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Fuzzy EPQ Model for Green Quality Products of Three Step Production Processes

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Abstract: Most commercial industries now encourage environmentally friendly or green products since they are safe for the environment in their creation, use, and disposal. Eco-friendly items serve an important role in decreasing pollution both during production and recycling. To ensure client happiness, most branded products are developed in an environmentally friendly manner. This research proposes a cost-effective manufacturing quantity model that promotes high-quality green products. To solve the model in a fuzzy sense, an extension of the lagrangian method is applied. The parameters are expressed as a fuzzy pentagonal number. Defuzzification is accomplished using the graded mean integration approach. To demonstrate the model, a numerical example is provided.
Keywords: environmental cost, green products, fuzzy, recycling.

I. INTRODUCTION

The management of manufacturing inventory is heavily weighted in favour of the company's profit and customer acquisition. Controlling the quantity of a product is a method of inspecting raw materials, work-in-progress, and finished goods in a methodical manner. Apart from inspecting for defective materials as part of quality control, the degree of pollution must also be assessed to make environmentally friendly products, as most consumers are now interested in eco-friendly commodities. This model incorporates the concept of green quality product into a fuzzy EPQ model. The product cost parameters are represented as pentagonal fuzzy numbers.

Inventory models were created with the primary goal of calculating the best quantity to order and the best timing to place the orders. In 1913, Harris [2] created the first economic ordering quantity (EOQ) inventory model, which includes ordering and holding expenses. By incorporating the fraction of real ideal time spent in the production process, Taft [3] changed this model to the economic production quantity (EPQ) model in 1918. Deterministic inventory models were developed first, then probabilistic and fuzzy inventory models were developed to deal with unpredictable scenarios. Inventory fluctuations result in shortages or surpluses, which is a very typical occurrence. To deal with such situations, Drenzer [4], Goyal [5], and Gurgani [6] outlined the shortage, partial backlog, and complete backlog inventory models, and Goyal fused the notion of trade credit and price measures to the inventory model in 1985. Jamal [7], Chang [8], Jaggi [9], and Shah [10] expanded these trade credit inventory models. In general, a production organisation obtains input from a variety of sources; but, if customer satisfaction declines, procurement is moved to a different source for quality control, resulting in product switching costs. In addition, the manufacturing process is not a one-step process. It is divided into three stages: pre-production, production, and post-production. All of the production-related activities in the three processes must be coordinated and closely monitored in order to produce green-quality products.

II. DEFINITIONS

A. Fuzzy Set

A fuzzy set $\tilde{A}$ in a universe of discourse X is defined as the following set of pairs $\tilde{A} = \{(x, \mu_{\tilde{A}}(x): x \epsilon X)\}$.Here $\mu_{\tilde{A}}: X \to [0,1]$is a mapping called the membership value of $x \in X$ in a fuzzy set $\tilde{A}$.

B. Graded Mean Integration Representation Method

If $\tilde{A} = (a_1, a_2, a_3, a_4, a_5)$ is a pentagonal fuzzy number then the graded mean representation (GMIR) method of $\tilde{A}$ is defined as

$$\mathcal{P}(\tilde{A}) = \frac{1}{12}(b_1 + 3b_2 + 4b_3 + 3b_4 + b_5)$$

C. Pentagonal Fuzzy Number

A fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4, a_5)$ where $a_1 < a_2 < a_3 < a_4 < a_5$ are defined on R is called pentagonal fuzzy number if its membership function is

$$\mu_{\tilde{A}}(x) = \begin{cases} L_1(x) = \dfrac{x-a}{b-a}, & a \le x \le b \\ L_2(x) = \dfrac{x-a}{b-a}, & b \le x \le c \\ 1 & x = c \\ L_2(x) = \dfrac{d-x}{d-c}, & c \le x \le d \\ L_2(x) = \dfrac{e-x}{e-d}, & d \le x \le e \\ 0 & otherwise \end{cases}$$

D. Arithmetic Operations under Function Principle

The arithmetic operations between pentagonal fuzzy numbers proposed are given below.

Let us consider $\tilde{A} = (a_1, a_2, a_3, a_4, a_5)$and $\tilde{B} = (b_1, b_2, b_3, b_4, b_5)$be two pentagonal fuzzy numbers.

- The addition of $\tilde{A} \oplus \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5)$
- The subtraction of $\tilde{A} \ominus \tilde{B} = (a_1 - b_5, a_2 - b_4, a_3 - b_3, a_4 - b_2, a_5 - b_1)$
- The multiplication of $\tilde{A}$ and $\tilde{B}$ is $\tilde{A} \otimes \tilde{B} = (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4, a_5 b_5)$
- The division of $\tilde{A}$ and $\tilde{B}$ is $\tilde{A} \oslash \tilde{B}\left(\frac{a_1}{b_5}, \frac{a_2}{b_4}, \frac{a_3}{b_3}, \frac{a_4}{b_2}, \frac{a_5}{b_1}\right)$

E. Extension of the Lagrangean Method

Taha discussed how to solve the optimum solution of nonlinear programming problem with equality constraints by using Lagrangean Method, and showed how the Lagrangean method may be extended to solve inequality constraints. The general idea of extending the Lagrangean procedure is that in the unconstrained optimum the problem does not satisfy all constraints, the constrained optimum must occur at a boundary point of the solution space. Suppose that the problem is given by Minimize y = f(x) Subject to $g_i(x) \geq 0$, i = 1,2, …. m. The non-negativity constraints x ≥0 if any are included in them constraints. Then the procedure of the Extension of the Lagrangean method involves the following steps.

