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Extended Nested Array Configuration with Increased Degrees of Freedom for DOA Estimation

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Abstract: In this paper, a new linear array configuration inspired by the two-level Nested Array concept has been proposed. The proposed geometry comprises of two Uniform Linear Arrays (ULAs) with different spacings and an additional element, this geometry results in a hole-free Difference Co-Array (DCA). In comparison to most of the existing sparse array configurations, the proposed geometry has simple closed-form expressions for the element positions and Degrees of Freedom (DoFs), also providing consecutive DoFs hence, avoiding the need for spatial interpolation. Spatially Smoothed MUltiple SIgnal Classification (SS-MUSIC) algorithm has been deployed for Direction of Arrival (DOA) estimation. Numerical simulations using MATLAB are performed to exhibit the superior performance of the proposed geometry.
Keywords: DOA estimation, Co-array Processing, Spatial Smoothing, Degrees of Freedom, Non-Uniform Linear Arrays

I. INTRODUCTION

Array signal processing [1-2] has gained immense popularity as a valuable research topic due to the endless potential of improvisation owing to the ever-growing need for enhancing the capacity of the wireless communication systems. Direction of Arrival (DOA) estimation is a crucial aspect of various array signal processing applications, such as RADAR, Radio Astronomy, and Satellite communication, where the antenna arrays are essential for accumulating the spatial samples of incident signals. The simplest geometry employed in Antenna Arrays is the Uniform Linear Arrays (ULAs), which have limited Degrees of Freedom (DOFs) i.e number of signals that can be identified by the array (L-1 DoFs for a L element ULA). Thus, requiring elements to be increased in the array to detect more signals. As an alternative to increasing the number of elements in the arrays, several Non-Uniform Linear Array (NULA) geometries [1-9] were proposed. NULA requires the processing of the signals in the Co-Array Domain; hence the Difference Co-Array (DCA) plays a vital role. DCA is a difference set, containing all possible spatial lags that can be generated using the elements in the NULA. It leads to a virtual linear array with virtual elements at locations marked by the difference set, this virtual array has a much larger aperture than the NULA. Thus, NULAs have proven to have higher DoFs than the ULAs containing same number of elements due to the enhanced aperture of the resulting DCA. The DCA with no missing lags (holes) are preferred as presence of holes reduces the effective aperture of the virtual array and requires the additional process of 'Spatial Interpolation' to fill in the missing measurements to enable utilising the entire virtual aperture.

The first NULAs to be developed were the Minimum Redundancy Array (MRA) [1] and the Minimum Hole Array (MHA) [2], which increase DoFs and result in hole-free DCA. MRAs and MHAs do not have closed-form expressions for their geometrical configurations and rely on extensive computer simulations of various combinations of element placements. Over the last decade, two types of novel NULA geometries have been proposed by Vaidyanathan and Pal in [3,5]. These geometries called: Co-Prime Arrays (CPAs) [3] and Nested Arrays (NAs) [5], have proven to achieve increased DoFs over the ULAs with same number of elements and have exact closed-form expressions for element locations. NA consists of a dense ULA and a sparse ULA with different spacings, concatenated to result in NULA. NAs offer N^2 DOF with N physical sensors. CPAs are constructed by combining two ULAs with coprime number of elements, they suffer from holes in the DCA and hence have less consecutive DOF than the NA. On basis of these NULA prototypes, several modifications have been proposed, including Extended Co-Prime Arrays (Ext. CPAs) [5], Super NAs (SNAs) [6], Augmented NAs (ANAs) [7], Huang NA (HNAs) [9], SNAs and ANAs rely on relocating the elements from the parent NA geometry to locations within the NULA and on either ends of the NULA with the aim of lowering the effects of mutual coupling by reducing the number of consecutive elements in the dense ULA. Ext. CPAs extend the idea of parent CPAs by increasing the number of elements in one of the ULA by k-times, though it increases the DoFs beyond the parent CPAs it still suffers from holes in the DCA. HNAs are constructed by increasing the inter-element spacings of both ULAs as well as interleaving the two ULAs by a pre-set distance. HNAs have increased DoFs compared to the parent NAs, but suffer from holes in the DCA.

