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Finite Element Based Static and Dynamic Vibration Analysis of a Beam with Axial Variation of Material Properties

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Abstract: The study examines the vibration analysis of a damped cantilever beam with nonuniform variation in material properties using finite element methods. With power modification of material parameters in the axial direction, the static and dynamic response of a prismatic rectangular beam with damping has been examined. For the finite element approach, two noded beam elements with two degrees of freedom at each node were evaluated. The power law variation of material properties was used throughout the investigation. This research investigated the effects of proportional damping on displacement, velocity, and acceleration responses. The proposed beam's mass, stiffness, and damping matrices were calculated using Hamilton's concept. In the temporal domain, the Newmark Method has been utilized.
Keywords: Finite Element Analysis; Hamilton's principle, Dynamic analysis

I. INTRODUCTION

Nowadays, functionally graded (FG) materials are regarded as among the most advanced materials, with mechanical properties that gradually vary with respect to a chosen spatial coordinate. In comparison to laminated composites, using FG materials in structural systems eliminates stress concentration and enhances the structure's strength and toughness. Most of the literature on FG beams deals with beams whose mechanical properties vary through thickness[1][2][3].There are relatively few works on axially FG beams whose mechanical properties vary along the axis of the beam where most of them concern the special case of uniform beams. Due to varying cross-sectional area, modulus of elasticity and mass density along the beam axis, the governing differential equations of axially FG tapered beams for transverse and longitudinal vibrations and buckling are differential equations with variable coefficients for which closed-form solutions could be hardly found or even impossible to obtain; hence application of numerical techniques is essential. FEM has been used to study the free vibration of an AFG-tapered beam based on Euler-Bernoulli and Timoshenko beam theory[4][5].The free vibration analysis of AFG-tapered Euler-Bernoulli beams employing the differential transform element method has been studied[6].The free bending vibration of rotating AFG-tapered Euler-Bernoulli beams with different boundary conditions using the differential transformation method and differential quadrature element method[7].Further, the free vibration analysis of AFG Timoshenko beams using the same method has been studied [8].The free vibration of variable cross-sectional axially functionally graded beam has been studied [9].The differential equation with variable coefficients is combined with the boundary conditions and transformed into Fredholm integral equation. By solving Fredholm integral equation, the natural frequencies of axially functionally graded beams can be obtained. The free vibration analysis of a functionally graded ordinary (FGO) twisted Timoshenko beam of cantilever type was investigated. The shape functions were derived from differential equations of static equilibrium. The mass and stiffness matrices were obtained from the energy equation. The various material properties along the thickness direction are assumed to vary according to a power law. It was observed from the analysis that increasing the pretwist angle, the first natural frequency increased whereas the second natural frequency decreased. The simultaneous effects of power law index and pretwist angle on first natural frequency were conducted and observed that it was marginal[10].The bending analysis of a simply supported FG beam subjected to uniformly distributed load (UDL) was investigated. The material properties of the FG beam varied continuously in the thickness direction based on power law. The position of the natural surface of the FG beam was obtained, and its influence on the deflection of the beam under UDL was studied[11].The numerical calculations for natural frequencies of FG simply supported beams were presented. The first order Timoshenko beam theory and third-order shear deformation theory were applied for the analysis of FG beam. The nonlinear forced vibration analysis of a beam made of FG material was presented.

The modelling of the beam was carried out using Euler-Bernoulli beam theory and von Karman geometric nonlinearity. The effects of material properties on the nonlinear dynamic behavior of FG beam were discussed. The frequency response equation of the system was presented, and the effects of different parameters on the response of the system were investigated[12].The free vibration analysis of a functionally graded ordinary (FGO) twisted Timoshenko beam of cantilever type was investigated. The shape functions were derived from differential equations of static equilibrium. The mass and stiffness matrices were obtained from the energy equation. The various material properties along the thickness direction are assumed to vary according to a power law. It was observed from the analysis that increasing the pretwist angle, the first natural frequency increased whereas the second natural frequency decreased. The simultaneous effects of power law index and pretwist angle on first natural frequency were conducted and observed that it was marginal. The bending analysis of a simply supported FG beam subjected to uniformly distributed load (UDL) was investigated. The material properties of the FG beam varied continuously in the thickness direction based on power law. The position of the natural surface of the FG beam was obtained, and its influence on the deflection of the beam under UDL was studied. The bending analysis of a simply supported FG beam subjected to uniformly distributed load (UDL) was investigated. The material properties of the FG beam varied continuously in the thickness direction based on power law. The position of the natural surface of the FG beam was obtained, and its influence on the deflection of the beam under UDL was studied[13].The free vibration analysis of a simply supported FG beam with piezoelectric layers subjected to axial compressive loads was studied. The various effects of volume fractions, the effects of applied voltage and axial compressive loads on the vibration frequency were presented. It was concluded from the analysis that the piezoelectric actuators induce tensile piezoelectric force produced by applying negative voltages that significantly affect the free vibration of the FG beam. The vibration frequency increases when the applied voltage is negative[14].The differential transformation method (DTM) was applied for investigating the free vibration analysis of FG beams with arbitrary boundary conditions, including various types of elastically end constraints. By using DTM, the natural frequencies and mode shapes were presented. For free vibration of the beam, Al_2O_3/Al was considered for the study. It was seen that there was considerable variation of frequencies and mode shapes when the stiffness of spring becomes more[15].A new approach has been initiated based on Chebyshev polynomial theory to investigate the free vibration of AFG Euler-Bernoulli and Timoshenko beams with nonuniform cross sections[16].Even though several research works has been commenced on axial functionally graded beam still there is some gap in the vibration analysis of beams with variation of material properties in axial direction through finite element method. The present paper is an initiation towards the analysis of static and dynamic response of such beam with proposed power law variation of material properties in axial direction. The material properties are an essential aspect in design consideration of any beam, which need an attention for study. The behaviour of such beam has been analysed in Matlab environment.