1) *Step 1:* Solve the unconstrained problem Min y = f(x) If the resulting optimum satisfies all the constraints, stop because all constraints are redundant. Otherwise, set k = 1 and go to step 2.
2) *Step 2:* Activate any k constraints (i. e., convert them into equality) and optimize f(x) subject to the k active constraints by the Lagrangean method. If the resulting solution is feasible with respect to the remaining constraints and repeat the step. If all sets of active constraints taken k at a time are considered without encountering a feasible solution, go to step 3.
3) *Step 3:* If K = m, stop; no feasible solution exists. Otherwise, set k = k + 1 and go to step 2.

F. Assumptions

- The pace of demand is constant and higher than the rate of production.
- The switching costs occur regardless of the nature of the input or the machinery.
- There is a predetermined lead time.

G. Notations

$s_{\mathscr{k}}$ – Demand per unit time

$\mathcal{A}$ – Production per unit of time

$\mathscr{g}$ - ${}^{s_{\mathscr{k}}}/_{\mathcal{A}}$

$1 - \mathscr{g}$ - The production of time the production process spends actually idling

$\mathcal{H}$ - Fixed ordering cost

$\mathscr{f}$ – Holding cost per unit per unit of time

$\mathcal{I}_d$ – Initialisation cost for pre production process

$\mathcal{M}_i$ – Inspection costs of inputs

$\mathcal{L}_y$ – Inspection cost of machinery
$\mathcal{R}_i$ – Procedural switching cost for input
$\mathcal{J}_y$ – Procedural switching cost for machinery
$\mathcal{B}_i$ – Relative switching cost for input
$\mathcal{Q}_y$ – Relative switching cost for machinery
$\mathcal{F}_i$ – financial switching cost for input
$\mathcal{N}_y$ – financial switching cost for machinery
$\mathcal{W}_i$ – Cost of procuring new inputs
$\mathcal{V}_y$ – Cost of installing efficient machinery
$\mathcal{S}_z$ - Initialization cost for production process
$\mathcal{P}_\ell$ - Production cost per cycle
$\mathcal{I}_\mathcal{C}$ – Quality evaluation cost before packing
$\mathcal{M}_u$ - Procedural switching cost for enhancing the product's quality after production process
$\mathcal{D}_w$ - Relative switching cost for enhancing the product's quality after production process
$\mathcal{G}_q$ - Financial switching cost for enhancing the product's quality after production process
$\mathcal{T}_\sigma$ - Initialization cost for post-production process
$\mathcal{K}_e$ - Screening cost of the products
$\mathcal{W}_m$ - Rework costs of the defective items
$\mathcal{Z}_\hbar$ - Delivery cost of the products per unit per unit of time
$\mathcal{S}_x$ - Waste processing costs before disposal
$\mathcal{N}_r$ - Disposal costs of the waste of all forms

H. Formulation of the crisp Model

The total cost per unit of time is $\boldsymbol{TC}$ =

$$\frac{f\, \mathcal{C}_b\,(1-g)}{2} + \frac{s_k}{\mathcal{C}_b}\left(\mathcal{H} + \mathcal{I}_d + \mathcal{M}_i + \mathcal{L}_y + \mathcal{R}_i + \mathcal{J}_y + \mathcal{B}_i + \mathcal{Q}_y + \mathcal{F}_i + \mathcal{N}_y + \mathcal{W}_i + \mathcal{V}_y + \mathcal{S}_z + \mathcal{P}_\ell + \mathcal{I}_\mathcal{C} + \mathcal{M}_u + \mathcal{D}_w + \mathcal{G}_q + \mathcal{T}_\sigma + \mathcal{K}_e + \mathcal{W}_m + \mathcal{Z}_\hbar + \mathcal{S}_x + \mathcal{N}_r\right) \text{--------------------} (1)$$

Differentiating (1) with respect to $\mathcal{C}_b$, $\frac{\partial TC}{\partial \mathcal{C}_b} = 0$

$$\frac{f\,(1-g)}{2} = \frac{s_k}{{\mathcal{C}_b}^2}\left(\mathcal{H} + \mathcal{I}_d + \mathcal{M}_i + \mathcal{L}_y + \mathcal{R}_i + \mathcal{J}_y + \mathcal{B}_i + \mathcal{Q}_y + \mathcal{F}_i + \mathcal{N}_y + \mathcal{W}_i + \mathcal{V}_y + \mathcal{S}_z + \mathcal{P}_\ell + \mathcal{I}_\mathcal{C} + \mathcal{M}_u + \mathcal{D}_w + \mathcal{G}_q + \mathcal{T}_\sigma + \mathcal{K}_e + \mathcal{W}_m + \mathcal{Z}_\hbar + \mathcal{S}_x + \mathcal{N}_r\right)$$

$$\mathcal{C}_b = \sqrt{\frac{2\, s_k \begin{pmatrix} \mathcal{H} + \mathcal{I}_d + \mathcal{M}_i + \mathcal{L}_y + \mathcal{R}_i + \mathcal{J}_y + \mathcal{B}_i + \mathcal{Q}_y + \mathcal{F}_i + \mathcal{N}_y + \mathcal{W}_i + \mathcal{V}_y \\ + \mathcal{S}_z + \mathcal{P}_\ell + \mathcal{I}_\mathcal{C} + \mathcal{M}_u + \mathcal{D}_w + \mathcal{G}_q + \mathcal{T}_\sigma + \mathcal{K}_e + \mathcal{W}_m + \mathcal{Z}_\hbar + \mathcal{S}_x + \mathcal{N}_r \end{pmatrix}}{f\,(1-g)}} \text{--------------------} (2)$$