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There is a need to develop a NULA geometry which will result in increased virtual aperture with no missing lags, resulting in increased DoFs. Hence, the proposed 'Extended Nested Array' (Ext. NA) geometry is proposed, which has simple closed-form expressions for the element locations, increases DoFs by providing a hole-free DCA. SS-MUSIC Algorithm is deployed for DOA estimation to demonstrate the effectiveness of the proposed geometry. SS-MUSIC can only be applied to the contiguous portion of the virtual ULA aperture.

II. SIGNAL MODEL

A NULA with L antenna elements and M source signals incident on the array from directions [θ_1, θ_2,…, θ_M]. $S_i(t)$, i=1,2,..,M be the baseband source signals as shown in Fig. 1

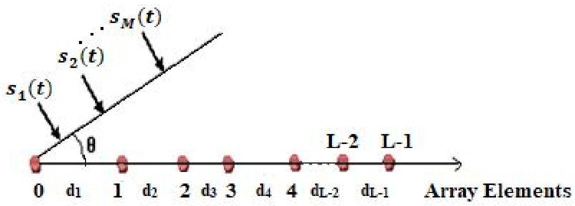

Fig. 1 Array signal model

Signal model is explained for the proposed geometry based on the DCA of the physical sparse array to account for the increase in Degrees of Freedom. $a(\theta)$ is the Lx1 steering vector corresponding to the angle θ as:

$$a(\theta) = [exp(j\frac{2\pi}{\lambda}d_n sin\theta] \quad n = 1,2,\ldots L-1 \tag{1}$$

Where, d_n corresponds to the spacing between the n^{th} and $(n-1)^{th}$ element, λ is the operating wavelength.

The signal received by the array is: $$x[l] = Asig[l] + noise[l] \tag{2}$$

Where, $A = [a(\theta_1), a(\theta_2), \ldots\ldots, a(\theta_M)]$is the Array Manifold

$sig[l] = [sig_1[l], sig_2[l], \ldots\ldots, sig_M[l]]$ is the source signal matrix

$noise[l]$ corresponds to the noise

Source signals are assumed to be uncorrelated, hence the covariance matrix R_{xx} becomes a diagonal matrix

$$R_{xx} = E[xx^H] = AR_{ss}A^H + \sigma^2{}_n I \tag{3}$$

$$R_{xx} = A\begin{bmatrix}\sigma^2{}_1 & \cdots & 0\\ \vdots & \ddots & \vdots\\ 0 & \cdots & \sigma^2{}_M\end{bmatrix}A^H + \sigma^2{}_n I \tag{4}$$

Vectorised R_{xx} yields a vector z: $$z = vec(R_{xx}) = vec[\textstyle\sum_{i=1}^{M}\sigma^2{}_i(a(\theta_i)a(\theta_i)^H)] + \sigma^2{}_n\tilde{1}_n \tag{5}$$

$$z = (A^* \odot A)p + \sigma^2{}_n\tilde{1}_n \tag{6}$$

Where, p is the signal power vector

$\sigma^2{}_n$ is the noise variance

$\tilde{1}_n = [e_1{}^T\ e_2{}^T \ldots\ldots e_N{}^T]$ is the Identity Matrix in vectorized representation

From equations (2) and (6), vector z is equated to the received signal by array with the Array Manifold ($A^*\odot A$), which denotes the Khatri-Rao product of the Array Manifold matrix of the physical NULA, with its own conjugate. ($A^*\odot A$) is the Array Manifold matrix of the larger virtual array, with elements placed as per the corresponding DCA. Thus, the DOA estimation is done using equation (6) to enhance the DoFs. Signal model in equation (6), is similar to received signal at the array having longer aperture denoted by the DCA. Covariance matrix constructed using this received signal, is rank deficit. Several approaches are available to complete the rank, this paper employs the very efficient 'Spatial Smoothing' method along with the MUSIC algorithm to complete the rank of the covariance matrix and estimate the DOA accurately.

