^{(k^2-1)t} (a^{(k^2-1)t} + 1)$$

Verification of solutions with arithmetic values for little bit Kaprekar numbers are illustrated in the table 3.3

Table 3.3

| k | a | t | l | m | ω |
|-----|-----|-----|-------------------------------------|------------------------------------|-----------------------------|
| 9 | 2 | 1 | $2^{820} (2^{80} + 1)^{10}$ | $2^{648} (2^{80} + 1)^8$ | $2^{80} (2^{80} + 1)$ |
| 45 | 3 | 2 | $3^{186392} (3^{4048} + 1)^{46}$ | $3^{178200} (3^{4048} + 1)^{44}$ | $3^{4048} (3^{4048} + 1)$ |
| 55 | 2 | 2 | $2^{338912} (2^{6048} + 1)^{56}$ | $2^{326700} (2^{6048} + 1)^{54}$ | $2^{6048} (2^{6048} + 1)$ |
| 99 | 2 | 3 | $2^{2940600} (2^{29400} + 1)^{100}$ | $2^{2881494} (2^{29400} + 1)^{98}$ | $2^{29400} (2^{29400} + 1)$ |

The values of (3.2) on either side are shown in the table below

Table 3.3(i)

| LHS of (3.2) | RHS of (3.2) |
|--|--|
| $2^{6480} (2^{80} + 1)^{81}$ | $2^{6480} (2^{80} + 1)^{81}$ |
| $3^{8197200} (3^{4048} + 1)^{2025}$ | $3^{8197200} (3^{4048} + 1)^{2025}$ |
| $2^{18295200} (2^{6048} + 1)^{3025}$ | $2^{18295200} (2^{6048} + 1)^{3025}$ |
| $2^{288149400} (2^{29400} + 1)^{9801}$ | $2^{288149400} (2^{29400} + 1)^{9801}$ |

2) Case 2:

Let $\alpha = k + 1, \beta = k - 1$ and $\gamma = k^2$ where k is a kaprekar number. Then the equation (3.1) becomes,

$$l^{k+1} + m^{k-1} = \omega^{k^2} \tag{3.5}$$

Subcase 2(i):

Assume that $l = a^x(a^x + 1)^{k-1}$, $m = a^y(a^y + 1)^{k+1}$ and $\omega = a^z(a^z + 1)$ in the place l, m and ω in (3.5) and appealing the corresponding methodology labelled in subcase 1(i), the reasonable value of $x, y,$ and z are scrutinized by

$$x = (k - 1)^2 kt, y = (k - 1)k^2 t \text{ and } z = (k - 1)^2 t$$

Hence the value of l, m and ω satisfying (3.5) are given by

$$l = a^{(k-1)^2 kt} (a^{(k-1)^2 kt} + 1)^{k-1}$$

$$m = a^{(k-1)k^2 t} (a^{(k-1)^2 kt} + 1)^{k+1}$$

$$\omega = a^{(k-1)^2 t} (a^{(k-1)^2 kt} + 1)$$

The following Table 3.4 shows some numerical instances for the Kaprekar numbers

Table 3.4

| k | a | t | l | m | ω |
|-----|-----|-----|------------------------------------|-------------------------------------|-----------------------------|
| 9 | 3 | 2 | $3^{1152} (3^{1152} + 1)^8$ | $3^{1296} (3^{1152} + 1)^{10}$ | $3^{128} (3^{1152} + 1)$ |
| 45 | 2 | 3 | $2^{261360} (2^{261360} + 1)^{44}$ | $2^{267300} (2^{261360} + 1)^{46}$ | $2^{5808} (2^{261360} + 1)$ |
| 55 | 2 | 2 | $2^{320760} (2^{320760} + 1)^{54}$ | $2^{326700} (2^{320760} + 1)^{56}$ | $2^{5832} (2^{320760} + 1)$ |
| 99 | 3 | 1 | $3^{950796} (3^{950796} + 1)^{98}$ | $3^{960498} (3^{950796} + 1)^{100}$ | $3^{9604} (3^{950796} + 1)$ |

The table below offers illustrations illustrating the values for each side of (3.5)

