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Extreme Value Charts & Analysis of Means Based on the Lomax Distribution

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Abstract: It is assumed that the probabilistic model of the quality characteristics follows the new weighted exponential distribution. Control charts based on each subgroup's extreme values are established. The constants in the control chart are determined by the probability distribution of the extreme value order statistics of the sub-group and the sub-group size. The proposed chart is thus referred to as an extreme values chart. A biased overall mean analysis method (ANOM for truncated population) is used for the Lomax Distribution. Examples based on real time data are used to explain the findings.
Keywords: ANOM, Equi-tailed, In-control, LD.

I. INTRODUCTION

A. Lomax Distribution

Let $x_1, x_2,..., x_n$ be a random sample of size n from LD(α, λ), its Probability Density Function (pdf) and cumulative distribution function(CDF) are given by

$$f(x)=\frac{\alpha}{\lambda}\left[1+\frac{x}{\lambda}\right]^{-(\alpha+1)}; \qquad x \geq 0, \alpha > 0, \lambda>0 \qquad \ldots(1)$$

α = shape parameter

λ=scale parameter

$$\text{CDF} \quad F(X)=1-\left(1+\frac{x}{\lambda}\right)^{-\alpha} \qquad \ldots(2)$$

$\text{Mean}=\frac{\lambda}{\alpha-1}$ for $\alpha>1$

$\text{Variance}=\frac{\lambda^2\alpha}{(\alpha-1)^2(\alpha-2)}$ for $\alpha>2$

The extreme order statistical percentiles of the LD sample are required to create a control chart that uses extreme observations from a subset of manufacturing processes with LD quality options. Specifically, the first test vector $X=(x_1,x_{2,\ldots\ldots},x_n)$ from the continuous processing is used as the test statistic on the extreme value control chart. The control chart in extreme value chart displays entire sample observations, but no statistic(s) is/are calculated from it. According to one or both extreme values of the sample, x_1 (test least) and n x_n (test most extreme), fall below or above two defined lines (limits), a corrective action is taken. Therefore, this chart is called an extreme value control chart[9].

Many professionals use Shewart control charts as a statistical method [5] for quality control. If the solution is found, the technique shall be adjusted when such charts indicate that an assignable cause exists [1]. In the abstract group statistical for which the control chart is built, the existence of an assignable cause is understood as a signal of heterogeneity [1, 8]. For example, the mean process would be heterogeneous when the figures are sample mean, which would signify differences from the goal mean [4]. Such an analysis is often done by means of means to split a collection of different subset mean into categories [2], so that means are homogenous within a category and heterogeneous between categories and the technique is known as an analysis of means (ANOM) as described by Ott.E.R [7]. The control chart for the mean is read differently using the ANOM technique [6, 10]: grouping of the plotted means within or beyond the control limits. The two means must fall under the control limits in order for all of them to be homogeneous. The probability of any sub-group is equal to the coefficient of confidence, take is as (1-α) . This probability statement will be the n th power of the likelihood that the mean of a subgroup fall within the boundaries, provided it is supposed to be independent. I.e. the confidence interval of x for the distribution of samples should be equivalent to $(1-\alpha)^{1/n}$ between two specified bounds. In the rest of this article, the same principle is also applied by LD. We only looked at ANOM control charts [3] in this research since it intends to examine ANOM by employing extreme value statistical control limits.

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No new ANOM tables or procedures have been examined by us. However, there is a thorough documentary on ANOM by Rao.C.V [9] and certain similar works are in this direction [11-14] are mentioned in references.

The rest of the paper is described below. Section 2 gives a fundamental exposure to extreme value control diagrams that are supported by average runtime (ARL) and to ANOM. In Section 3, LD is used with an ANOM in conjunction with numerical examples employing extreme value control limits of LD. The findings and conclusions of Section 4 are provided.

II. MATERIALS AND METHODS

The mathematical and statistical research background of Extreme value charts & ANOM and the methods for the study of LD model are discussed in this part.