I. Formulation of the Equation in Fuzzy Model

We fuzzify the total cost in the equation (1) $\boldsymbol{TC}$ =

$$\frac{\tilde{f}\, \widetilde{\mathcal{C}_b}\,(1-\tilde{g})}{2} + \frac{s_k}{\widetilde{\mathcal{C}_b}}\left(\widetilde{\mathcal{H}} + \widetilde{\mathcal{I}_d} + \widetilde{\mathcal{M}_i} + \widetilde{\mathcal{L}_y} + \widetilde{\mathcal{R}_i} + \widetilde{\mathcal{J}_y} + \widetilde{\mathcal{B}_i} + \widetilde{\mathcal{Q}_y} + \widetilde{\mathcal{F}_i} + \widetilde{\mathcal{N}_y} + \widetilde{\mathcal{W}_i} + \widetilde{\mathcal{V}_y} + \widetilde{\mathcal{S}_z} + \widetilde{\mathcal{P}_\ell} + \widetilde{\mathcal{I}_\mathcal{C}} + \widetilde{\mathcal{M}_u} + \widetilde{\mathcal{D}_w} + \widetilde{\mathcal{G}_q} + \widetilde{\mathcal{T}_\sigma} + \widetilde{\mathcal{K}_e} + \widetilde{\mathcal{W}_m} + \widetilde{\mathcal{Z}_\hbar} + \widetilde{\mathcal{S}_x} + \widetilde{\mathcal{N}_r}\right)$$

Then, $\widetilde{\mathcal{H}} = (\mathcal{H}_1, \mathcal{H}_2, \mathcal{H}_3, \mathcal{H}_4, \mathcal{H}_5)$ $\quad \widetilde{\mathcal{I}_d} = (\mathcal{I}_{d_1}, \mathcal{I}_{d_2}, \mathcal{I}_{d_3}, \mathcal{I}_{d_4}, \mathcal{I}_{d_5})$

$\mathcal{M}_i = (\mathcal{M}_{i_1}, \mathcal{M}_{i_2}, \mathcal{M}_{i_3}, \mathcal{M}_{i_4}, \mathcal{M}_{i_5})$ $\quad \widetilde{\mathcal{L}_y} = (\mathcal{L}_{y_1}, \mathcal{L}_{y_2}, \mathcal{L}_{y_3}, \mathcal{L}_{y_4}, \mathcal{L}_{y_5})$

$\widetilde{\mathcal{R}_i} = (\mathcal{R}_1, \mathcal{R}_2, \mathcal{R}_3, \mathcal{R}_4, \mathcal{R}_5)$ $\quad \widetilde{\mathcal{J}_y} = (\mathcal{J}_{y_1}, \mathcal{J}_{y_2}, \mathcal{J}_{y_3}, \mathcal{J}_{y_4}, \mathcal{J}_{y_5})$

$\widetilde{\mathcal{B}_i} = (\mathcal{B}_{i_1}, \mathcal{B}_{i_2}, \mathcal{B}_{i_3}, \mathcal{B}_{i_4}, \mathcal{B}_{i_5})$ $\quad \widetilde{\mathcal{Q}_y} = (\mathcal{Q}_{y_1}, \mathcal{Q}_{y_2}, \mathcal{Q}_{y_3}, \mathcal{Q}_{y_4}, \mathcal{Q}_{y_5})$

$\widetilde{\mathcal{F}_i} = (\mathcal{F}_{i_1}, \mathcal{F}_{i_2}, \mathcal{F}_{i_3}, \mathcal{F}_{i_4}, \mathcal{F}_{i_5})$ $\quad \widetilde{\mathcal{N}_y} = (\mathcal{N}_{y_1}, \mathcal{N}_{y_2}, \mathcal{N}_{y_3}, \mathcal{N}_{y_4}, \mathcal{N}_{y_5})$

