iJRASET
International Journal For Research in
Applied Science and Engineering Technology

INTERNATIONAL JOURNAL
FOR RESEARCH
IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 11 Issue: VIII Month of publication: Aug 2023

DOI: https://doi.org/10.22214/ijraset.2023.55379

Flow of Third Grade Fluid and Vogel's Model Viscosity in Cylindrical Pipe

Obi Boniface Inalu
Department of Mathematics, Imo State University, Owerri, Nigeria.

***Abstract:** The flow of non-Newtonian fluid and Vogel's model viscosity in cylindrical pipe is treated. The coupled nonlinear equations of motion solved and the effects of sundry parameters of the non-Newtonian fluid and the Vogel model viscosity examined. Results show that the gravitational parameter has a great influence on the flow field. It is observed that as the viscosity index increases, the velocity of the fluid flow reduces. This indicates that the shear strain which reduces the flow velocity, increases the viscosity. Results further show that increase in β increases the temperature of the cylinder indicates a low viscosity. This is because high temperature means that particles have more thermal energy and are easily able to overcome the attractive forces holding them together.*

***Keywords:** Non-Newtonian, Third grade fluid, Viscosity, Heat Transfer, Vogel model*

I. INTRODUCTION

In the past years, generalization of the Navier-Stoke's model which is highly non-linear constitutive laws have been proposed and examined by many researchers. Because of the deficiency of the classical Navier-Stokes theory in describing rheological complex fluids such as paints, blood, oils and greases and the applications in engineering and technology as well as the pulp industries, has led to the development of many theories of non-Newtonian fluids.

Many researchers have done some works to explain some of the complex nature of the non-Newtonian fluids of the differential types, amongst whom are: Rivlin and Erickson [12], on stress deformation relations for isotropic materials. Fosdic and Rajagopal [4], on thermodynamics, stability of fluids of third grade. Okedayo *et al* [7] carried out computational study of reactive flow of an electrically conducting fluid with temperature dependent viscosity and axial magnetic field using the semi-implicit finite difference scheme.

Aksoy and Pakdemirli [2]. They dealt with the flow of a non-Newtonian fluid through a porous medium in between two parallel plates at different temperatures. They considered different cases: constant viscosity, Reynold's model viscosity and Vogel's model viscosity and derived the criteria for validity, for approximate solutions. Obi *et al* [8], analyzed the flow of incompressible MHD third grade fluid in an inclined rotating cylindrical pipe with isothermal wall and joule heating. Obi [9], applied perturbation technique to analyze magnetohydrodynamic flow of third grade fluid in an inclined cylindrical pipe.

Hayat *et al* [5] applied homotopy perturbation and numerically obtained the solution of the third grade fluid past a porous channel with suction and injection at the walls. Massoudi and Christie [6] examined the effects of variable viscosity and viscous dissipation on the flow of a third grade fluid in a pipe. Shirkhani *et al* [13], examined the unsteady time-dependent incompressible Newtonian fluid flow between two parallel plates by homotopy analysis method (HAM), homotopy perturbation method (HPM) and collocation method (CM). They transformed the Navier-Stokes equation into ordinary differential equation using similarity transformation and investigated the effects of Reynolds number and suction or injection characteristic parameter on the velocity field.

Pakdemirli and Yilbas [10] examined entropy generation in a pipe due to non-Newtonian fluid flow, a case of constant viscosity. They formulated the entropy generation number due to heat transfer and fluid friction. The influences of non-Newtonian parameters and Brinkman number on entropy generation number were examined and results revealed that increase in the non-Newtonian parameter reduces the fluid friction in the region close to the wall of the pipe, given rise to low entropy generation. They further discovered that increase in the Brinkman number enhances the fluid friction and heat transfer rate thereby increases the entropy number.

Aiyesimi *et al* [1] on the analysis of unsteady MHD thin film flow of a third grade fluid with heat transfer down an inclined plane. They discovered that the variation of the velocity and temperature profiles with the magnetic fields and gravitational field parameters depended on time. Ayub *et al* [3] examined the exact flow of third grade fluid past a porous plat e. the applied homotopy perturbation method for their analysis.