II. MATHEMATICAL MODELLING OF THE PROPOSED BEAM

The theory of beam and mathematical formulations involve the modeling of beam with finite element analysis (FEA) from vibrating of such beam. The detail of the above formulation is presented in the following subsections. A cantilever beam is shown Figure 1 for dynamic analysis when subjected to an impulse force.

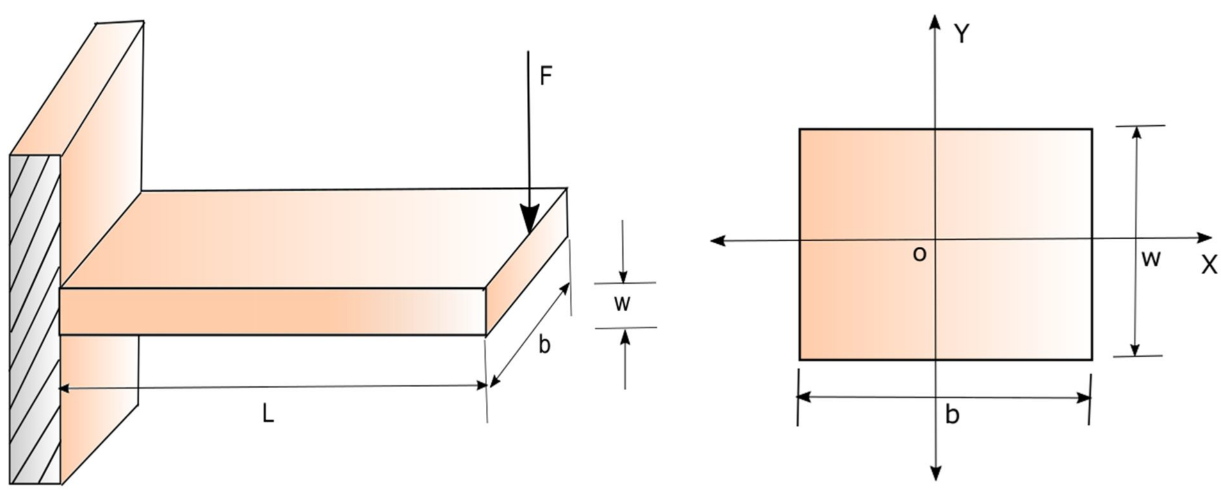

Figure 1. Proposed Cantilever beam with rectangular cross section (a) front view of the beam (b) side view of the beam

The length of the proposed beam is considered to be L .The width and thickness of the beam is b and w. An impulse load of F is applied at the free end of the cantilever beam. Euler-Bernoulli formulation has been incorporated in the static and dynamic formulation of the beam. All mathematical formulation has been coded in Matlab environment. The beam is modelled as FG, i.e., non-homogeneity of material properties (such as density, Young's modulus, and Poisson's ratio) in the axial direction. The following mathematical expression has been proposed to determine such FG properties of the beam in the axial direction which is continuously decreasing towards the tip of the cantilever beam[17]

$$J(x) = J\left[1 - \frac{x}{(p+1)L}\right]^n \tag{1}$$

Where, *J(x)* denotes the material property such as density, Young's modulus, poison's ratio and shear modulus respectively, which is position dependent. The terms p and n are the positive integer parameter (to avoid the material properties to be zero at the tip of cantilever beam) and the power gradient index.

A. Displacement Field

The beam element with two degrees of freedom at each node is shown in. In finite element (FE) modeling, each nodal point assumed to experience two degrees of freedom i.e. transverse displacement (v) and rotation (θ) which are supposed to act due to the shear force and bending moment.