III. THE PROPOSED ARRAY

The proposed method called 'Extended Nested Arrays (Ext. NAs)' is a class of NULA, designed to increase the DoFs achieved by parent NA. Ext. NAs provide a larger continuous aperture of DCA. The preliminary construction of the proposed geometry is very much alike the parent NA, consisting of a dense ULA and a sparse ULA. Dense ULA with N_1 elements has an inter-element spacing d and the Sparse ULA with $(N_2\text{-}1)$ elements has an inter-element spacing $(N1+2)d$, the last element in the sparse ULA is shifted away from the remainder of the array by an amount $(N_1+1)d$. Ext. NA has an advantage over the parent NA and the other modified variants. This effectiveness will be demonstrated by comparing the existing geometries in terms of some crucial evaluation metrics. Table I gives an insight into the number of elements and the obtainable DoFs.

TABLE I
OPTIMAL NO. OF ELEMENTS AND DOFS

No. of elements N	Optimal No. of elements in each level	Optimal DoFs
Even	$N_1=N_2=N/2$	$(N^2\text{-}2)/2+2N\text{-}3$
Odd	$N_1=(N\text{-}1)/2$ $N_2=(N+1)/2$	$(N^2\text{-}1)/2+2N\text{-}4$

A 6-element Ext. NA has $N_1=3$ elements with Inter-element spacing: d, Sparse ULA has $N_2\text{-}1=2$ elements with Inter-element spacing: $(N_1+2)d=5d$. The additional element is placed at $(N_1+1)d=4d$ from the Sparse ULA. The proposed Geometry with six elements positioned at: $\{0,1,2,3,8,12\}d$ and the corresponding DCA are as shown in Fig. 2 and Fig. 3 respectively.

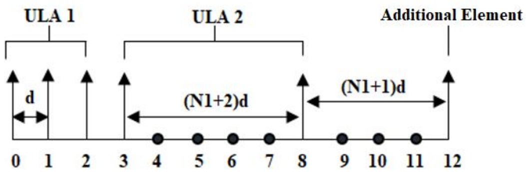

Fig. 2 Extended Nested Array Geometry

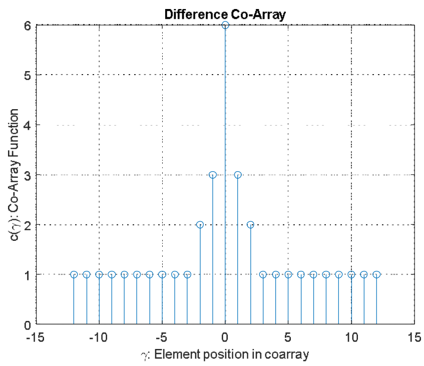

Fig. 3 Corresponding Difference Co-Array

DCA of the proposed geometry extends from *-12d to 12d*, with no holes, thus increasing DoFs in comparison to the parent NAs. The proposed array has increased aperture length, causing an increase in the number of spatial lags over the parent NA.

IV. SPATIALLY SMOOTHED MUSIC ALGORITHM

The Spatial Smoothing procedure using MUSIC Algorithm is listed:

1) The vector z from equation (6), has a number of repeated rows, identifying, deleting repeated rows and re-arranging the remaining rows results in vector z_1 which has spatial lags extending from -N, (-N-1), .. , (N-1), N
2) The virtual array is partitioned into (N+1) overlapping sub-arrays, the received signal z_i at every sub-array is evaluated.
3) Covariance matrix is calculated as:

$$R = \frac{1}{N+1}\sum_{i=1}^{L+1} z_i z_i^H \tag{7}$$

4) R is positive definite of dimension (N+1) x (N+1) subjecting it to Eigen Value Decomposition to evaluate Eigen Vectors E_N corresponding to (N-M) noise Eigen Values.
5) Compute Power spectrum of SS-MUSIC algorithm

$$P(\theta) = \frac{1}{a(\theta)E_N E_N^H a(\theta)^H} \tag{8}$$

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V. PERFORMANCE EVALUATION

A. Difference Co-Array (DCA):

All the geometries with 6 elements are considered. The element positions of each geometry are depicted in Fig. 4, Aperture lengths and the corresponding DCA details are listed in Table II.