II. MATHEMATICAL FORMULATION

Considering a steady incompressible MHD flow of third grade fluid in cylindrical pipe. The velocity field is of the form

$$v = (0, 0, u(0)) \tag{1}$$

$$\nabla . v = 0 \tag{2}$$

$$S = s(r) \tag{3}$$

$$\rho \frac{Dv}{Dt} = -\nabla p + \rho B + divT \tag{4}$$

The equation in a third grade fluid is expressed as in Rajagopal and Na [11] as

$$T = -p_1 + \mu A_1 + \alpha_1 A_2 + \alpha_2 A_1^2 + \beta_1 A_3 + \beta_2 (A_1 A_2 + A_2 A_1) + \beta_3 (trA_1^2) A_1 \tag{5}$$

Where

ρ is the fluid density, B is the body force, V is the velocity, p is pressure, T is the Cauchy stress tensor, μ is the dynamic viscosity, $\frac{Dv}{Dt}$ is the material –time derivative, ∇ is the gradient operator and S is the extra tensor for the third grade.

$$A_0 = I, A_1 = L + L^T \tag{6}$$

$$A_n = \frac{dA_{n-1}}{dt} + A_{n-1} L + L^T A_{n-1},\ n = 2, 3, 4, ... \tag{7}$$

$$L = \nabla v \tag{8}$$

The incompressibility criterion is satisfied by eqn (1). In applying the eqns (6-8) and substituting the values of v and T in eqns (1) and (2), neglecting body force, the momentum equation is given as

$$\frac{d^2u}{dr^2} + 6(\beta_2 + \beta_3)\left(\frac{du}{dr}\right)^2 \frac{d^2u}{dr^2} - \frac{1}{\ell}\frac{dp}{dr} - \sigma B_0^2 u = 0 \tag{9}$$

$$k\frac{d^2T}{dr^2} + \mu\left(\frac{du}{dr}\right)^2 + 2\beta\left(\frac{du}{dr}\right)^4 + \sigma B_0^2 u^2 = 0 \tag{10}$$

$$\frac{du}{dr} = \frac{dT}{dr} = 0, u(a) = 0, T(a) = T_0 \tag{11}$$

Introducing the following dimensionless variables for as:

$$\overline{r} = \frac{r}{a}, \overline{u} = \frac{u}{u_0}, \theta = \frac{T}{T_0} \tag{12}$$

Using eqn (4) in eqns (9-11), yields

$$\frac{d^2u}{dr^2} + 6\beta\left(\frac{du}{dr}\right)^2 \frac{d^2u}{dr^2} - \tau - Mu = 0 \tag{13}$$

$$\frac{d^2\theta}{dr^2} + B_r\left(\frac{du}{dr}\right)^2 + 2\beta\left(\frac{du}{dr}\right)^4 + Mu^2 = 0 \tag{14}$$

$$u(0) = \theta(0) = 0, u(1) = \theta(1) = 1 \tag{15}$$

III. METHOD OF SOLUTION

$$u(r)=u_0(r)+\beta u_1(r)+O(\beta^2),\theta(r)=\theta_0(r)+\beta\theta_1(r)+O(\beta^2),\tau=\beta G, M=\beta M \tag{16}$$

Substituting the following perturbation series (16) into eqns (13) and (14),

$$\beta^0:\frac{d^2u_0}{dr^2}=0 \tag{17}$$

$$\beta:\frac{d^2u_1}{dr^2}+6\left(\frac{du}{dr}\right)^2\frac{d^2u_0}{dr^2}-G-Mu_0=0 \tag{18}$$

$$\beta^0:\frac{d^2\theta_0}{dr^2}+B_r\left(\frac{du_0}{dr}\right)^2=0 \tag{19}$$

$$\beta:\frac{d^2\theta_1}{dr^2}+2B_r\left(\frac{du_0}{dr}\frac{du_1}{dr}\right)+2\left(\frac{du_0}{dr}\right)^4+Mu_0^2=0 \tag{20}$$