Table 3.4(i)

| LHS of (3.5) | RHS of (3.5) |
|--|--|
| $3^{10368} (3^{1152} + 1)^{81}$ | $3^{10368} (3^{1152} + 1)^{81}$ |
| $2^{11761200} (2^{261360} + 1)^{2025}$ | $2^{11761200} (2^{261360} + 1)^{2025}$ |
| $2^{17641800} (2^{320760} + 1)^{3025}$ | $2^{17641800} (2^{320760} + 1)^{3025}$ |
| $3^{9694128804} (3^{950796} + 1)^{9801}$ | $3^{9694128804} (3^{950796} + 1)^{9801}$ |

Subcase 2(ii):

Adopting $l = a^x(a^y + 1)^{k-1}$, $m = a^y(a^y + 1)^{k+1}$ and $\omega = a^z(a^y + 1)$ instead of the values of l, m and ω in (3.5) and applying the procedure using in subcase 1(i), the above system of equation is fulfilled

$$x = k^3 t, y = (k + 1)k^2 t \text{ and } z = (k^2 - 1)t$$

Therefore, the values of l, m and ω satiating (3.5) are given by,

$$l = a^{k^3 t} (a^{(k+1)k^2 t} + 1)^{k-1}$$

$$m = a^{(k+1)k^2 t} (a^{(k+1)k^2 t} + 1)^{k+1}$$

$$\omega = a^{(k^2-1)t} (a^{(k+1)k^2 t} + 1)$$

Table 3.5 provides numerical verification for specific values of the Kaprekar number.

Table 3.5

| | a | t | l | m | ω |
|----|-----|-----|------------------------------------|-------------------------------------|-----------------------------|
| 9 | 3 | 2 | $3^{1458} (3^{1620} + 1)^8$ | $3^{1296} (3^{1620} + 1)^{10}$ | $3^{160} (3^{1620} + 1)$ |
| 45 | 2 | 3 | $2^{273375} (2^{279450} + 1)^{44}$ | $2^{267300} (2^{279450} + 1)^{46}$ | $2^{132} (2^{279450} + 1)$ |
| 55 | 2 | 2 | $2^{332750} (2^{338800} + 1)^{54}$ | $2^{267300} (2^{338800} + 1)^{56}$ | $2^{132} (2^{338800} + 1)$ |
| 99 | 3 | 1 | $3^{970299} (3^{980100} + 1)^{98}$ | $3^{980100} (3^{980100} + 1)^{100}$ | $3^{9800} (3^{980100} + 1)$ |

The table below displays the values of the equation (3.5) on both sides

Table 3.5(i)

| LHS of (3.5) | RHS of (3.5) |
|--|--|
| $3^{12960} (3^{1620} + 1)^{81}$ | $3^{12960} (3^{1620} + 1)^{81}$ |
| $2^{12295800} (2^{279450} + 1)^{2025}$ | $2^{12295800} (2^{279450} + 1)^{2025}$ |
| $2^{18295200} (2^{338800} + 1)^{3025}$ | $2^{18295200} (2^{338800} + 1)^{3025}$ |
| $3^{96049800} (3^{980100} + 1)^{9801}$ | $3^{96049800} (3^{980100} + 1)^{9801}$ |

Subcase 2(iii):

Agreeing $l = a^x(a^z + 1)^{k-1}$, $m = a^y(a^z + 1)^{k+1}$ and $\omega = a^z(a^z + 1)$ in (3.5) and

recapping the analogous procedure as illuminated in subcase1(i), the significant values of x, y and z are

$$x = (k - 1)(k^2 + 1)t, y = (k + 1)k^2t \text{ and } z = (k^2 - 1)t$$

Consequently, the assessment of l, m and ω gratifying (3.5) are indicated by

$$l = a^{(k-1)(k^2+1)t} (a^{(k^2-1)t} + 1)^{k-1}$$

$$m = a^{(k+1)k^2t} (a^{(k^2-1)t} + 1)^{k+1}$$

$$\omega = a^{(k^2-1)t} (a^{(k^2-1)t} + 1)$$

Integer solution for few numerical cases of Kaprekar numbers are presented in Table 3.6 below