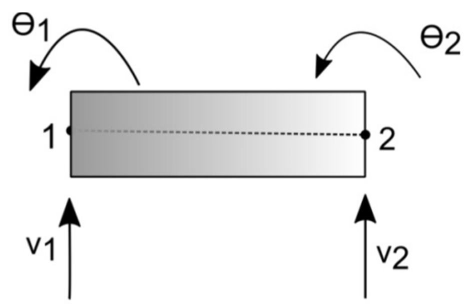

Figure 2. Nodal degrees of freedom of a beam element

The displacement field of the beam in x, y and z-direction can be written as

$$\begin{aligned} u(x,y,z,t) &= -z\theta(x,t) = -z\left(\frac{\partial w}{\partial x}\right) \\ v(x,y,z,t) &= 0 \\ w(x,y,z,t) &= w(x,t) \end{aligned} \tag{2}$$

Where u, v, w are the time-dependent axial, lateral and transverse displacements along x, y, z-axes respectively which is shown in Figure 2. The terms $w(x, t)$ is the transverse displacement of any point in the midplane ($z=0$). The term θ is the rotation of the midplane about y-axis whereas t denotes the time. The axial displacement at any point in the midplane ($z=0$) is neglected as its effect is negligible compared to transverse displacement. Moreover, as output power is greatly influenced by bending strain hence membrane strain is neglected for the above expressions.

B. Shape Function

The displacement field could be interpolated in terms of degrees of freedom of nodes and shape functions based on the concept of FEM as

$$\{r\} = [N_w]\{\delta\} \tag{3}$$

Here q_w and N_w signify the nodal degrees of freedom and the bending shape functions respectively. The accuracy of the result is governed by how well the shape function is selected.

$$\{\delta\} = \begin{Bmatrix} w_1 \\ \theta_1 \\ w_2 \\ \theta_2 \end{Bmatrix} \text{where}\, \theta_1 = \frac{\partial w_1}{\partial x} \text{ and } \theta_2 = \frac{\partial w_2}{\partial x} \tag{4}$$

Since there are four nodal values, we select polynomial with four constants. Thus

$$[N_w]=[N_1 \quad N_2 \quad N_3 \quad N_4] \tag{5}$$

By applying the boundary conditions the shape functions can be determined as

$$\begin{aligned} N_1 &= 1-\frac{3x^2}{L^2}+\frac{2x^3}{L^3} \\ N2 &= x-\frac{2x^2}{L}+\frac{x^3}{L^2} \\ N3 &= \frac{3x^2}{L^2}-\frac{2x^3}{L^3} \\ N4 &= \frac{-x^2}{L}+\frac{x^3}{L^2} \end{aligned} \tag{6}$$

C. *Governing Equation*

The proposed cantilever beam with variation in material properties is subjected to load F at the free end. Using Hamilton's principle, the dynamic equations of motion is represented as[18]

$$\partial\Pi = \int_{t_1}^{t_2}[\partial(E_K - E_P + W)]dt = 0 \tag{7}$$

Where E_K is the kinetic energy, E_P is the total electromechanical enthalpy and W is the total work done by the external mechanical force respectively. The terms t_1 and t_2 represent the initial and final time. The expressions for the above terms can be written as

$$E_k = \frac{1}{2}\int_V \rho_b \dot{r}^T \dot{r}\, dV \tag{8}$$

$$E_p = \frac{1}{2}\int_V \varepsilon_1{}^T \sigma_1 dV \tag{9}$$

$$W = \sum_{i=1}^{nf} r(x_i)Q(x_i) \tag{10}$$

From the equation the elemental mass matrix for the beam and the piezoelectric patch can be expressed as

$$[M_b^e] = \int_0^{L_b} [N_w]^T \rho(x) A [N_w] dx \tag{11}$$

Similarly, the elemental stiffness matrices of the beam can be articulated as

$$[K_b^e] = \int_0^{L_b} \left[\frac{\partial[N_\theta]}{\partial x}\right]^T E(x) I \left[\frac{\partial[N_\theta]}{\partial x}\right] dx \tag{12}$$

Where $[N_\theta] = \partial[N_w]/\partial x$ .The dynamic equation of the proposed beam is obtained by using the above equation The length of entire beam is divided into number of finite elements. The mass and stiffness matrices are assembled together using the finite element technique and the global matrices are obtained. The equation of motion of the discretized structure is represented by

$$\lfloor M^e \rfloor\{\ddot{r}\} + \lfloor K^e \rfloor\{r\} = \{f^e\} \tag{13}$$

The stiffness matrices, mass matrices are evaluated by numerical integration using two points Gauss quadrature. Apart from this, the system should have some supplementary structural damping which needs to be accounted for. This is done by using proportional damping methods. The damping ratio is predicted from the computed fundamental frequency as

$$C^e = \alpha\lfloor M^e \rfloor + \beta\lfloor K^e \rfloor \tag{14}$$

Where α and β are found out from

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$$\zeta_i = \frac{\alpha}{2\omega_i} + \frac{\beta\omega_i}{2}, \quad i=1, 2, 3..n \tag{15}$$

Where ζ_i is the damping ratio of the proposed beam. The global set of equation can be found by assembling the elemental mass, stiffness and damping matrices. The final equation of motion of the beam can be found by assembling the elemental matrices as follows.

$$[M]\{\ddot{r}\}+[C]\{\dot{r}\}+[K]\{r\}=\{Q^e\} \tag{16}$$

Where the terms *[M]*is the global mass matrices of beam,*[K]*is the global stiffness matrices for beam and *[C]* is the global proportional damping matrix respectively.