Solving eqns (17-20) for the case of constant viscosity, with the condition (15), yields

$$u(r)=r+\beta\left(\frac{1}{2}r^2G+\frac{1}{6}r^3M+r-\frac{1}{2}rG-\frac{1}{6}rM\right) \tag{21}$$

$$\theta(r)=r+\left(\frac{1}{2}r-\frac{1}{2}r^2\right)B_r+\beta\left(-B_r\left(\frac{1}{3}r^3G+\frac{1}{12}r^4M+r^2-\frac{1}{2}r^2G-\frac{1}{6}r^2M\right)-r^2-\frac{1}{9}r^3M\right.$$
$$\left.+B_r\left(1-\frac{1}{6}G-\frac{1}{12}M\right)r+2+\frac{1}{9}M\right) \tag{22}$$

A. *The Vogel's Model Case*

The dimensionless Vogel model momentum equation and energy balance are:

$$\frac{d\mu}{dr}\frac{du}{dr}+\frac{d^2u}{dr^2}\left(\mu+6\beta\left(\frac{du}{dr}\right)^2\right)+K\theta+\tau u=-1 \tag{23}$$

$$\frac{d^2\theta}{dr^2}+\mu\left(\frac{du}{dr}\right)^2+2\beta\left(\frac{du}{dr}\right)^4+\tau u^2=0 \tag{24}$$

Assuming the expansion for the velocity and energy equations is of the form eqn (16),
where β is a small parameter. In Vogel's model, viscosity can be considered in the direction of Massoudi and Christie [6] as

$$\mu=\mu\exp\left(\frac{A}{B+\theta}-\theta_w\right) \tag{25}$$

Applying Taylor's series expansion, eqn (25), yields

$$\mu=\alpha\left(1-\frac{A\theta}{B^2}\right) \tag{26}$$

where

$$\alpha = \mu\left(\exp\frac{A}{B} - \theta_w\right) \tag{27}$$

A and B, being parameters relating to Vogel's model.

$$\frac{d\mu}{dr} \equiv -\alpha\frac{A}{B^2}\frac{d\theta}{dr} \tag{28}$$

Substituting eqns (16), (26) and (28) into eqns (23) and (24), expanding and choosing the zeroth order and the order 1 of the small parameter β, for the Vogel's model momentum equation and energy balance, yields:

$$\beta^0 : \frac{d^2u_0}{dr^2} = 0 \tag{29}$$

$$\beta : \frac{d^2u_1}{dr^2} - \alpha\frac{n}{B^2}\frac{d\theta_0}{dr}\frac{du_0}{dr} + 6\left(\frac{du_0}{dr}\right)^2\frac{d^2u_0}{dr^2} - G - Mu_0 \tag{30}$$

$$\beta^0 : \frac{d^2\theta_0}{dr^2} + \alpha\left(\frac{du_0}{dr}\right)^2 = 0 \tag{31}$$

$$\beta : \frac{d^2\theta_1}{dr^2} - 2\alpha\frac{du_0}{dr}\frac{du_1}{dr} + \alpha\frac{n\theta_0}{B^2}\left(\frac{du_0}{dr}\right)^2 + 2\left(\frac{du_0}{dr}\right)^4 + Mu_0^2 \tag{32}$$