A. *Extreme Value Chart for LD Model*

LD model is considered to be followed by the sample observations. The theory of extreme order statistics, based on the LD model, determines the control lines. The control lines should be chosen such that an arbitrarily chosen x_i of $X=(x_1,x_2,\ldots,x_n)$ lies inside the limits with probability $(1-\alpha)^{1/n}$. The following formula can be used to express this as a probability inequality:

$P(x_i \le L) = \alpha/2$ & $P(x_n \le U) = \alpha/2$. The cumulative distribution functions of the lowest and largest order statistics in a sample of size n from any continuous population are $[F(x)]^n$ along with $1-[1-F(x)]^n$ commonly, according to the theory of order statistics, here $F(x)$ is the population's distribution function. The value of α will be 0.0027, if $(1-\alpha)$ were needed at 0.9973. Using $F(x)$ as the CDF of a LD model, we can find solutions to the two equations $1-[1-F(x)]^n = 0.00135$and $[F(x)]^n = 0.99865$, which can then be used to establish the extreme value chart's control limits.

B. *Analysis of Means (ANOM) for LD Model*

Assume that $\bar{x}_1, \bar{x}_2, \ldots\ldots \bar{x}_n$, are arithmetical means of a LD model in k subgroups of the size n. When control charts are established by employing these sub-group means, whether acceptable qualitative changes exist can be assessed in the original population from which these sub-groups come to be drawn. Depending on the elementary population distribution, one can adopt the constants of control diagram we created or the usual Shewart constants from statistical textbook. Broadly speaking, when all means in the subsets are inside the control limits, the mechanism is considered to be beneath control. Otherwise, the mechanism would be excluded from control. If α is the amount of significance of the above decisions, the following likely claims can be made.

$$P(LCL < \bar{x}_i < UCL) = (1-\alpha), \quad \forall i = 1,2,\ldots\ldots k \qquad (3)$$

Using the notion of independent subgroups Eq. (3) becomes

$$P(LCL < \bar{x}_i < UCL) = [1-\alpha]^{1/k} \qquad (4)$$

With Equi-tailed probability for each subgroup mean, we can find two constants say U^* *and* L^*such that

$$P(\bar{x}_i < L^*) = P(\bar{x}_i > U^*) = \frac{1-(1-\alpha)^{1/k}}{2} \qquad (5)$$

L^* and U^* satisfy $U^* = -L^*$, when the population drawn from a Normal distribution. We must measure L^* & U^* separately from sampling distribution of $\bar{x}_i$ for skewed populations like LD. As a consequence, these are dependent on the number of subgroups k and sub-groups size n. Percentiles of the sampling distribution for $\bar{x}$ in LD model were calculated using simulation process (Monte-Carlo) and are shown in Table 3 and Table 4. For stated n and k, we use the percentiles in Eq. (5) to get L^* and U^* for α = 0.01,0.05,and 0.10 . Table 3 and Table 4 contain this detail.

III. RESULTS AND DISCUSSION

The solutions to the two equations $1-[1-F(x)]^n = 0.00135$and $[F(x)]^n = 0.99865$ for n = 2,3,...,10 are denoted as $Z_{(1)\,0.00135}$& $Z_{(n)\,0.99895}$ are listed in Table 1.

Table 1. Control Limits of Extreme value charts.

n	2	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
$Z_{(1)\,0.00135}$	0.0243	0.0214	0.0198	0.0145	0.0137	0.0121	0.0117	0.0116	0.0104
$Z_{(n)\,0.99895}$	32.8129	34.0816	36.9894	36.6767	39.2679	42.688	47.1467	48.5163	48.6755

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The values from Table 1 express the subsequent probability statement;

$$P\left(Z_{(1)0.00135} < Z_i < Z_{(n)0.99895}\right) = 0.9973 \quad \forall i = 1,2, \ldots \ldots k \qquad (6)$$

$$P\left(\sigma Z_{(1)0.00135} < x_i < \sigma Z_{(n)0.99895}\right) = 0.9973 \quad \forall i = 1,2, \ldots \ldots k \qquad (7)$$