Table 3.6

| k | a | t | l | m | ω |
|-----|-----|-----|------------------------------------|-------------------------------------|-----------------------------|
| 9 | 2 | 1 | $2^{656} (2^{80} + 1)^8$ | $2^{810} (2^{80} + 1)^{10}$ | $2^{80} (2^{80} + 1)$ |
| 45 | 3 | 2 | $3^{178288} (3^{4048} + 1)^{44}$ | $3^{186300} (3^{4048} + 1)^{46}$ | $3^{4048} (3^{4048} + 1)$ |
| 55 | 2 | 2 | $2^{326808} (2^{6048} + 1)^{54}$ | $2^{338800} (2^{6048} + 1)^{56}$ | $2^{6048} (2^{6048} + 1)$ |
| 99 | 3 | 3 | $3^{2881788} (3^{29400} + 1)^{98}$ | $3^{2940300} (3^{29400} + 1)^{100}$ | $3^{29400} (3^{29400} + 1)$ |

The values of (3.5) on either side are shown in the following table

Table 3.6(i)

| LHS of (3.5) | RHS of (3.5) |
|--|--|
| $2^{6480} (2^{80} + 1)^{81}$ | $2^{6480} (2^{80} + 1)^{81}$ |
| $3^{8197200} (3^{4048} + 1)^{2025}$ | $3^{8197200} (3^{4048} + 1)^{2025}$ |
| $2^{18295200} (2^{6048} + 1)^{3025}$ | $2^{18295200} (2^{6048} + 1)^{3025}$ |
| $3^{288149400} (3^{29400} + 1)^{9801}$ | $3^{288149400} (3^{29400} + 1)^{9801}$ |

B. Theorem 3.2

If k is an even perfect number in base 2, a special sort of Kaprekar number, then all positive integers solution for $l^{k-1} + m^{k+1} = \omega^{k^2}$ are

$$\begin{pmatrix} l \\ m \\ \omega \end{pmatrix} = \begin{pmatrix} a^{(2^{2p-1}-2^{p-1}+1)(2^{2p-1}-2^{p-1})^2 t} (a^{(2^{2p-1}-2^{p-1}+1)(2^{2p-1}-2^{p-1})^2 t} + 1)^{2^{2p-1}-2^{p-1}+1}, \\ a^{(2^{2p-1}-2^{p-1}-2)(2^{2p-1}-2^{p-1})^2 t} (a^{(2^{2p-1}-2^{p-1}+1)(2^{2p-1}-2^{p-1})^2 t} + 1)^{2^{2p-1}-2^{p-1}-1}, \\ a^{(2^{2p-1}-2^{p-1}+1)(2^{2p-1}-2^{p-1}-2)t} (a^{(2^{2p-1}-2^{p-1}+1)(2^{2p-1}-2^{p-1})^2 t} + 1) \end{pmatrix}$$