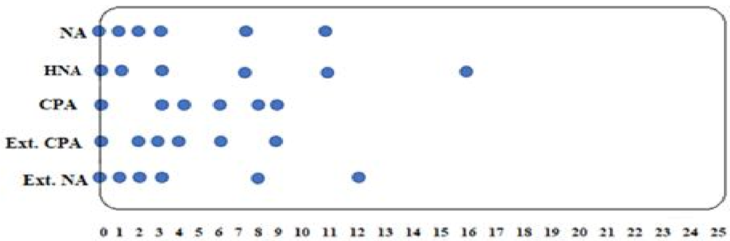

Fig. 4 Various NULA geometries considered

TABLE III
GEOMETRIES WITH 6 ELEMENTS, DCA LENGTH AND MISSING LAGS

NULA Configuration	Element positions	Aperture length	DCA length	Missing Lags
Co-Prime Arrays	[0 3 4 6 8 9] d	18d	-9d to 9d	Yes, ±7d
Extended Co-Prime Arrays	[0 2 3 4 6 9] d	18d	-9d to 9d	Yes, ±8d
Nested Arrays	[0 1 2 3 7 11] d	22d	-11d to 11d	No
Huang Nested Arrays	[0 1 3 7 11 16] d	32d	-16d to 16d	Yes, ±12d, ±14d
Extended Nested Arrays	[0 1 2 3 8 12] d	24d	-12d to 12d	No

Ext. CPA, CPA and HNA have holes in the corresponding DCA, this needs additional process of Spatial Interpolation to fill in the missing measurements. Since, the Spatially Smoothed MUSIC Algorithm can only be applied to a continuous set of lags, either the missing lags must be interpolated or the aperture must be reduced to omit the missing lags in DCA, hence reducing the effective aperture length of the DCA.

B. Degrees of Freedom (DoFs)

The DoFs obtainable for a 6-Element NULA possessing different geometries are listed in Table III. Fig. 5. Gives the graphical representation of the DoFs for different number of elements in the NULA.

TABLE IIIII
DOFS OF ALL GEOMETRIES WITH 6 ELEMENTS

No. of sensors	CPA	Ext. CPA	NA	HNA	Proposed Array (Ext. NA)
5	9	11	17	15	19
7	15	23	31	15	35
8	17	29	39	15	43
12	25	47	83	23	91
13	27	53	97	27	107
16	33	69	143	31	155
19	39	79	199	39	215
21	43	119	241	43	259
23	47	143	287	47	307
25	51	167	337	51	359
31	63	195	511	63	539
Gaps/Holes	Yes	Yes	No	Yes	No

Theoretical Limit [4] is defined as the Maximum DoFs that can be obtained from the Co-array of a NULA having any geometry:

$$DoF_{max}=L(L-1)+1 \quad (9)$$

Fig. 5. proves that the proposed Extended Nested Array has the highest DoFs among the geometries considered. The proposed geometry is the closest to the theoretical limit. Followed by the parent Nested Array. Thus, it is proven that the proposed geometry has the most DoFs for the given number of elements under consideration.