Taking $\bar{x}/2.156$ as an unbiased estimate of σ for a specific parametric combination α = 0.3 and λ = 0.5 of NWED by the simulation process, the above equation becomes

$$P(L\bar{x} < x_i < U\bar{x}) = 0.9973, \qquad \forall i = 1,2, \ldots \ldots k \qquad (8)$$

Where $L = \frac{Z_{(1)0.00135}}{2.156}$ and $= \frac{Z_{(n)0.99895}}{2.156}$. Thus L & U would establish the control chart constants for extreme value charts. Table 2 display these values for n=2(1)10.

Table 2. Constants of Extreme value charts.

n	2	3	4	5	6	7	8	9	10
L	0.0243	0.0214	0.0198	0.0145	0.0137	0.0121	0.0117	0.0116	0.0104
U	12.8169	14.0816	16.9564	16.6767	16.3678	17.5688	17.1467	18.6163	18.6785

A. Examples Under Study

Example 1: Take into consideration the following data from the 25 observations concerning the production of metal products suspected of variations in raw material iron content from five suppliers. Of each of the five suppliers, five ingots were randomly chosen. The below observations (Data 1) includes percentage by weight for the iron determination on each ingot.

Data 1:

Supplier	% weight				
1	3.46	3.48	3.56	3.39	3.4
2	3.59	3.46	3.42	3.49	3.5
3	3.51	3.64	3.46	3.52	3.49
4	3.38	3.4	3.37	3.46	3.39
5	3.29	3.46	3.37	3.32	3.38

Example 2: The study includes three battery bands. The life of the three brands is suspected (in weeks). The following findings (Data 2) are tested for five batteries of each brand. Test whether the lives of these battery brands are different at 5% level of significance.

Data 2:

Brands	Life in Weeks				
1	100	96	92	96	92
2	76	80	75	84	82
3	108	100	96	98	100

Table 3. LD constants for Analysis of Means when α = 0.01

k/n	2	3	4	5	6	7	8	9	10
1	0.4356	0.4568	0.4813	0.5235	0.5637	0.7589	0.7986	0.8769	0.8956
	10.4536	10.2342	9.6758	9.6547	9.0876	8.2346	7.3462	7.1786	7.0984
2	0.4256	0.4448	0.4756	0.5146	0.5467	0.7483	0.7789	0.8459	0.8851
	10.4936	10.4542	9.3458	9.6542	9.0826	8.2343	7.0862	7.1453	7.0453
3	0.4256	0.4448	0.4756	0.5146	0.5467	0.7483	0.7789	0.8459	0.8851
	10.8936	10.4542	9.3458	9.6542	9.0826	8.2343	7.0862	7.1453	7.0453
4	0.4116	0.4167	0.4098	0.5098	0.5345	0.7198	0.7546	0.8345	0.8675

	10.9238	10.7642	9.7654	9.5467	9.6976	8.2843	7.1852	7.1124	7.1098
5	0.3958	0.4893	0.4876	0.5178	0.5489	0.5688	0.5857	0.6893	0.6965
	10.9862	10.8942	9.4722	9.4567	9.5676	8.7863	7.4562	7.1334	7.1296
10	0.3638	0.4120	0.4430	0.4977	0.5231	0.5408	0.5638	0.6773	0.6203
	12.4706	10.1287	7.7855	6.9325	6.8845	5.9669	5.6970	5.5982	5.0432
15	0.3396	0.3756	0.3939	0.4461	0.4455	0.4589	0.5346	0.5893	0.5575
	13.3413	10.0287	7.8439	6.9614	7.0127	6.2414	5.9245	5.9131	5.0973
20	0.3396	0.3756	0.3959	0.4451	0.4365	0.4599	0.5016	0.5343	0.5585
	13.3563	10.0677	7.7639	6.8714	7.0145	6.2424	5.9565	5.9167	5.0943
25	0.3196	0.3346	0.3859	0.4471	0.4369	0.4699	0.5057	0.5389	0.5685
	13.3963	10.0687	7.7639	6.8814	7.0145	6.2424	5.9565	5.9167	5.0843
50	0.2987	0.2996	0.3098	0.3245	0.3567	0.3765	0.3876	0.3971	0.3975
	13.5863	10.0677	7.7839	6.8818	7.0139	6.2443	5.9569	5.9188	5.0853