$$\begin{aligned} \binom{l}{m} &= \binom{a^{(2^{2p-1}-2^{p-1}+2)(2^{2p-1}-2^{p-1})^2 t} (a^{(2^{2p-1}-2^{p-1}-1)(2^{2p-1}-2^{p-1})^2 t} + 1)^{(2^{2p-1}-2^{p-1}+1)},}{a^{(2^{2p-1}-2^{p-1}-1)(2^{2p-1}-2^{p-1})^2 t} (a^{(2^{2p-1}-2^{p-1}-1)(2^{2p-1}-2^{p-1})^2 t} + 1)^{(2^{2p-1}-2^{p-1}-1)},}{a^{((2^{2p-1}-2^{p-1})^2-1)t} (a^{(2^{2p-1}-2^{p-1}-1)(2^{2p-1}-2^{p-1})^2 t} + 1)} \\ \binom{l}{m} &= \binom{a^{(2^{2p-1}-2^{p-1}+1)((2^{2p-1}-2^{p-1})^2+1)t} (a^{((2^{2p-1}-2^{p-1})^2-1)t} + 1)^{2^{2p-1}-2^{p-1}+1},}{a^{(2^{2p-1}-2^{p-1}-1)(2^{2p-1}-2^{p-1})^2 t} (a^{((2^{2p-1}-2^{p-1})^2-1)t} + 1)^{2^{2p-1}-2^{p-1}-1},}{a^{((2^{2p-1}-2^{p-1})^2-1)t} (a^{((2^{2p-1}-2^{p-1})^2-1)t} + 1)} \\ \binom{l}{m} &= \binom{a^{(2^{2p-1}-2^{p-1}-1)^2(2^{2p-1}-2^{p-1})t} (a^{(2^{2p-1}-2^{p-1}-1)^2(2^{2p-1}-2^{p-1})t} + 1)^{2^{2p-1}-2^{p-1}-1},}{a^{(2^{2p-1}-2^{p-1}-1)k^2 t} (a^{(2^{2p-1}-2^{p-1}-1)^2(2^{2p-1}-2^{p-1})t} + 1)^{2^{2p-1}-2^{p-1}+1}},}{a^{(2^{2p-1}-2^{p-1}-1)^2 t} (a^{(2^{2p-1}-2^{p-1}-1)^2(2^{2p-1}-2^{p-1})t} + 1)} \\ \binom{l}{m} &= \binom{a^{(2^{2p-1}-2^{p-1})^3 t} (a^{(2^{2p-1}-2^{p-1}+1)(2^{2p-1}-2^{p-1})^2 t} + 1)^{2^{2p-1}-2^{p-1}-1},}{a^{(2^{2p-1}-2^{p-1}+1)(2^{2p-1}-2^{p-1})^2 t} (a^{(2^{2p-1}-2^{p-1}+1)k^2 t} + 1)^{2^{2p-1}-2^{p-1}+1}},}{a^{((2^{2p-1}-2^{p-1})^2-1)t} (a^{(2^{2p-1}-2^{p-1}+1)(2^{2p-1}-2^{p-1})^2 t} + 1)} \\ \binom{l}{m} &= \binom{a^{(2^{2p-1}-2^{p-1}-1)((2^{2p-1}-2^{p-1})^2+1)t} (a^{((2^{2p-1}-2^{p-1})^2-1)t} + 1)^{2^{2p-1}-2^{p-1}-1},}{a^{(2^{2p-1}-2^{p-1}+1)(2^{2p-1}-2^{p-1})^2 t} (a^{((2^{2p-1}-2^{p-1})^2-1)t} + 1)^{2^{2p-1}-2^{p-1}+1}},}{a^{((2^{2p-1}-2^{p-1})^2-1)t} (a^{((2^{2p-1}-2^{p-1})^2-1)t} + 1)} \end{aligned}$$

Proof

The equation to be analysed is $l^\alpha + m^\beta = \omega^\gamma$ (3.6)

where l, m, ω belongs to Z^+ .

Let us deliberate on the subsequent cases.

1) Case 1

Let $\alpha = k - 1$, $\beta = k + 1$ and $\gamma = k^2$ where k is a Kaprekar number which is an even perfect number in base 2. That is $k = 2^{p-1}(2^p - 1)$ in base 2 where p is a prime number.

Then the above equation (3.6) becomes,

$$l^{k-1} + m^{k+1} = \omega^{k^2} \tag{3.7}$$

Subcase 1(i):

Acknowledge that $l = a^x(a^x + 1)^{2^{2p-1}-2^{p-1}+1}$, $m = a^y(a^y + 1)^{2^{2p-1}-2^{p-1}-1}$

$\omega = a^z(a^z + 1)$ in (3.7) and repeating the corresponding technique as illuminated in the Theorem 3.1, the preference of x, y and z are

$$\begin{aligned} x &= (2^{2p-1} - 2^{p-1} + 1)(2^{2p-1} - 2^{p-1})^2 t \\ y &= (2^{2p-1} - 2^{p-1} - 2)(2^{2p-1} - 2^{p-1})^2 t \\ z &= (2^{2p-1} - 2^{p-1} + 1)(2^{2p-1} - 2^{p-1} - 2)t \end{aligned}$$