$$\widetilde{\mathcal{W}_{\iota}} = \left(\mathcal{W}_{i_1}, \mathcal{W}_{i_2}, \mathcal{W}_{i_3}, \mathcal{W}_{i_4}, \mathcal{W}_{i_5}\right) \qquad \widetilde{\mathcal{V}_{y}} = \left(\mathcal{V}_{y_1}, \mathcal{V}_{y_2}, \mathcal{V}_{y_3}, \mathcal{V}_{y_4}, \mathcal{V}_{y_5}\right)$$
$$\widetilde{\mathcal{S}_{z}} = \left(\mathcal{S}_{z_1}, \mathcal{S}_{z_2}, \mathcal{S}_{z_3}, \mathcal{S}_{z_4}, \mathcal{S}_{z_5}\right) \qquad \widetilde{\mathcal{P}_{\ell}} = \left(\mathcal{P}_{\ell_1}, \mathcal{P}_{\ell_2}, \mathcal{P}_{\ell_3}, \mathcal{P}_{\ell_4}, \mathcal{P}_{\ell_5}\right)$$
$$\widetilde{\mathcal{J}_{\mathcal{C}}} = \left(\mathcal{J}_{\mathcal{C}_1}, \mathcal{J}_{\mathcal{C}_2}, \mathcal{J}_{\mathcal{C}_3}, \mathcal{J}_{\mathcal{C}_4}, \mathcal{J}_{\mathcal{C}_5}\right) \qquad \widetilde{\mathcal{M}_{u}} = \left(\mathcal{M}_{u_1}, \mathcal{M}_{u_2}, \mathcal{M}_{u_3}, \mathcal{M}_{u_4}, \mathcal{M}_{u_5}\right)$$
$$\widetilde{\mathcal{D}_{w}} = \left(\mathcal{D}_{w_1}, \mathcal{D}_{w_2}, \mathcal{D}_{w_3}, \mathcal{D}_{w_4}, \mathcal{D}_{w_5}\right) \qquad \widetilde{\mathcal{G}_{q}} = \left(\mathcal{G}_{q_1}, \mathcal{G}_{q_2}, \mathcal{G}_{q_3}, \mathcal{G}_{q_4}, \mathcal{G}_{q_5}\right)$$
$$\widetilde{\mathcal{T}_{\sigma}} = \left(\mathcal{T}_{\sigma_1}, \mathcal{T}_{\sigma_2}, \mathcal{T}_{\sigma_3}, \mathcal{T}_{\sigma_4}, \mathcal{T}_{\sigma_5}\right) \qquad \widetilde{\mathcal{K}_{e}} = \left(\mathcal{K}_{e_1}, \mathcal{K}_{e_2}, \mathcal{K}_{e_3}, \mathcal{K}_{e_4}, \mathcal{K}_{e_5}\right)$$
$$\widetilde{\mathcal{W}_{m}} = \left(\mathcal{W}_{m_1}, \mathcal{W}_{m_2}, \mathcal{W}_{m_3}, \mathcal{W}_{m_4}, \mathcal{W}_{m_5}\right) \qquad \widetilde{\mathcal{Z}_{h}} = \left(\mathcal{Z}_{h_1}, \mathcal{Z}_{h_2}, \mathcal{Z}_{h_3}, \mathcal{Z}_{h_4}, \mathcal{Z}_{h_5}\right)$$
$$\widetilde{\mathcal{S}_{x}} = \left(\mathcal{S}_{x_1}, \mathcal{S}_{x_2}, \mathcal{S}_{x_3}, \mathcal{S}_{x_4}, \mathcal{S}_{x_5}\right) \qquad \widetilde{\mathcal{N}_{r}} = \left(\mathcal{N}_{r_1}, \mathcal{N}_{r_2}, \mathcal{N}_{r_3}, \mathcal{N}_{r_4}, \mathcal{N}_{r_5}\right)$$
$$\tilde{f} = (f_1, f_2, f_3, f_4, f_5) \qquad \tilde{g} = (g_1, g_2, g_3, g_4, g_5)$$

Are non- negative pentagonal fuzzy numbers

By graded mean integration formula, $\mathcal{P}\left(\widetilde{\mathcal{B}}\right) = \frac{1}{12}\left(b_1 + 3b_2 + 4b_3 + 3b_4 + b_5\right)$