The constants mentioned in the above Table 3 are in particular used to illustrate the Example 1. For Data 1, the sample means are $\bar{x}_1$=3.458,$\bar{x}_2$=3.492,$\bar{x}_3$=3.524,$\bar{x}_4$=3.400 and $\bar{x}_5$=3.364. The overall mean or population mean is $\bar{x} = 3.4476$. For stated n=5 and k=5, we use the percentiles in Eq. (5) to get $L^* = L\bar{x}(LDL)\ and\ \ U^* = U\bar{x}(UDL)$ for α = 0.0.1 (assumed). Those values (decision limits) are presented in Table 3. Here from Table 5, we can observe that, for n=5 and k=5, L = 0.5178 and U =9.4567 . These constants are useful in general, for specified n and k.

Table 4. LD constants for Analysis of Means when α = 0.05

k/n	2	3	4	5	6	7	8	9	10
1	0.5459	0.6142	0.6716	0.6992	0.7294	0.7565	0.7738	0.8033	0.8137
	7.7094	6.9488	6.3640	6.0360	5.9085	5.6288	5.4859	5.3875	5.2365
2	0.5079	0.5723	0.6266	0.6641	0.5848	0.7201	0.5361	0.7605	0.7753
	8.7877	7.7089	7.1072	6.5788	6.3758	6.0549	5.9040	5.7448	5.5865
3	0.4929	0.4605	0.5105	0.5417	0.5722	0.6050	0.6218	0.6413	0.6564
	9.3963	8.2439	7.4008	6.9023	4.7266	4.2877	4.1333	4.0069	3.7727
4	0.4848	0.4471	0.4965	0.5299	0.5562	0.5887	0.6028	0.6325	0.6477
	10.2010	8.5604	7.7067	7.2001	8.8805	6.4533	4.3297	4.1791	3.9887
5	0.4766	0.4397	0.5827	0.5205	0.6511	0.5815	0.5972	0.6260	0.6407
	10.4269	8.9514	7.8997	5.3867	5.0590	4.8643	4.4056	4.2640	3.9676
10	0.4639	0.4130	0.4440	0.5877	0.5221	0.5508	0.5628	0.6003	0.6103
	11.5115	9.7933	8.3599	7.8573	5.7302	4.8636	4.8489	4.7880	4.2090
15	0.4579	0.4058	0.4345	0.4800	0.5055	0.5399	0.5477	0.5892	0.5890
	11.9812	10.2613	8.6065	8.0942	5.9519	5.0645	5.0405	4.7397	4.4420
20	0.4528	0.3998	0.6270	0.4708	0.4912	0.5222	0.5372	0.5779	0.5840
	12.1611	11.4739	8.7615	8.2244	6.0377	5.3210	5.1372	4.8682	4.5567
25	0.4502	0.3969	0.4264	0.4603	0.4841	0.5019	0.5330	0.5718	0.5827
	12.5592	11.5502	8.8370	8.3731	6.0921	5.3670	5.1678	4.9812	4.6578
50	0.4449	0.3900	0.4149	0.4541	0.4563	0.4746	0.5220	0.4551	0.5740
	13.9862	11.4422	9.3689	8.7793	6.3743	5.5213	5.4787	5.3052	4.9895

In specifically for Example 2 the constants specified in the above-described Table 4 are employed. For Data 2, the sample means are $\bar{x}_1$=95.2,$\bar{x}_2$=79.4 and $\bar{x}_3$=100.4. The overall mean or population mean is $\bar{x} = 91.6667$. For stated n=5 and k=3, and for α = 0.05 , from Table 4 we can find L = 0.5417 and U = 6.9023 .