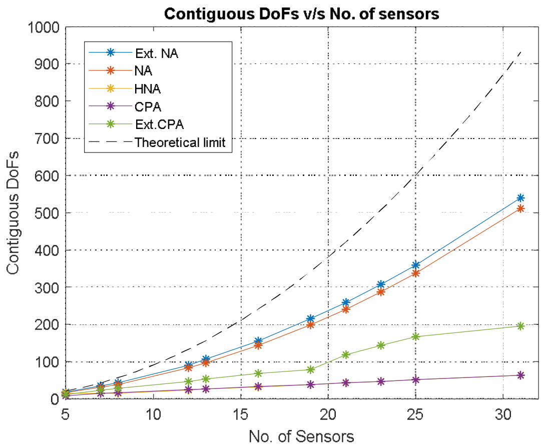

Fig. 5 DoF versus No. of sensors

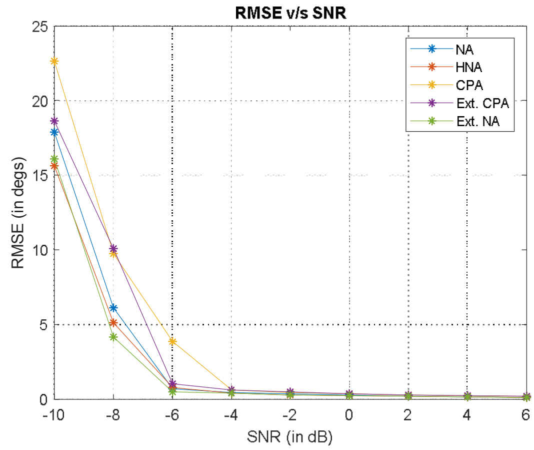

Fig. 6 RMSE v/s SNR of all configurations considered

C. Root Mean Square Error (RMSE)

This the measure of the error in estimating the DOA using SS-MUSIC algorithm, for a single incident signal at 30^0 with Signal to Noise Ratios (SNR) [-10 -8 -6 -4 -2 0 2 4 6] dB.

$$RMSE = \sqrt{\frac{1}{T}\sum_{k=1}^{T}(\hat{\theta}_k - \theta)^2} \quad (10)$$

Where, $\hat{\theta}_k$ represents the DOA estimate of the incident signal at the k^{th} trial and T is the no. of trials.

Fig. 6 shows that the proposed Ext. NAs has the least RMSE, closely followed by the parent NAs. Hence, Ext. NA proves to be the best geometry among those considered since it has the least RMSE with no holes in DCA.

D. DOA Estimation of more signals than elements

Detection of more signals than the no. of elements is demonstrated. M=10 incident signals equally spaced between [-60^0, 60^0] are considered. NULA with L=6 elements is considered possessing different geometries. SNR is 0 dB, DOA of the incident signals is: [-60^0, -46.6667^0, -33.3333^0, -20^0, -6.6667^0, 6.6667^0, 20^0, 33.3333^0, 46.6667^0, 60^0]. Fig. 7 demonstrates that NA, HNA and the proposed Ext. NA detect 10 signals using 6 elements in the geometry. CPA and Ext. CPA fail to detect 10 signals.

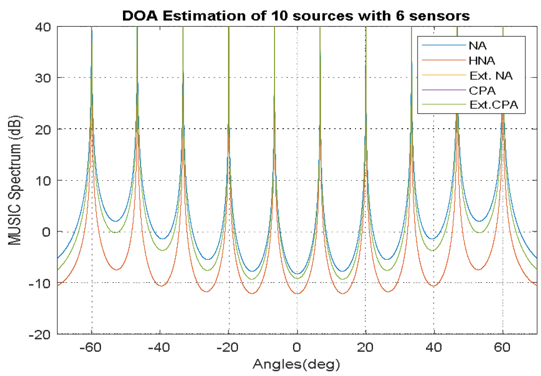

Fig. 7 MUSIC Spectrum of 10 incident signals

VI. CONCLUSIONS

A Non-Uniform Linear Array geometry is proposed to increase Degrees of Freedom (DoFs). The proposed Extended Nested Array geometry is compared with the existing configurations to demonstrate the increase in DoFs and DOA estimation of more signals than the number of elements in the array is carried out using SS-MUSIC. It is demonstrated that the proposed geometry increased DoFs compared to the existing geometries without any additional spatial interpolation and has Closed-form expressions for the DoFs and the exact element positions.