we write the fuzzified total cost as,

$$\left(\widetilde{TC}\right) = \left\{\begin{matrix}
\frac{f_1 \mathcal{C}_{b_1}(1-g_1)}{2} + \frac{s_k}{\mathcal{C}_{b_5}}\begin{pmatrix} \mathcal{H}_1 + \mathcal{I}_{d_1} + \mathcal{M}_{i_1} + \mathcal{L}_{y_1} + \mathcal{R}_{i_1} + \mathcal{J}_{y_1} + \mathcal{B}_{i_1} + \mathcal{Q}_{y_1} + \mathcal{F}_{i_1} + \mathcal{N}_{y_1} + \mathcal{W}_{i_1} + \mathcal{V}_{y_1} \\ + \mathcal{S}_{z_1} + \mathcal{P}_{\ell_1} + \mathcal{J}_{\mathcal{C}_1} + \mathcal{M}_{u_1} + \mathcal{D}_{w_1} + \mathcal{G}_{q_1} + \mathcal{T}_{\sigma_1} + \mathcal{K}_{e_1} + \mathcal{W}_{m_1} + \mathcal{Z}_{h_1} + \mathcal{S}_{x_1} + \mathcal{N}_{r_1} \end{pmatrix}, \\
\frac{f_2 \mathcal{C}_{b_2}(1-g_2)}{2} + \frac{s_k}{\mathcal{C}_{b_4}}\begin{pmatrix} \mathcal{H}_2 + \mathcal{I}_{d_2} + \mathcal{M}_{i_2} + \mathcal{L}_{y_2} + \mathcal{R}_{i_2} + \mathcal{J}_{y_2} + \mathcal{B}_{i_2} + \mathcal{Q}_{y_2} + \mathcal{F}_{i_2} + \mathcal{N}_{y_2} + \mathcal{W}_{i_2} + \mathcal{V}_{y_2} \\ + \mathcal{S}_{z_2} + \mathcal{P}_{\ell_2} + \mathcal{J}_{\mathcal{C}_2} + \mathcal{M}_{u_2} + \mathcal{D}_{w_2} + \mathcal{G}_{q_2} + \mathcal{T}_{\sigma_2} + \mathcal{K}_{e_2} + \mathcal{W}_{m_2} + \mathcal{Z}_{h_2} + \mathcal{S}_{x_2} + \mathcal{N}_{r_2} \end{pmatrix}, \\
\frac{f_3 \mathcal{C}_{b_3}(1-g_3)}{2} + \frac{s_k}{\mathcal{C}_{b_3}}\begin{pmatrix} \mathcal{H}_3 + \mathcal{I}_{d_3} + \mathcal{M}_{i_3} + \mathcal{L}_{y_3} + \mathcal{R}_{i_3} + \mathcal{J}_{y_3} + \mathcal{B}_{i_3} + \mathcal{Q}_{y_3} + \mathcal{F}_{i_3} + \mathcal{N}_{y_3} + \mathcal{W}_{i_3} + \mathcal{V}_{y_3} \\ + \mathcal{S}_{z_3} + \mathcal{P}_{\ell_3} + \mathcal{J}_{\mathcal{C}_3} + \mathcal{M}_{u_3} + \mathcal{D}_{w_3} + \mathcal{G}_{q_3} + \mathcal{T}_{\sigma_3} + \mathcal{K}_{e_3} + \mathcal{W}_{m_3} + \mathcal{Z}_{h_3} + \mathcal{S}_{x_3} + \mathcal{N}_{r_3} \end{pmatrix}, \\
\frac{f_4 \mathcal{C}_{b_4}(1-g_4)}{2} + \frac{s_k}{\mathcal{C}_{b_2}}\begin{pmatrix} \mathcal{H}_4 + \mathcal{I}_{d_4} + \mathcal{M}_{i_4} + \mathcal{L}_{y_4} + \mathcal{R}_{i_4} + \mathcal{J}_{y_4} + \mathcal{B}_{i_4} + \mathcal{Q}_{y_4} + \mathcal{F}_{i_4} + \mathcal{N}_{y_4} + \mathcal{W}_{i_4} + \mathcal{V}_{y_4} \\ + \mathcal{S}_{z_4} + \mathcal{P}_{\ell_4} + \mathcal{J}_{\mathcal{C}_4} + \mathcal{M}_{u_4} + \mathcal{D}_{w_4} + \mathcal{G}_{q_4} + \mathcal{T}_{\sigma_4} + \mathcal{K}_{e_4} + \mathcal{W}_{m_4} + \mathcal{Z}_{h_4} + \mathcal{S}_{x_4} + \mathcal{N}_{r_4} \end{pmatrix}, \\
\frac{f_5 \mathcal{C}_{b_5}(1-g_5)}{2} + \frac{s_k}{\mathcal{C}_{b_1}}\begin{pmatrix} \mathcal{H}_5 + \mathcal{I}_{d_5} + \mathcal{M}_{i_5} + \mathcal{L}_{y_5} + \mathcal{R}_{i_5} + \mathcal{J}_{y_5} + \mathcal{B}_{i_5} + \mathcal{Q}_{y_5} + \mathcal{F}_{i_5} + \mathcal{N}_{y_5} + \mathcal{W}_{i_5} + \mathcal{V}_{y_5} \\ + \mathcal{S}_{z_5} + \mathcal{P}_{\ell_5} + \mathcal{J}_{\mathcal{C}_5} + \mathcal{M}_{u_5} + \mathcal{D}_{w_5} + \mathcal{G}_{q_5} + \mathcal{T}_{\sigma_5} + \mathcal{K}_{e_5} + \mathcal{W}_{m_5} + \mathcal{Z}_{h_5} + \mathcal{S}_{x_5} + \mathcal{N}_{r_5} \end{pmatrix}
\end{matrix}\right\}$$

Now let us defuzzify the total cost, then we have, $p[\widetilde{TC(\mathcal{C}_b)}] =$