The same technique is used and the calculated values (decision limits) are presented in Table 5. In general, for stated n and k, these constants are helpful.

We have determined and submitted Table 5, the decision limits (DL) for the Normal Population and the LD Population utilizing these findings in data as a single sample.

Table 5. Comparison between Normal Distribution and LD

	Normal Dist.			LD		
Example No. (n k, ,α)	Decision Limits [LDL, UDL]	Count i (Within Limits)	Probability (p = i/k)	Decision Limits [LDL, UDL]	Count i (Within Limits)	Probability (p = i/k)
1 (5, 5, 0.01)	[3.517, 3.879]	3	0.6	[1.587, 21.972]	5	1
2 (5, 3, 0.05)	[87.82, 95.52]	2	0.7	[49.656, 449.378]	3	1

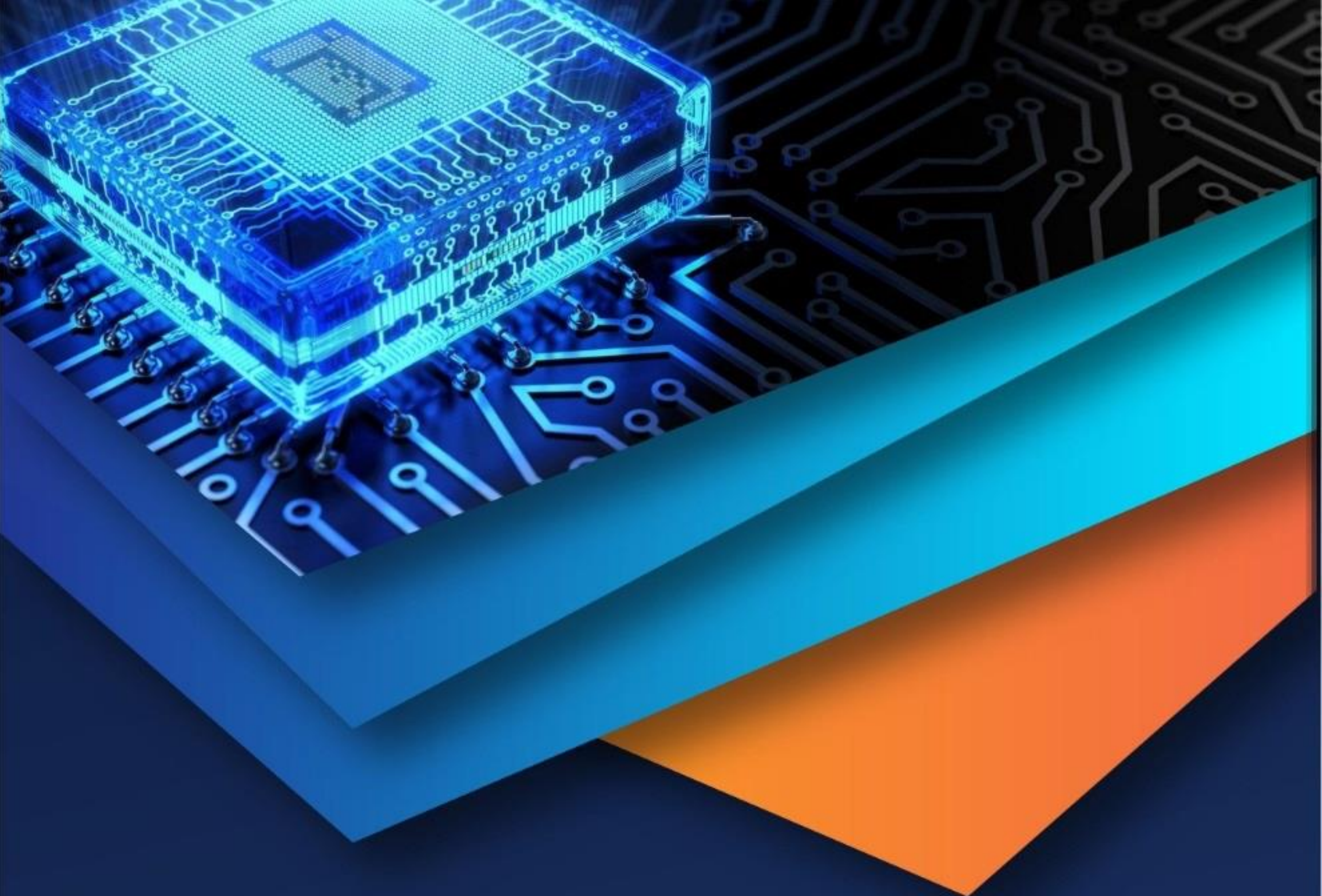
n : Size of Subgroup, k : No. of Subgroups, α : level of significance, LDL: Lower Decision Limit, UDL:Upper Decision Limit.