III. RESULT AND DISCUSSION

Based on the mathematical formulations of finite element method, MATLAB code has been developed for analysis of the proposed beam for output responses. The developed MATLAB code is validated and various results are presented in the following sub sections. As per the analysis the variation of Young's modulus and density of the beam has been taken as proposed power law variation. In static analysis the frequency and mode shapes of the beam have been analyzed. For free vibration analysis an initial displacement has been given to the beam at the free end. The first four natural frequencies have been calculated. The convergence result has been presented in the table .From the table it has been found that for 16numbers of elements the first four natural frequencies are converged properly. The present code is validated by considering a cantilever beam of rectangular cross section. The dimensions of the beam are (500×68×3.9) mm. The convergence result has been obtained for first four natural frequencies with different number of elements.

Table 1 Convergence result of first four natural frequencies of the proposed beam with p=2 and n=2.

	No of elements					
frequency	4	8	12	16	20	21
ω_1	82.98	82.16	82.01	81.96	81.93	81.93
ω_2	519.87	514.89	513.96	513.64	513.49	513.49
ω_3	1452.22	1441.99	1439.18	1438.23	1437.80	1437.80
ω_4	2730.98	2827.31	2820.70	2818.53	2817.59	2817.59

From the Table it has been found that the convergence will take place at 20 numbers of elements. Hence for further calculation same amount of beam element numbers are taken into consideration.

A. Variation Of Material Properties

The variations of material properties such as density (ρ), Young's modulus (E), Shear modulus (G) and Poison's ratio (υ) have been presented in Figure 3 and Figure 4 by using the proposed power law formula. The various material properties such as density (ρ) as 7850g/m^{3}Young's modulus (E) as 210GPa,Shear modulus (G) as 140GPaandPoison's ratio (υ) as 0.3 are taken as initial properties for the analysis.

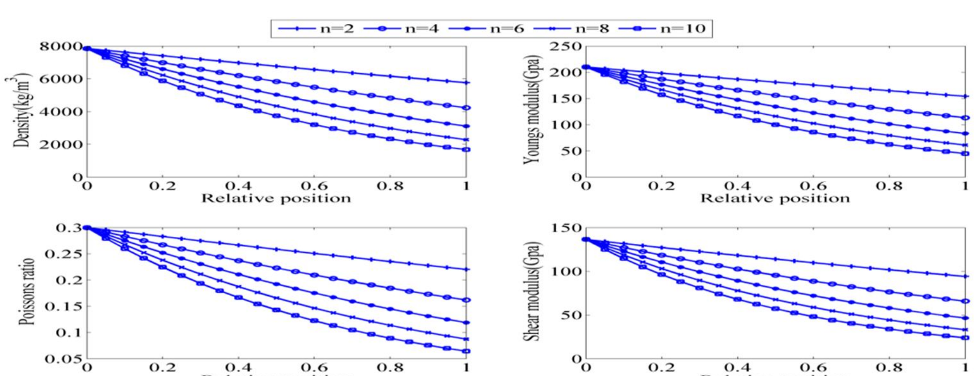


Figure 3 Variation of Density, Young's Modulus, Poison's ratio and Shear modulus with p=2

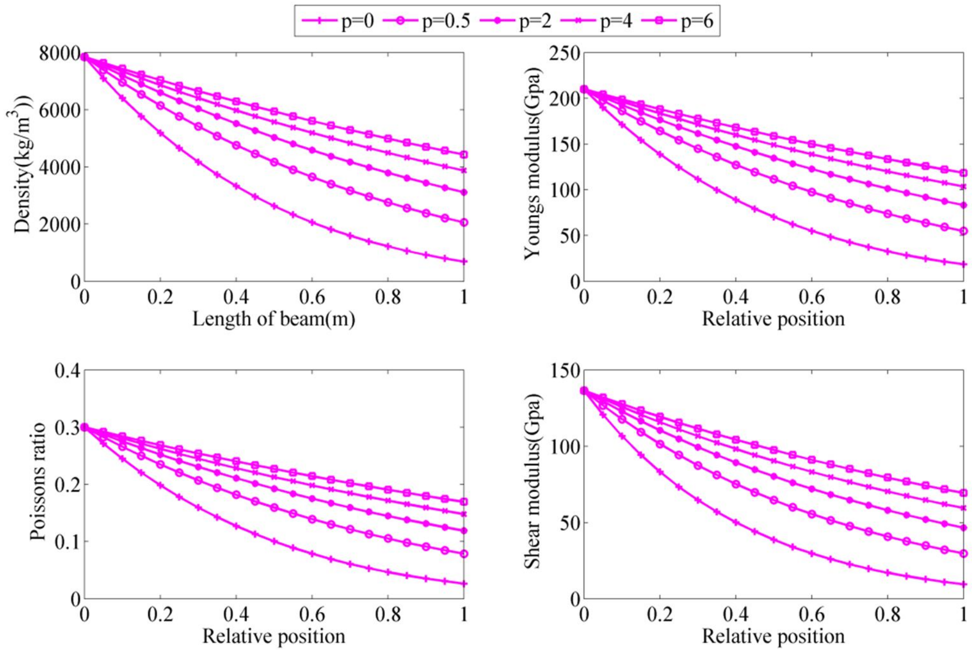


Figure 4 Variation of Density, Young's Modulus, Poison's ratio and Shear modulus with n=2