and hence

$$\begin{aligned} l &= a^{(2^{2p-1}-2^{p-1}+1)(2^{2p-1}-2^{p-1})^2 t} (a^{(2^{2p-1}-2^{p-1}+1)(2^{2p-1}-2^{p-1})^2 t} + 1)^{2^{2p-1}-2^{p-1}+1} \\ m &= a^{(2^{2p-1}-2^{p-1}-2)(2^{2p-1}-2^{p-1})^2 t} (a^{(2^{2p-1}-2^{p-1}+1)(2^{2p-1}-2^{p-1})^2 t} + 1)^{2^{2p-1}-2^{p-1}-1} \\ \omega &= a^{(2^{2p-1}-2^{p-1}+1)(2^{2p-1}-2^{p-1}-2)t} (a^{(2^{2p-1}-2^{p-1}+1)(2^{2p-1}-2^{p-1})^2 t} + 1) \end{aligned}$$

For relevant choices of k , a few statistical examples are explicitly described in the next table 3.7

Table 3.7

| p | a | t | k | $\binom{l}{m}{\omega}$ | LHS and RHS of (3.7) |
|-----|-----|-----|-----|---|---|
| 2 | 2 | 1 | 6 | $\binom{2^{252}(2^{252} + 1)^7}{2^{144}(2^{252} + 1)^5}{2^{28}(2^{252} + 1)}$ | $2^{1008}(2^{252} + 1)^{36}$ |
| 3 | 3 | 3 | 28 | $\binom{3^{68208}(3^{68208} + 1)^{29}}{3^{61152}(3^{68208} + 1)^{27}}{3^{2262}(3^{68208} + 1)}$ | $3^{96049800}(3^{9800} + 1)^{9801}$ |
| 5 | 2 | 1 | 496 | $\binom{2^{122269952}(2^{122269952} + 1)^{497}}{2^{121531904}(2^{122269952} + 1)^{495}}{2^{245518}(2^{122269952} + 1)}$ | $2^{1992009996000}(2^{1996000} + 1)^{998001}$ |

Subcase 1(ii):

$$\text{Implementing } = a^x(a^y + 1)^{(2^{2p-1}-2^{p-1}+1)}, m = a^y(a^y + 1)^{(2^{2p-1}-2^{p-1}-1)}$$

$\omega = a^z(a^z + 1)$ in (3.7) and the possible values of x, y and z are determined by using the similar style presented in Theorem 3.1,

$$x = (2^{2p-1} - 2^{p-1} + 2)(2^{2p-1} - 2^{p-1})^2 t$$

$$y = (2^{2p-1} - 2^{p-1} - 1)(2^{2p-1} - 2^{p-1})^2 t$$

$$z = ((2^{2p-1} - 2^{p-1})^2 - 1)$$

The subsequent solutions l, m , and ω satisfying the given equation $l^{k-1} + m^{k+1} = \omega^{k^2}$ are

$$l = a^{(2^{2p-1}-2^{p-1}+2)(2^{2p-1}-2^{p-1})^2 t} (a^{(2^{2p-1}-2^{p-1}-1)(2^{2p-1}-2^{p-1})^2 t} + 1)^{(2^{2p-1}-2^{p-1}+1)}$$

$$m = a^{(2^{2p-1}-2^{p-1}-1)(2^{2p-1}-2^{p-1})^2 t} (a^{(2^{2p-1}-2^{p-1}-1)(2^{2p-1}-2^{p-1})^2 t} + 1)^{(2^{2p-1}-2^{p-1}-1)}$$

$$\omega = a^{((2^{2p-1}-2^{p-1})^2 - 1)t} (a^{(2^{2p-1}-2^{p-1}-1)(2^{2p-1}-2^{p-1})^2 t} + 1)$$

The next table 3.8 explicitly lists a few statistical evidence for applicable Kaprekar numeral selections.