Solving the dimensionless momentum and energy equations (29-32), with the condition (15), yields

$$u(r) = -r + \beta\left(\frac{\alpha^2 n}{2B^2}r^2 - \frac{\alpha^2 n}{6B^2}r^3 + \frac{1}{2}r^2G - \frac{1}{6}r^3M - \frac{\alpha^2 n}{3}r - \frac{1}{2}rG + \frac{1}{6}rM + r\right) \tag{33}$$

$$\theta(r) = \frac{1}{2}\alpha\left(r^2 - r\right) + 1 + \beta\left(\frac{\alpha^3 n}{3B^2}r^3 - \frac{\alpha^3 n}{12B^2}r^4 + \frac{\alpha}{3}r^3G - \frac{\alpha}{12}r^4M - \frac{\alpha^2}{6}r^3 - \frac{\alpha}{2}r^3G + \frac{\alpha}{6}r^2M\right.$$
$$+\alpha r^2 - \frac{\alpha^2 n}{24}r^4 + \frac{\alpha n}{12}r^3 - \frac{\alpha n}{2}r^2 - \frac{1}{2}r^2 - \frac{1}{12}r^4M + r - \frac{\alpha^3 n}{4B^2}r + \frac{\alpha}{6}rG - \frac{\alpha}{12}rM + \frac{\alpha^4 n}{6}r + \frac{\alpha^2 n}{24}r$$
$$\left.+\frac{5\alpha n}{12}r - \alpha r + \frac{1}{2}r + \frac{1}{12}rM\right) \tag{34}$$

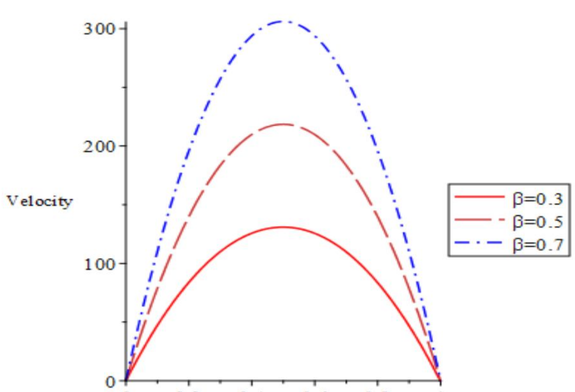

Figure 1: Velocity Profile For Various Values Of The Third Grade Parameter (β)

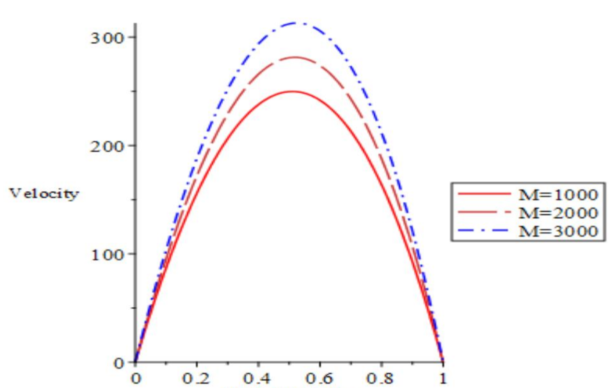

Figure 2: Velocity Profile For Various Values Of Magnetic Field Parameter (M)

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.538
Volume 11 Issue VIII Aug 2023- Available at www.ijraset.com

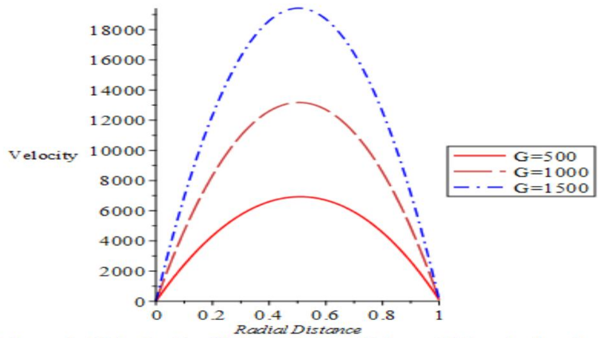

Figure 3: Velocity Profile For Various Values Of Gravitational Parameter (G)