The material property variation with axial direction i.e. along the length of the beam is presented in Figure 3 and Figure 4.In Figure 3 the variation of material properties such as density (ρ), Young's modulus (E), Shear modulus (G) and Poison's ratio (υ) are presented with keeping *p=6* and *n=2,4,6,8,10*. From the figure it has been observed that for a given value of *p*, if *n* increases all the values of material properties decreases towards the free end of the cantilever beam. Further, with increase in *n* the material properties decreases. Similarly Figure 4 shows the variation of material properties such as density (ρ), Young's modulus (E), Shear modulus (G) and Poison's ratio (υ) are presented with keeping *n=6* and *p=2,4,6,8,10*. From the figure it has been observed that for a given value of *n*, if *p* increases all the values of material properties increases towards the free end of the cantilever beam. Further, with increase in *p* the material properties increases.

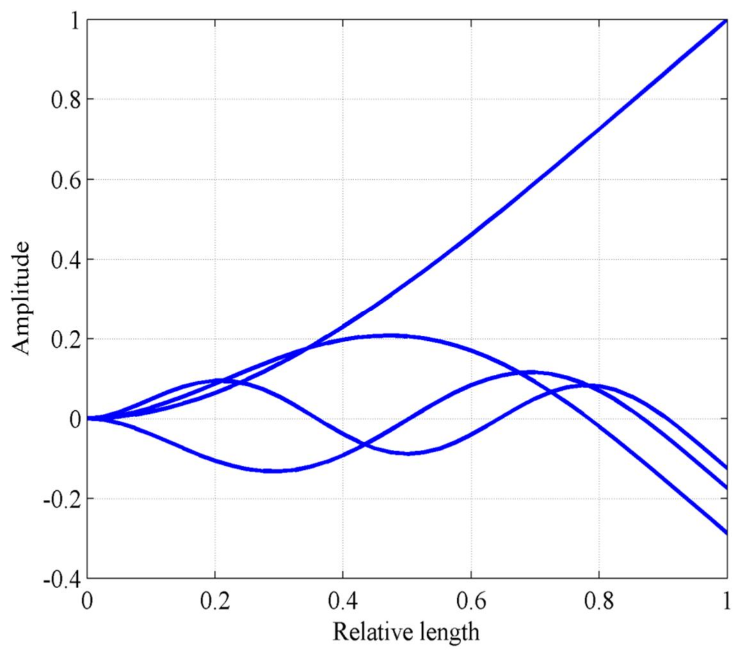


Figure 5. 1st, 2nd 3rd& 4th mode shapes of the proposed cantilever beam with p=6 and n=6.

The first four mode shapes for the proposed beam has been presented in Figure 5. From the figure it has been observed that the orientation of the beam is equivalent to the beam with homogeneous prismatic condition. The variation of first four natural frequencies of the proposed beam for different integer constant (*p*) and power index (*n*) are represented in the following table.

Table 2 First four natural frequencies of the proposed cantilever beam

p=2

Frequency	n=2	n=4	n=6	n=8	n=10
ω_1	81.93	81.97	81.99	82.017	82.02
ω_2	513.49	513.70	513.87	513.99	514.06
ω_3	1437.80	1438.40	1438.87	1439.20	1439.40
ω_4	2817.59	2818.76	2819.67	2820.32	2820.72

p=4

Frequency	n=2	n=4	n=6	n=8	n=10
ω_1	81.92	81.94	81.96	81.98	81.99
ω_2	513.39	513.53	513.66	513.77	513.87
ω_3	1437.52	1437.93	1438.29	1438.60	1438.86
ω_4	2817.05	2817.84	2818.54	2819.15	2819.66

p=6

Frequency	n=2	n=4	n=6	n=8	n=10
ω_1	81.91	81.93	81.94	81.96	81.97
ω_2	513.34	513.45	513.55	513.65	513.73
ω_3	1437.40	1437.70	1437.98	1438.24	1438.47
ω_4	2816.81	2817.40	2817.94	2818.44	2818.89

p=8

Frequency	n=2	n=4	n=6	n=8	n=10
ω_1	81.91	81.92	81.93	81.94	81.96
ω_2	513.32	513.41	513.49	513.56	513.64
ω_3	1437.33	1437.57	1437.80	1438.01	1438.21
ω_4	2816.67	2817.14	2817.58	2818.00	2818.39

p=10

Frequency	n=2	n=4	n=6	n=8	n=10
ω_1	81.90	81.91	81.93	81.94	81.95
ω_2	513.30	513.37	513.44	513.51	513.57
ω_3	1437.28	1437.48	1437.67	1437.86	1438.03
ω_4	2816.58	2816.97	2817.34	2817.70	2818.04

Table 1 represents the first four natural frequencies of the proposed beam for different values of *p* and *n*. It has been observed that for a given value of *p*; with increase in *n* the natural frequencies increase. But with increasing value of *p*, thenatural frequency decreases for a given value of *n*.