Table 3.8

| p | a | t | k | $\binom{l}{m}{\omega}$ | LHS and RHS of (3.7) |
|-----|-----|-----|-----|---|---|
| 2 | 2 | 3 | 6 | $\binom{2^{864}(2^{540} + 1)^7}{2^{540}(2^{540} + 1)^5}{2^{105}(2^{540} + 1)}$ | $2^{3780}(2^{540} + 1)^{36}$ |
| 3 | 3 | 2 | 28 | $\binom{3^{47040}(3^{42336} + 1)^{29}}{3^{42336}(3^{42336} + 1)^{27}}{3^{1566}(3^{42336} + 1)}$ | $3^{1227744}(3^{42336} + 1)^{784}$ |
| 5 | 2 | 1 | 496 | $\binom{2^{122515968}(2^{121777920} + 1)^{497}}{2^{121777920}(2^{121777920} + 1)^{495}}{2^{246015}(2^{121777920} + 1)}$ | $2^{60523626240}(2^{121777920} + 1)^{246016}$ |

Subcase 1(iii):

Approving $= a^x(a^z + 1)^{2^{2p-1}-2^{p-1}+1}$, $m = a^y(a^z + 1)^{2^{2p-1}-2^{p-1}-1}$, $\omega = a^z(a^z + 1)$ in (3.7) and following an approach described in the Theorem 3.1, the potential values of x, y and z are

$$x = (2^{2p-1} - 2^{p-1} + 1)((2^{2p-1} - 2^{p-1})^2 + 1)t,$$

$$y = (2^{2p-1} - 2^{p-1} - 1)(2^{2p-1} - 2^{p-1})^2 t \text{ and}$$

$$z = ((2^{2p-1} - 2^{p-1})^2 - 1)t$$

Accordingly, the calculation of l, m and ω are specified by

$$l = a^{(2^{2p-1}-2^{p-1}+1)((2^{2p-1}-2^{p-1})^2+1)t} \left(a^{((2^{2p-1}-2^{p-1})^2-1)t} + 1 \right)^{2^{2p-1}-2^{p-1}+1},$$

$$m = a^{(2^{2p-1}-2^{p-1}-1)(2^{2p-1}-2^{p-1})^2 t} \left(a^{((2^{2p-1}-2^{p-1})^2-1)t} + 1 \right)^{2^{2p-1}-2^{p-1}-1} \text{ and}$$

$$\omega = a^{((2^{2p-1}-2^{p-1})^2-1)t} \left(a^{((2^{2p-1}-2^{p-1})^2-1)t} + 1 \right)$$

A few precise cases in point for the Kaprekar numbers are listed in table 3.9 below.

Table 3.9

| p | a | t | k | $\begin{pmatrix} l \\ m \\ \omega \end{pmatrix}$ | LHS and RHS of (3.7) |
|-----|-----|-----|-----|--|---|
| 2 | 2 | 3 | 6 | $\begin{pmatrix} (2^{259}(2^{35} + 1)^7) \\ 2^{180}(2^{35} + 1)^5 \\ 2^{35}(2^{35} + 1) \end{pmatrix}$ | $2^{1260}(2^{35} + 1)^{36}$ |
| 3 | 3 | 2 | 28 | $\begin{pmatrix} (3^{45530}(3^{1566} + 1)^{29}) \\ 3^{42336}(3^{1566} + 1)^{27} \\ 3^{1566}(3^{1566} + 1) \end{pmatrix}$ | $3^{1227744}(3^{1566} + 1)^{784}$ |
| 5 | 2 | 1 | 496 | $\begin{pmatrix} 2^{366811347}(2^{738045} + 1)^{497} \\ 2^{365333760}(2^{738045} + 1)^{495} \\ 2^{738045}(2^{738045} + 1) \end{pmatrix}$ | $2^{181570878720}(2^{738045} + 1)^{246016}$ |

2) Case 2:

Let $\alpha = k + 1, \beta = k - 1$ and $\gamma = k^2$ where $k = 2^{2p-1} - 2^{p-1}$.

Then the given equation turns out to be

$$l^{k+1} + m^{k-1} = \omega^{k^2} \tag{3.8}$$

Subcase 2(i):