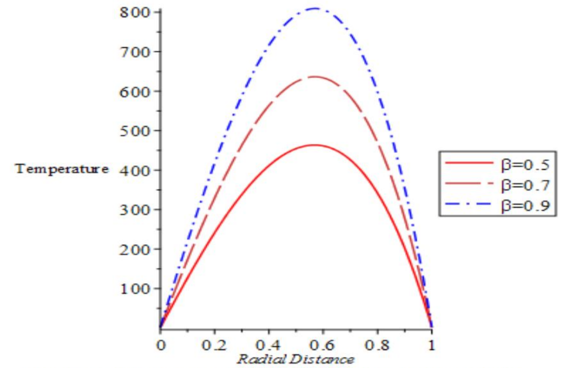

Figure 4: Temperature Profiles For Various Values Of The Third Grade Parameter (β)

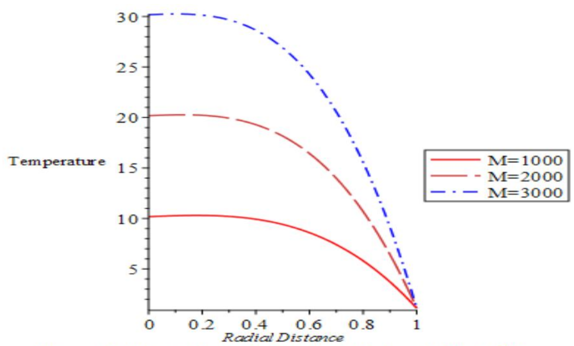

Figure 5: Temperature Profiles For Various Values Of Magnetic Field Parameter (M)

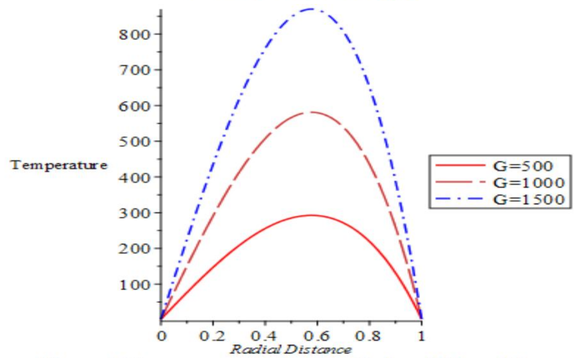

Figure 6: Temperature Profiles For Various Values Of Gravitational Parameter (G)

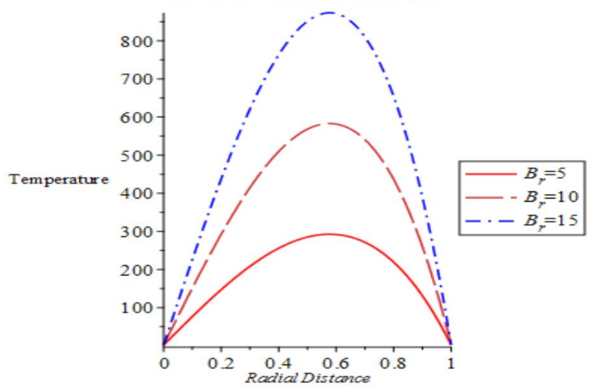

Figure 7: Temperature Profiles For Various Values Of Brinkmann Parameter (B_r)

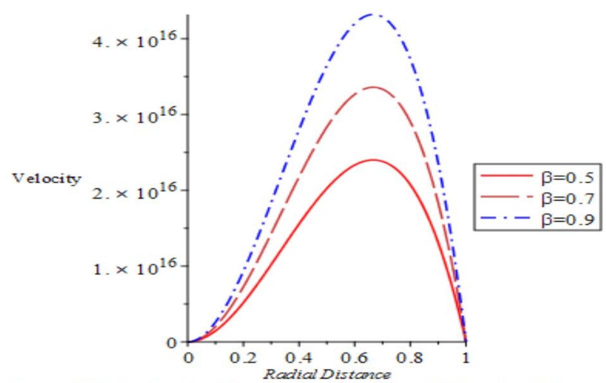

Figure 8: Velocity Profiles For Various Vogel Viscosity Index (β), When α=9000,B_r = 0.0005, n=2000, G=1000 and M=50