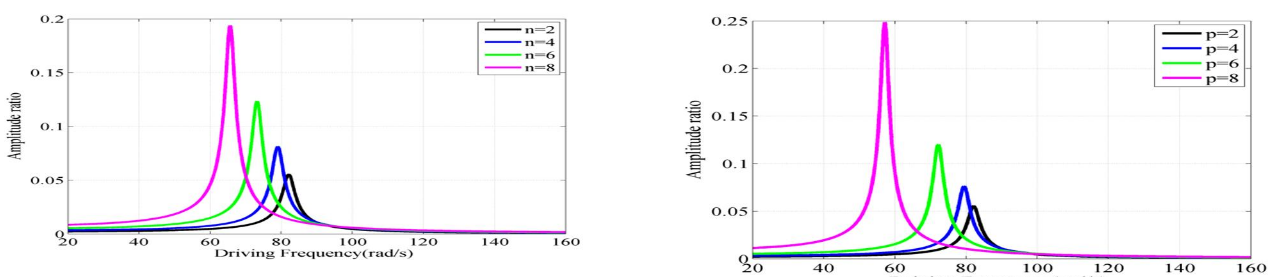
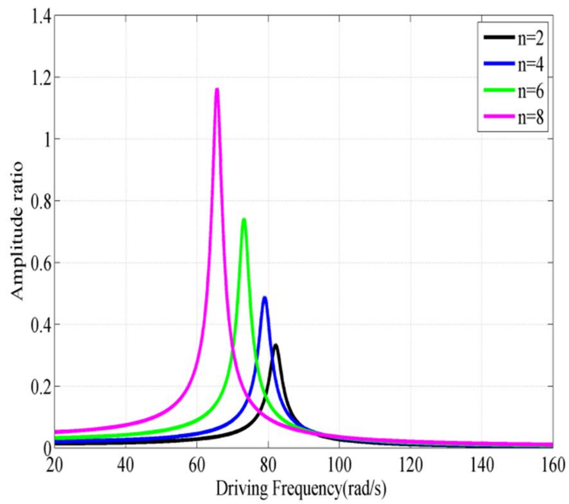


Figure 6 Frequency domain response of the proposed cantilever beam with (a) *p*=2and *n*=2, 4, 6, 8.(b) *n=2and p*=2,4,6,8 for impulse load free end is 2N

The frequency domain analysis of the proposed beam for *p=2* and four different values of *n* such as 2, 4, 6 and 8has been carried out. The dynamic responses have been carried out with an impulse load of 2 N. The obtained response is shown in Figure 6 Frequency domain response of the proposed cantilever beam with (a) *p*=2and *n*=2, 4, 6, 8.Figure 6(a). From the figure it has been observed that the peak response of the proposed beam increases as the power gradient index increases. It has been observed further that there is 68% increase of amplitude as the value increases from 2 to 8. This is due to the fact that with increase in power gradient index the amount of damping decreases. Further, it has been perceived that the resonant frequency of the proposed beam decreases as the value of *n* increases. This is due to the fact that as the value of *n* changes, there is a variation in material properties which affects both the stiffness and mass matrices of the beam. Similarly, the frequency domain analysis has been carried out for *n=2* and four different values of *p* such as 2, 4, 6 and 8. The response of such beam is shown in Figure 6(b).From the figure it has been observed that with increase in the value of p there is an increase in amplitude of the proposed beam for a given value of n. It has been noticed that there is an increase in 80% of amplitude when the value of p changes from 2 to 8. This is due to the fact that the amount of damping decreases when the value of p increases. Further, the increase in amplitude is more for the beam when compared with the previous case. Moreover, the resonant frequency is also decreases asthe value of p increases. The value of damping is more with increase in k as compared to increase in n.