Assume that

$$l = a^x(a^x + 1)^{2^{2p-1}-2^{p-1}-1},$$

$$m = a^y(a^y + 1)^{2^{2p-1}-2^{p-1}+1},$$

$\omega = a^z(a^z + 1)$ in equation (3.8) and employing the corresponding process established by the Theorem 3.1, the suitable

values of x, y and z are deliberated as

$$x = (2^{2p-1} - 2^{p-1} - 1)^2(2^{2p-1} - 2^{p-1})t,$$

$$y = (2^{2p-1} - 2^{p-1} - 1)(2^{2p-1} - 2^{p-1})^2 t \text{ and}$$

$$z = ((2^{2p-1} - 2^{p-1}) - 1)^2 t$$

Hence the value of l, m and ω satisfying (3.8) are given by

$$l = a^{(2^{2p-1}-2^{p-1}-1)^2(2^{2p-1}-2^{p-1})t} \left(a^{(2^{2p-1}-2^{p-1}-1)^2(2^{2p-1}-2^{p-1})t} + 1 \right)^{2^{2p-1}-2^{p-1}-1}$$

$$m = a^{(2^{2p-1}-2^{p-1}-1)k^2 t} \left(a^{(2^{2p-1}-2^{p-1}-1)^2(2^{2p-1}-2^{p-1})t} + 1 \right)^{2^{2p-1}-2^{p-1}+1}$$

$$\omega = a^{(2^{2p-1}-2^{p-1}-1)^2 t} \left(a^{(2^{2p-1}-2^{p-1}-1)^2(2^{2p-1}-2^{p-1})t} + 1 \right)$$

The following table 3.10 shows some numerical instances for the Kaprekar numbers.

Table 3.10

| p | a | t | k | $\begin{pmatrix} l \\ m \\ \omega \end{pmatrix}$ | LHS and RHS of (3.8) |
|-----|-----|-----|-----|---|--|
| 2 | 2 | 3 | 6 | $\begin{pmatrix} 2^{450}(2^{450} + 1)^5 \\ 2^{540}(2^{450} + 1)^7 \\ 2^{75}(2^{450} + 1) \end{pmatrix}$ | $2^{2700}(2^{450} + 1)^{36}$ |
| 3 | 3 | 2 | 28 | $\begin{pmatrix} 3^{40824}(3^{40824} + 1)^{27} \\ 3^{42336}(3^{40824} + 1)^{29} \\ 3^{1458}(3^{40824} + 1) \end{pmatrix}$ | $3^{1143072}(3^{40824} + 1)^{784}$ |
| 5 | 4 | 2 | 496 | $\begin{pmatrix} 4^{243064800}(4^{243064800} + 1)^{495} \\ 4^{243555840}(4^{243064800} + 1)^{497} \\ 4^{490050}(4^{243064800} + 1) \end{pmatrix}$ | $4^{120560140800}(4^{243064800} + 1)^{246016}$ |

Subcase 2(ii):

Adopting $l = a^x(a^y + 1)^{2^{2p-1}-2^{p-1}-1}$, $m = a^y(a^y + 1)^{2^{2p-1}-2^{p-1}+1}$, $\omega = a^z(a^y + 1)$ in (3.8) and applying the procedure used in Theorem.3.1, the above system is satisfied when

$$x = (2^{2p-1} - 2^{p-1})^3 t,$$

$$y = (2^{2p-1} - 2^{p-1} + 1)k^2 t \text{ and}$$

$$z = ((2^{2p-1} - 2^{p-1})^2 - 1)t$$

Therefore, the value of l, m and ω gratifying (3.8) are given by

$$l = a^{(2^{2p-1}-2^{p-1})^3 t} \left(a^{(2^{2p-1}-2^{p-1}+1)(2^{2p-1}-2^{p-1})^2 t} + 1 \right)^{2^{2p-1}-2^{p-1}-1}$$

$$m = a^{(2^{2p-1}-2^{p-1}+1)(2^{2p-1}-2^{p-1})^2 t} \left(a^{(2^{2p-1}-2^{p-1}+1)k^2 t} + 1 \right)^{2^{2p-1}-2^{p-1}+1}$$

$$\omega = a^{((2^{2p-1}-2^{p-1})^2-1)t} \left(a^{(2^{2p-1}-2^{p-1}+1)(2^{2p-1}-2^{p-1})^2 t} + 1 \right)$$

The next table 3.11 explicitly displays a few pieces of evidence for suitable Kaprekar numeral choices.