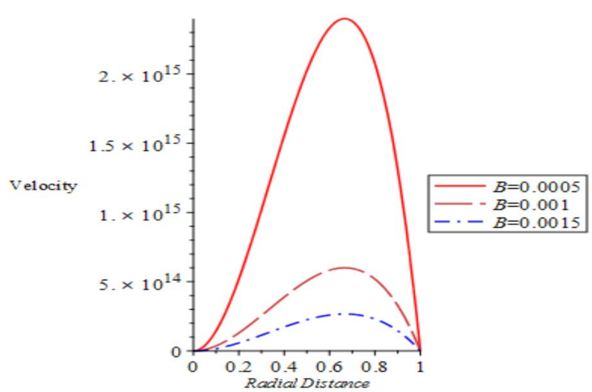

Figure 9: Velocity Profiles For Various Vogel Viscosity Index (B), When β=0.05,α=9000,n=2000, G=1000 and M=1000

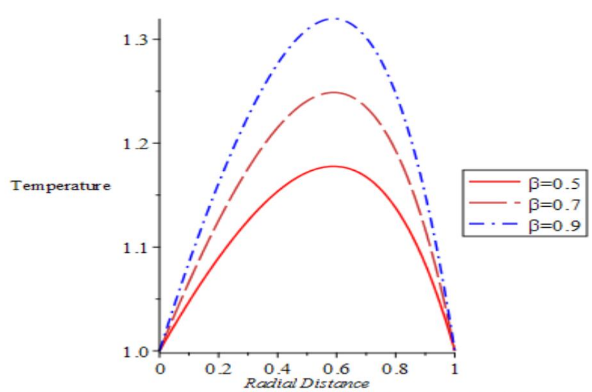

Figure 10: Temperature Profiles For Various Vogel Viscosity Index (B), When B=5,α=0.00009,n=0.5, G=0.5 and M=6

IV. RESULT AND DISCUSSIONS

In this section, the solutions of the velocity and temperature profiles are obtained by specifying the values of the thermo-solutal parameters to see the effects of various dimensionless numbers on the velocity field and energy balance.
The velocity and energy equations presented in section (3.0) eqns (9) and (10), with the solutions in equations (21) and (22). The Vogel model equations are presented in equations (23) and (24) with the results in equations (33) and (34). Figure 1 shows the velocity profiles for different values of the non-Newtonian parameter β . Results show that increase in the non-Newtonian parameter increases the velocity of the fluid flow. Figure 2 is the variation of the magnetic field parameter while other parameters are held constant. Results indicate that a little increase in the velocity profiles as the magnetic field increases. Figure 3 shows the velocity profiles for variation of the gravitational field parameter G, with other parameters kept constant. Results show that the gravitational parameter has a great influence on the flow field. It is observed that velocity increased with increase in the gravitational parameter. Figure 4 is the temperature profiles for variation of the non-Newtonian parameter β in which results indicate that as the parameter increases, the temperature of the system also increases proportionately.
Figures 5 and 6 show the effects of the magnetic and gravitational field parameters on the temperature profiles. Results show that as the magnetic and gravitational field parameters increases, the temperature of the system also increases at the same rate. In figure 7, the Brinkman parameter is varied while other parameters held constant. It is observed that increase in the Brinkman parameter increases the temperature of the cylinder. The effects of the dimensionless parameter β is shown in figure 8. Results reveals that as the flow velocity increases, indicates a low rate of strain which in turn results in low mean and maximum velocity.
In figure 9, the effects of the viscosity index B is presented with its resulting influence. It is observed that as the viscosity index increases, the velocity of the fluid flow reduces. This indicates that the rate of shear strain which reduces the flow velocity, increases the viscosity. In figure 10. The influence of the non-Newtonian parameter β n the Vogel model viscosity is presented and the results show that increase in β increases the temperature of the cylinder indicates a low viscosity. This is because high temperature means that particles have higher thermal energy and are more easily able to pull through the attractive forces holding them together.