VII. ACKNOWLEDGMENT

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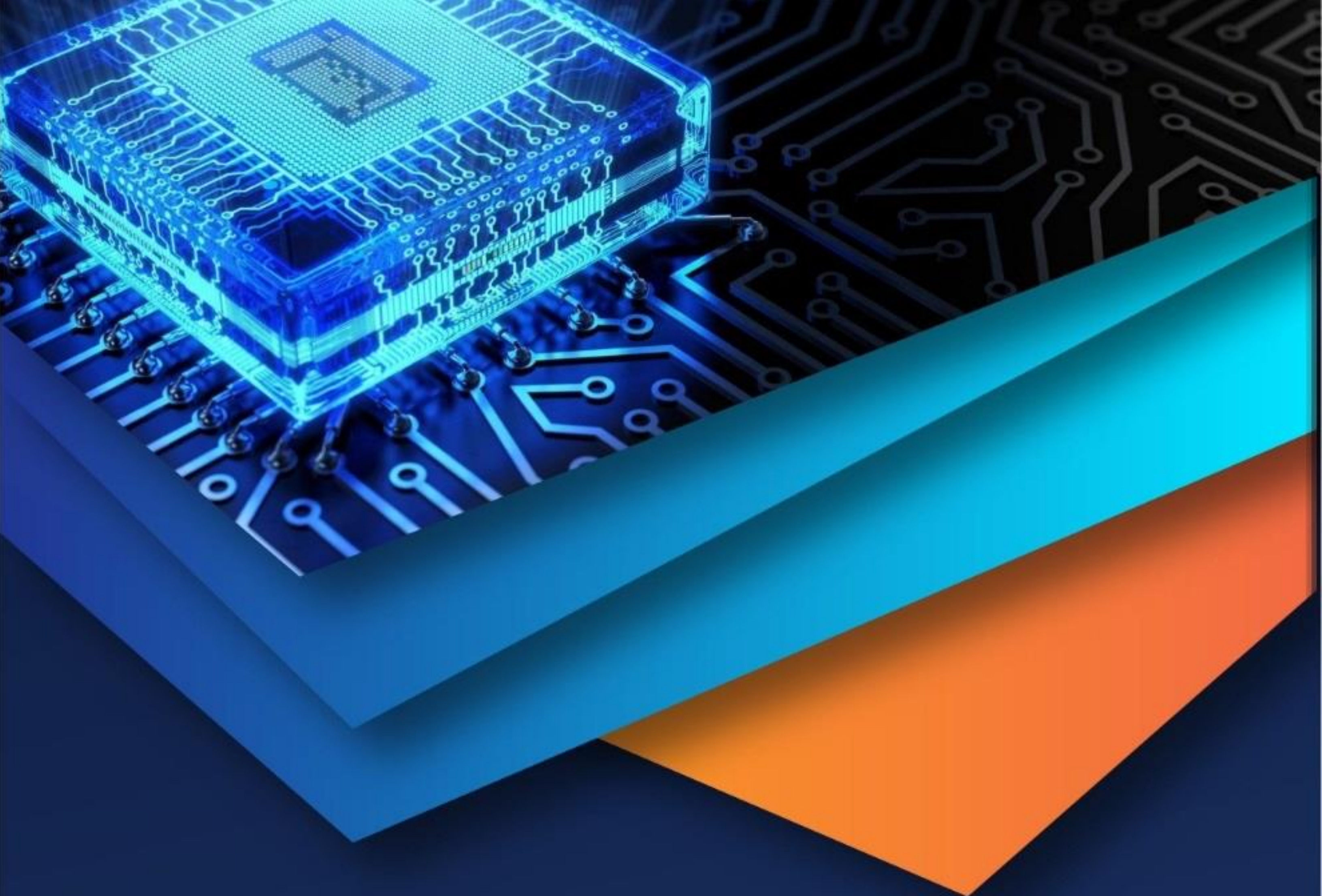

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