Table 3.11

| p | a | t | k | $\begin{pmatrix} l \\ m \\ \omega \end{pmatrix}$ | LHS and RHS of (3.8) |
|-----|-----|-----|-----|---|---|
| 2 | 2 | 3 | 6 | $\begin{pmatrix} 2^{648}(2^{756} + 1)^5 \\ 2^{756}(2^{756} + 1)^7 \\ 2^{105}(2^{756} + 1) \end{pmatrix}$ | $2^{3780}(2^{756} + 1)^{36}$ |
| 3 | 3 | 2 | 28 | $\begin{pmatrix} 3^{43904}(3^{45472} + 1)^{27} \\ 3^{45472}(3^{45472} + 1)^{29} \\ 3^{1566}(3^{45472} + 1) \end{pmatrix}$ | $3^{1227744}(3^{45472} + 1)^{784}$ |
| 5 | 5 | 1 | 496 | $\begin{pmatrix} 5^{122023936}(5^{122269952} + 1)^{495} \\ 5^{122269952}(5^{122269952} + 1)^{497} \\ 5^{246015}(5^{122269952} + 1) \end{pmatrix}$ | $5^{60523626240}(5^{122269952} + 1)^{246016}$ |

Subcase 2(iii):

Agreeing $l = a^x(a^z + 1)^{2^{2p-1}-2^{p-1}-1}$, $m = a^y(a^z + 1)^{2^{2p-1}-2^{p-1}+1}$ and $\omega = a^z(a^z + 1)$ in (3.8) and repeating the equivalent method as shown in the Theorem 3.1, x, y and z are acknowledged as

$$x = (2^{2p-1} - 2^{p-1} - 1)((2^{2p-1} - 2^{p-1})^2 + 1)t,$$

$$y = (2^{2p-1} - 2^{p-1} + 1)(2^{2p-1} - 2^{p-1})^2 t \text{ and}$$

$$z = ((2^{2p-1} - 2^{p-1})^2 - 1)t$$

Consequently, the assessment of l, m and ω satisfying (3.8) are indicated by,

$$l = a^{(2^{2p-1}-2^{p-1}-1)((2^{2p-1}-2^{p-1})^2+1)t} \left(a^{((2^{2p-1}-2^{p-1})^2-1)t} + 1 \right)^{2^{2p-1}-2^{p-1}-1}$$

$$m = a^{(2^{2p-1}-2^{p-1}+1)(2^{2p-1}-2^{p-1})^2t} \left(a^{((2^{2p-1}-2^{p-1})^2-1)t} + 1 \right)^{2^{2p-1}-2^{p-1}+1}$$

$$\omega = a^{((2^{2p-1}-2^{p-1})^2-1)t} \left(a^{((2^{2p-1}-2^{p-1})^2-1)t} + 1 \right)$$

Numerical verification for certain values of the Kaprekar number offered in the table 3.12

Table 3.12

| p | a | t | k | $\begin{pmatrix} l \\ m \\ \omega \end{pmatrix}$ | LHS and RHS of (3.8) |
|-----|-----|-----|-----|--|---|
| 2 | 4 | 2 | 6 | $\begin{pmatrix} (4^{370}(4^{70} + 1)^5) \\ 4^{504}(4^{70} + 1)^7 \\ 4^{70}(4^{70} + 1) \end{pmatrix}$ | $4^{2520}(4^{70} + 1)^{36}$ |
| 3 | 3 | 1 | 28 | $\begin{pmatrix} 3^{21195}(3^{783} + 1)^{27} \\ 3^{22736}(3^{783} + 1)^{29} \\ 3^{783}(3^{783} + 1) \end{pmatrix}$ | $3^{613872}(3^{783} + 1)^{784}$ |
| 5 | 2 | 2 | 496 | $\begin{pmatrix} 2^{243556830}(2^{492030} + 1)^{495} \\ 2^{244539904}(2^{492030} + 1)^{497} \\ 2^{492030}(2^{492030} + 1) \end{pmatrix}$ | $2^{121047252480}(2^{492030} + 1)^{246016}$ |

IV. CONCLUSION

In this transmission, a method for obtaining positive integer solution to the equation $l^a + m^b = \omega^c$ involving Kaprekar numbers is discovered. To conclude, one could look into another equation with different numbers.

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