$$\frac{1}{12}\left\{\begin{matrix}
\frac{f_1 \mathcal{C}_{b_1}(1-g_1)}{2} + \frac{s_k}{\mathcal{C}_{b_5}}\begin{pmatrix} \mathcal{H}_1 + \mathcal{I}_{d_1} + \mathcal{M}_{i_1} + \mathcal{L}_{y_1} + \mathcal{R}_{i_1} + \mathcal{J}_{y_1} + \mathcal{B}_{i_1} + \mathcal{Q}_{y_1} + \mathcal{F}_{i_1} + \mathcal{N}_{y_1} + \mathcal{W}_{i_1} + \mathcal{V}_{y_1} \\ + \mathcal{S}_{z_1} + \mathcal{P}_{\ell_1} + \mathcal{J}_{\mathcal{C}_1} + \mathcal{M}_{u_1} + \mathcal{D}_{w_1} + \mathcal{G}_{q_1} + \mathcal{T}_{\sigma_1} + \mathcal{K}_{e_1} + \mathcal{W}_{m_1} + \mathcal{Z}_{h_1} + \mathcal{S}_{x_1} + \mathcal{N}_{r_1} \end{pmatrix} + \\
3\left[\frac{f_2 \mathcal{C}_{b_2}(1-g_2)}{2} + \frac{s_k}{\mathcal{C}_{b_4}}\begin{pmatrix} \mathcal{H}_2 + \mathcal{I}_{d_2} + \mathcal{M}_{i_2} + \mathcal{L}_{y_2} + \mathcal{R}_{i_2} + \mathcal{J}_{y_2} + \mathcal{B}_{i_2} + \mathcal{Q}_{y_2} + \mathcal{F}_{i_2} + \mathcal{N}_{y_2} + \mathcal{W}_{i_2} + \mathcal{V}_{y_2} \\ + \mathcal{S}_{z_2} + \mathcal{P}_{\ell_2} + \mathcal{J}_{\mathcal{C}_2} + \mathcal{M}_{u_2} + \mathcal{D}_{w_2} + \mathcal{G}_{q_2} + \mathcal{T}_{\sigma_2} + \mathcal{K}_{e_2} + \mathcal{W}_{m_2} + \mathcal{Z}_{h_2} + \mathcal{S}_{x_2} + \mathcal{N}_{r_2} \end{pmatrix}\right] + \\
4\left[\frac{f_3 \mathcal{C}_{b_3}(1-g_3)}{2} + \frac{s_k}{\mathcal{C}_{b_3}}\begin{pmatrix} \mathcal{H}_3 + \mathcal{I}_{d_3} + \mathcal{M}_{i_3} + \mathcal{L}_{y_3} + \mathcal{R}_{i_3} + \mathcal{J}_{y_3} + \mathcal{B}_{i_3} + \mathcal{Q}_{y_3} + \mathcal{F}_{i_3} + \mathcal{N}_{y_3} + \mathcal{W}_{i_3} + \mathcal{V}_{y_3} \\ + \mathcal{S}_{z_3} + \mathcal{P}_{\ell_3} + \mathcal{J}_{\mathcal{C}_3} + \mathcal{M}_{u_3} + \mathcal{D}_{w_3} + \mathcal{G}_{q_3} + \mathcal{T}_{\sigma_3} + \mathcal{K}_{e_3} + \mathcal{W}_{m_3} + \mathcal{Z}_{h_3} + \mathcal{S}_{x_3} + \mathcal{N}_{r_3} \end{pmatrix}\right] + \\
3\left[\frac{f_4 \mathcal{C}_{b_4}(1-g_4)}{2} + \frac{s_k}{\mathcal{C}_{b_2}}\begin{pmatrix} \mathcal{H}_4 + \mathcal{I}_{d_4} + \mathcal{M}_{i_4} + \mathcal{L}_{y_4} + \mathcal{R}_{i_4} + \mathcal{J}_{y_4} + \mathcal{B}_{i_4} + \mathcal{Q}_{y_4} + \mathcal{F}_{i_4} + \mathcal{N}_{y_4} + \mathcal{W}_{i_4} + \mathcal{V}_{y_4} \\ + \mathcal{S}_{z_4} + \mathcal{P}_{\ell_4} + \mathcal{J}_{\mathcal{C}_4} + \mathcal{M}_{u_4} + \mathcal{D}_{w_4} + \mathcal{G}_{q_4} + \mathcal{T}_{\sigma_4} + \mathcal{K}_{e_4} + \mathcal{W}_{m_4} + \mathcal{Z}_{h_4} + \mathcal{S}_{x_4} + \mathcal{N}_{r_4} \end{pmatrix}\right] + \\
\frac{f_5 \mathcal{C}_{b_5}(1-g_5)}{2} + \frac{s_k}{\mathcal{C}_{b_1}}\begin{pmatrix} \mathcal{H}_5 + \mathcal{I}_{d_5} + \mathcal{M}_{i_5} + \mathcal{L}_{y_5} + \mathcal{R}_{i_5} + \mathcal{J}_{y_5} + \mathcal{B}_{i_5} + \mathcal{Q}_{y_5} + \mathcal{F}_{i_5} + \mathcal{N}_{y_5} + \mathcal{W}_{i_5} + \mathcal{V}_{y_5} \\ + \mathcal{S}_{z_5} + \mathcal{P}_{\ell_5} + \mathcal{J}_{\mathcal{C}_5} + \mathcal{M}_{u_5} + \mathcal{D}_{w_5} + \mathcal{G}_{q_5} + \mathcal{T}_{\sigma_5} + \mathcal{K}_{e_5} + \mathcal{W}_{m_5} + \mathcal{Z}_{h_5} + \mathcal{S}_{x_5} + \mathcal{N}_{r_5} \end{pmatrix}
\end{matrix}\right\}$$