IV. CONCLUSION

According to the decision limits using Normal distribution or the Shewart control limits and ANOM tables of Ott.E.R [7], the number of homogenous mean is 3 and 2 for each data set, and those who are not homogeneous are 2 and 1 respectively. When the ANOM tables of LD are utilized, the number of homogeneous means for the same data sets is 5 and 3, with no deviations from the homogeneity. This indicates that, when the normal distribution model has been applied, certain means have been homogenized, and others have deviated. This decision is valid, even if the data corresponds to the Normal distribution. In comparison, LD is a better model than normal. As a result, we concluded that the decision method of the Normal distribution will be correlated with more error. Henceforth, using proposed LD model is a better option rather than the usual, to achieve homogeneity for ANOM method in some cases.

REFERENCES

[1] Bakir.S.T., Means using ranks for randomised complete block designs. Communications in Statistics Simulation and Computation. 1994; 23:547-568.
[2] Bernard.A.J., and Wludyka.P.S. Robust I-sample Analysis of means type randomization tests for variances Journal of Statistical Computation and Simulation. 2001; 69:57-88.
[3] Farnum.N.R. Analysis of Means Tables using mathematical processors. Quality Engineering. 2004; 16:399-405.
[4] Guirguis.G.H., and Tobias.R.D. On the computation of the distribution for the analysis of means. Communications in Statistics- Simulation and Computation. 2004; 16:861-887.
[5] Montgomery.D.C. Design and Analysis of Experiments. Fifth edition, John Wiley and Sons, New York.
[6] Nelson.P.R., and Dudewicz.E.J. Exact Analysis of Means with Unequal Variances. Technometrics. 2002; 44:152-160.
[7] Ott.E.R. Analysis of Means- A graphical procedure. Industrial Quality Control. 1967; 24:101-109. [8] Ramig.P.F. Applications of Analysis of Means. Journal of Quality Technology. 1983; 15:19-25.
[8] Rao.C.V. Analysis of Means - A review. Journal of Quality Technology. 2005; 37:308-315.
[9] Rao.C.V., and Prankumar.M. ANOM-type Graphical Methods for testing the Equality of Several Correlation Coefficients. Gujarat Statistical Review. 2002; 29:47-56.
[10] Srinivasa Rao.B., Pratapa Reddy.J., and Rosaiah.K. Extreme value charts and ANOM based on inverse Rayleigh distribution. Pakistan Journal of Statistic & Operations Research. 2012; 8(4):759-766.
[11] Srinivasa Rao.B., and Kantam.R.R.L. Extreme value charts and Analysis of means based on half logistic distribution. International Journal of Quality, Reliability and Management. 2012; 29(5):501-511.
[12] Srinivasa Rao.B., and Sricharani.P. Extreme value charts and Analysis of means based on Dagum distribution. International Journal of Statisytics & Applied maythematics. 2018; 3(2):351-354.
[13] Srinivasa Rao.B., Pratapa Reddy.J., and Sarath Babu.G. Extreme value charts and ANOM based on log-logistic distribution. Modern Applied Statistical Methods. 2012; 493-505.



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