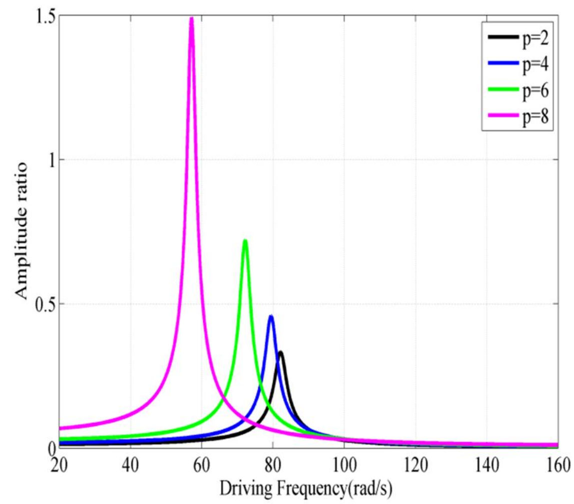
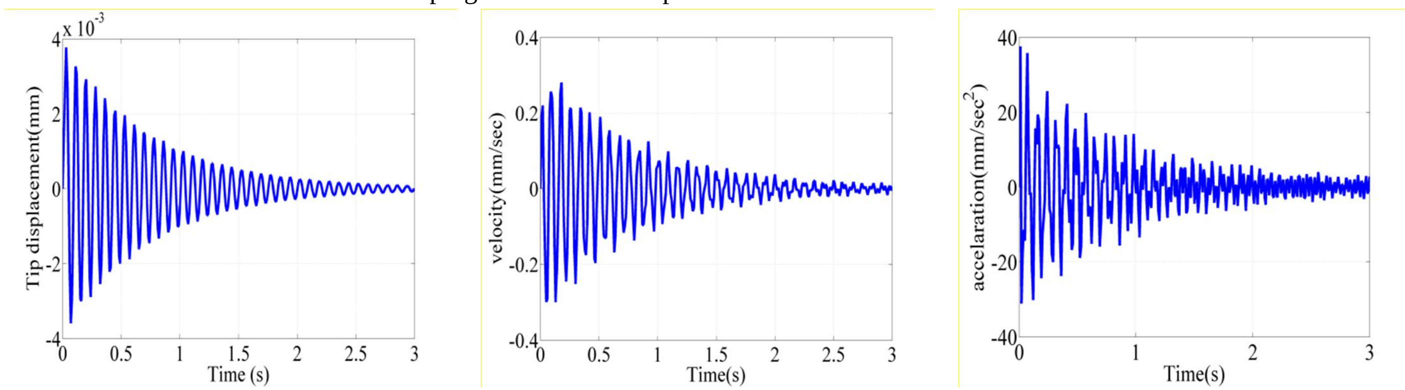


Figure 7 Frequency domain response of the proposed cantilever beam with (a) *p*=2and *n*=2, 4, 6, 8.(b) *n=2and p*=2,4,6,8 for impulse load free end is 6N

The frequency domain analysis of the proposed beam for *p=2* and four different values of *n* such as 2, 4, 6 and 8 has been carried out. The dynamic responses have been carried out with an impulse load of 6 N. The obtained response is shown in Figure 6 Frequency domain response of the proposed cantilever beam with (a) *p*=2and *n*=2, 4, 6, 8.Figure 7(a). From the figure it has been observed that the peak response of the proposed beam increases as the power gradient index (n) increases. It has been observed further that there is 76% increase of amplitude as the value increases from 2 to 8. This is due to the fact that with increase in power gradient index the amount of damping decreases. Further, it has been perceived that the resonant frequency of the proposed beam decreases as the value of *n* increases. This is due to the fact that as the value of *n* changes, there is a variation in material properties which affects both the stiffness and mass matrices of the beam. Similarly, the frequency domain analysis has been carried out for *n=2* and four different values of *p* such as 2, 4, 6 and 8. The response of such beam is shown in Figure 7(b).From the figure it has been observed that with increase in the value of p there is an increase in amplitude of the proposed beam for a given value of n. It has been noticed that there is an increase in 76% of amplitude when the value of *p* changes from 2 to 8. This is due to the fact that the amount of damping decreases when the value of p increases. Further, the increase in amplitude is more for the beam when compared with the previous case. Moreover, the resonant frequency is also decreases as the value of p increases. The value of damping is more with increase in k as compared to increase in n. Further from Figure 6 and Figure 7 it has been found that as the impulse force increases from 2N to 6 N the resonant frequency amplitude increases to 83% for both *p*=2; *n*=8 and *n*=2; *p*=8 case. This is due to the fact that as the impulse force increases the damping factor of the beam decreases which results in increase in amplitude.

The time domain analysis has been carried out using Newmark method in Matlab environment with 2N impulse load at the free end of the proposed cantilever beam.

The time domain responses such as displacement, velocity and acceleration have been carried out for 3 sec. The amount of proportional damping has been taken into account in the analysis. The responses have been shown in Figure 8 (a-c). From the figure it has been observed that the peak response of the beam is 3.8mm. The tip displacement response goes on decreasing with increasing in time. This is due to the structural damping for which the response diminishes with increase in time.



(a) (b) (c)

Figure 8 (a)Time domain displacement response at the free end with p=2 and n=6 (b) Time domain velocity response at the free end p=2 and n=6 (c) Time domain acceleration response at the free end p=2 and n=6.