III. SOLUTION METHODOLOGY USING LAGRANGIAN METHOD:

1) Step1

$$p[\widetilde{TC(\mathcal{C}_b)}] = \frac{1}{12}\left\{\begin{matrix}
\frac{f_1 \mathcal{C}_{b_1}(1-g_1)}{2} + \frac{s_k}{\mathcal{C}_{b_5}}\begin{pmatrix} \mathcal{H}_1 + \mathcal{I}_{d_1} + \mathcal{M}_{i_1} + \mathcal{L}_{y_1} + \mathcal{R}_{i_1} + \mathcal{J}_{y_1} + \mathcal{B}_{i_1} + \mathcal{Q}_{y_1} + \mathcal{F}_{i_1} + \mathcal{N}_{y_1} + \mathcal{W}_{i_1} + \mathcal{V}_{y_1} \\ + \mathcal{S}_{z_1} + \mathcal{P}_{\ell_1} + \mathcal{J}_{\mathcal{C}_1} + \mathcal{M}_{u_1} + \mathcal{D}_{w_1} + \mathcal{G}_{q_1} + \mathcal{T}_{\sigma_1} + \mathcal{K}_{e_1} + \mathcal{W}_{m_1} + \mathcal{Z}_{h_1} + \mathcal{S}_{x_1} + \mathcal{N}_{r_1} \end{pmatrix} + \\
3\left[\frac{f_2 \mathcal{C}_{b_2}(1-g_2)}{2} + \frac{s_k}{\mathcal{C}_{b_4}}\begin{pmatrix} \mathcal{H}_2 + \mathcal{I}_{d_2} + \mathcal{M}_{i_2} + \mathcal{L}_{y_2} + \mathcal{R}_{i_2} + \mathcal{J}_{y_2} + \mathcal{B}_{i_2} + \mathcal{Q}_{y_2} + \mathcal{F}_{i_2} + \mathcal{N}_{y_2} + \mathcal{W}_{i_2} + \mathcal{V}_{y_2} \\ + \mathcal{S}_{z_2} + \mathcal{P}_{\ell_2} + \mathcal{J}_{\mathcal{C}_2} + \mathcal{M}_{u_2} + \mathcal{D}_{w_2} + \mathcal{G}_{q_2} + \mathcal{T}_{\sigma_2} + \mathcal{K}_{e_2} + \mathcal{W}_{m_2} + \mathcal{Z}_{h_2} + \mathcal{S}_{x_2} + \mathcal{N}_{r_2} \end{pmatrix}\right] + \\
4\left[\frac{f_3 \mathcal{C}_{b_3}(1-g_3)}{2} + \frac{s_k}{\mathcal{C}_{b_3}}\begin{pmatrix} \mathcal{H}_3 + \mathcal{I}_{d_3} + \mathcal{M}_{i_3} + \mathcal{L}_{y_3} + \mathcal{R}_{i_3} + \mathcal{J}_{y_3} + \mathcal{B}_{i_3} + \mathcal{Q}_{y_3} + \mathcal{F}_{i_3} + \mathcal{N}_{y_3} + \mathcal{W}_{i_3} + \mathcal{V}_{y_3} \\ + \mathcal{S}_{z_3} + \mathcal{P}_{\ell_3} + \mathcal{J}_{\mathcal{C}_3} + \mathcal{M}_{u_3} + \mathcal{D}_{w_3} + \mathcal{G}_{q_3} + \mathcal{T}_{\sigma_3} + \mathcal{K}_{e_3} + \mathcal{W}_{m_3} + \mathcal{Z}_{h_3} + \mathcal{S}_{x_3} + \mathcal{N}_{r_3} \end{pmatrix}\right] + \\
3\left[\frac{f_4 \mathcal{C}_{b_4}(1-g_4)}{2} + \frac{s_k}{\mathcal{C}_{b_2}}\begin{pmatrix} \mathcal{H}_4 + \mathcal{I}_{d_4} + \mathcal{M}_{i_4} + \mathcal{L}_{y_4} + \mathcal{R}_{i_4} + \mathcal{J}_{y_4} + \mathcal{B}_{i_4} + \mathcal{Q}_{y_4} + \mathcal{F}_{i_4} + \mathcal{N}_{y_4} + \mathcal{W}_{i_4} + \mathcal{V}_{y_4} \\ + \mathcal{S}_{z_4} + \mathcal{P}_{\ell_4} + \mathcal{J}_{\mathcal{C}_4} + \mathcal{M}_{u_4} + \mathcal{D}_{w_4} + \mathcal{G}_{q_4} + \mathcal{T}_{\sigma_4} + \mathcal{K}_{e_4} + \mathcal{W}_{m_4} + \mathcal{Z}_{h_4} + \mathcal{S}_{x_4} + \mathcal{N}_{r_4} \end{pmatrix}\right] + \\
\frac{f_5 \mathcal{C}_{b_5}(1-g_5)}{2} + \frac{s_k}{\mathcal{C}_{b_1}}\begin{pmatrix} \mathcal{H}_5 + \mathcal{I}_{d_5} + \mathcal{M}_{i_5} + \mathcal{L}_{y_5} + \mathcal{R}_{i_5} + \mathcal{J}_{y_5} + \mathcal{B}_{i_5} + \mathcal{Q}_{y_5} + \mathcal{F}_{i_5} + \mathcal{N}_{y_5} + \mathcal{W}_{i_5} + \mathcal{V}_{y_5} \\ + \mathcal{S}_{z_5} + \mathcal{P}_{\ell_5} + \mathcal{J}_{\mathcal{C}_5} + \mathcal{M}_{u_5} + \mathcal{D}_{w_5} + \mathcal{G}_{q_5} + \mathcal{T}_{\sigma_5} + \mathcal{K}_{e_5} + \mathcal{W}_{m_5} + \mathcal{Z}_{h_5} + \mathcal{S}_{x_5} + \mathcal{N}_{r_5} \end{pmatrix}
\end{matrix}\right\}$$