V. CONCLUSIONS

The non-Newtonian fluid flow and Vogel's model viscosity in cylindrical pipe is treated. The coupled nonlinear equations of motion solved and the effects of sundry parameters of the non-Newtonian fluid and the Vogel model viscosity examined. Results show that the gravitational parameter has a great influence on the flow field. It is observed that as the viscosity index increases, the velocity of the fluid flow reduces. This indicates that the rate of shear strain which reduces the flow velocity, increases the viscosity. Results further show that increase in β increases the temperature of the cylinder indicates a low viscosity. This is because high temperature means that particles have higher thermal energy and are more easily able to pull through the attractive forces holding them together.

A. Declarations

1) Funding: Not applicable
2) Informed Consent Statement: Not applicable
3) Data Availability: Not applicable
4) Conflict of Interest Statement: No conflict of interest

REFERENCES

[1] Aiyesimi, Y.M., Okedayo, G.T., and Lawal, O.W.: Analysis of unsteady MHD thin film flow of a third grade fluid with heat transfer down an inclined plane. International Journal of Applied and Computational Mathematics, 2, 58-69(2014)
[2] Aksoy, Y. and Pakdemirli, M.: Approximate analytical solution for flow of a third grade fluid through a parallel plate channel filled with a porous medium. Transp. Porous. Med. 83,375-395(2010).
[3] Ayub, M., Rasheed, A. and Hayat, T.,: Exact flow of third grade fluid past a porous plate using homotopy analysis method. International Journal of Engineering and Sciences Vol 41,2091(2003)
[4] Fosdick R.L. and Rajagopal, K.R.: Thermodynamics and stability of fluids of third grade. Proc. R. Soc. Lond. 339, 351-377, (1980).
[5] Hayat,T., Ellahi, R., Ariel, P.D., Asghar, S.: Homotopy solution for the channel flow of a third grade fluid. Non-linear Dyn. 45, 55-64(2006).
[6] Massoudi, M. and Christie, I.: Effects of variable viscosity and viscous dissipation on the flow of a third –grade fluid in a pipe. Int. J. of Nonlinear Mech.,30(5): 687-699,(1995).

ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.538
Volume 11 Issue VIII Aug 2023- Available at www.ijraset.com

[7] Okedayo G. T., Abah S. O and Abah R. T.: Viscous dissipation effect on the reactive flow of a temperature dependent viscosity and thermal conductivity through a porous channel. Abacus the journal of the mathematical association of Nigeria 41(2),74-81, (2014).
[8] Obi B.I.,Okedayo, G.T., Jiya, M. And Aiyesimi, Y.M.: Analysis of Flow of An Incompressible MHD third Grade Fluid In An Inclined Rotating Cylindrical Pipe With Isothermal Wall And Joule Heating. International Journal For Research In Mathematics And Statistics. Vol 7 Issue 6,(2021)
[9] Obi B.I.: Perturbation Analysis Of Magnetohydrodynamic Flow of Third Grade Fluid In An Inclined Cylindrical Pipe: Journal Of Mathematical Science And Computational Mathematics (JMSCM) Vol 3, No. 3(2022)
[10] Pakdemirli, M.and Yilbas, B.S.: Entropy generation in a pipe due to non-Newtonian fluid flow: Constant viscosity case. Sadhana Vol 31 Part 1.(2006)
[11] Rajagopal K.R.and Na T.Y.: On Stoke's Problem For a Non-Newtonian Fluid. Acta. Mech. 48:233-239(1983)
[12] Rivlin, R.S., Ericksen, J.I.: Stress deformation for isotropic materials. J. Ration. Mech. Anal. 4,323-329(1955).
[13] Shirkhani, M.R., Hoshyar, H.A., Rahimipetroudi, I., Akhavan, H., Ganji, D.D. Unsteady time-dependent incompressible Newtonian fluid flow between two parallel plates by homotopy analysis method (HAM),homotopy perturbation method (HPM) and collocation method (CM). Propulsion and Power Research . 7(3): 247-256(2018).