The velocity response of the proposed beam has been shown in Figure 8 (b).From the figure it has been observed that the peak velocity response of the proposed beam for *p=2* and *n=6* is 0.28mm/sec. Due the presence of structural damping the velocity response diminishes after certain time. Similarly, the acceleration response of the proposed beam has been shown in Figure 8(c). From the figure it has been observed that, the peak response of acceleration is 31.28 mm/sec^2. The acceleration response diminishes with increase in time due to the structural damping present in the proposed beam.

IV. CONCLUSION

The present paper uses the finite element approach to analyse the static and dynamic vibrations of a cantilever prismatic beam with nonuniform variation of material properties in the axial direction. For the analysis, a hypothesized power variation of material attributes was used. In order to solve the governing equation, two noded beam elements with two degrees of freedom at each node are examined. The Euler-Bernoulli beam theory was investigated for solving the beam's governing equation. It has been discovered that material qualities play an important role in beam vibration analysis. It has been noticed that as the power gradient index grows, the amplitude of vibration increases while the fundamental frequency falls. It has once again been determined that structural dampening has a role.

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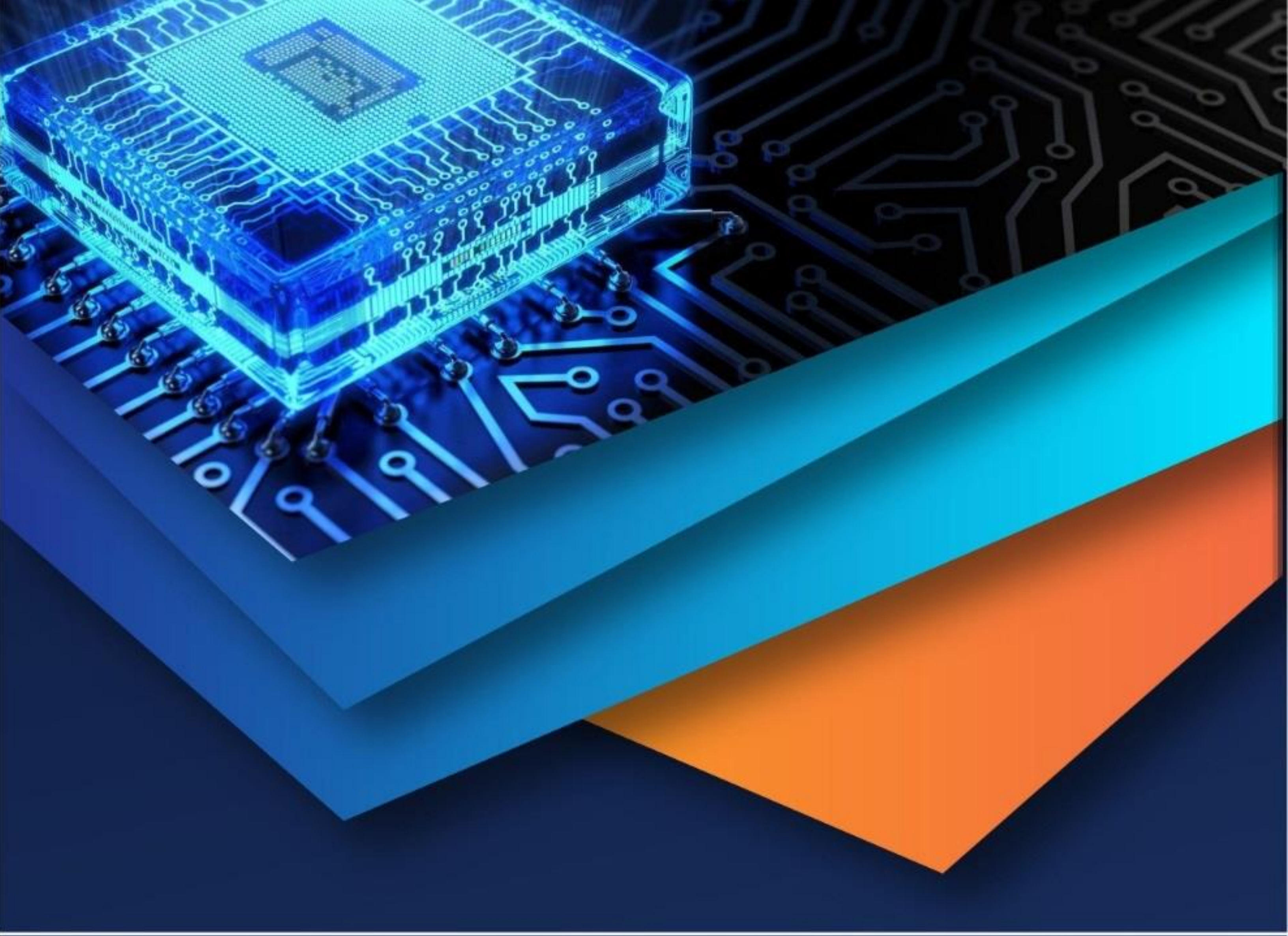

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