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With 0< $\mathcal{C}_{b_1} \le \mathcal{C}_{b_2} \le \mathcal{C}_{b_3} \le \mathcal{C}_{b_4} \le \mathcal{C}_{b_5}, \mathcal{C}_{b_2} - \mathcal{C}_{b_1} \ge 0$, $\mathcal{C}_{b_3} - \mathcal{C}_{b_2} \ge 0, \mathcal{C}_{b_4} - \mathcal{C}_{b_3} \ge 0, \mathcal{C}_{b_5} - \mathcal{C}_{b_4} \ge 0$,and $\mathcal{C}_{b_1} > 0$ ------------------------ (3)

Now differentiating (3) partially with respect to $\mathcal{C}_{b_1}, \mathcal{C}_{b_2}, \mathcal{C}_{b_3}, \mathcal{C}_{b_4}$ *and* $\mathcal{C}_{b_5}$ and equating to zero, we get

$$\mathcal{C}_{b_1} = \sqrt{\frac{2\ s_k \begin{pmatrix} \mathcal{H}_5+\mathcal{I}_{d_5}+\mathcal{M}_{i_5}+\mathcal{L}_{y_5}+\mathcal{R}_{i_5}+\mathcal{J}_{y_5}+\mathcal{B}_{i_5}+\mathcal{Q}_{y_5}+\mathcal{F}_{i_5}+\mathcal{N}_{y_5}+\mathcal{W}_{i_5}+\mathcal{V}_{y_5} \\ +\mathcal{S}_{z_5}+\mathcal{P}_{\ell_5}+\mathcal{I}_{\mathcal{C}_5}+\mathcal{M}_{u_5}+\mathcal{D}_{w_5}+\mathcal{G}_{q_5}+\mathcal{T}_{o_5}+\mathcal{K}_{e_5}+\mathcal{W}_{m_5}+\mathcal{Z}_{h_5}+\mathcal{S}_{x_5}+\mathcal{N}_{r_5} \end{pmatrix}}{f_1\ (1-g_1)}}$$

$$\mathcal{C}_{b_2} = \sqrt{\frac{2\ \left[3\ s_k \begin{pmatrix} \mathcal{H}_4+\mathcal{I}_{d_4}+\mathcal{M}_{i_4}+\mathcal{L}_{y_4}+\mathcal{R}_{i_4}+\mathcal{J}_{y_4}+\mathcal{B}_{i_4}+\mathcal{Q}_{y_4}+\mathcal{F}_{i_4}+\mathcal{N}_{y_4}+\mathcal{W}_{i_4}+\mathcal{V}_{y_4} \\ +\mathcal{S}_{z_4}+\mathcal{P}_{\ell_4}+\mathcal{I}_{\mathcal{C}_4}+\mathcal{M}_{u_4}+\mathcal{D}_{w_4}+\mathcal{G}_{q_4}+\mathcal{T}_{o_4}+\mathcal{K}_{e_4}+\mathcal{W}_{m_4}+\mathcal{Z}_{h_4}+\mathcal{S}_{x_4}+\mathcal{N}_{r_4} \end{pmatrix}\right]}{3\ f_2\ (1-g_2)}}$$

$$\mathcal{C}_{b_3} = \sqrt{\frac{2\ \left[4\ s_k \begin{pmatrix} \mathcal{H}_3+\mathcal{I}_{d_3}+\mathcal{M}_{i_3}+\mathcal{L}_{y_3}+\mathcal{R}_{i_3}+\mathcal{J}_{y_3}+\mathcal{B}_{i_3}+\mathcal{Q}_{y_3}+\mathcal{F}_{i_3}+\mathcal{N}_{y_3}+\mathcal{W}_{i_3}+\mathcal{V}_{y_3} \\ +\mathcal{S}_{z_3}+\mathcal{P}_{\ell_3}+\mathcal{I}_{\mathcal{C}_3}+\mathcal{M}_{u_3}+\mathcal{D}_{w_3}+\mathcal{G}_{q_3}+\mathcal{T}_{o_3}+\mathcal{K}_{e_3}+\mathcal{W}_{m_3}+\mathcal{Z}_{h_3}+\mathcal{S}_{x_3}+\mathcal{N}_{r_3} \end{pmatrix}\right]}{4\ f_3\ (1-g_3)}}$$

$$\mathcal{C}_{b_4} = \sqrt{\frac{2\ \left[3\ s_k \begin{pmatrix} \mathcal{H}_2+\mathcal{I}_{d_2}+\mathcal{M}_{i_2}+\mathcal{L}_{y_2}+\mathcal{R}_{i_2}+\mathcal{J}_{y_2}+\mathcal{B}_{i_2}+\mathcal{Q}_{y_2}+\mathcal{F}_{i_2}+\mathcal{N}_{y_2}+\mathcal{W}_{i_2}+\mathcal{V}_{y_2} \\ +\mathcal{S}_{z_2}+\mathcal{P}_{\ell_2}+\mathcal{I}_{\mathcal{C}_2}+\mathcal{M}_{u_2}+\mathcal{D}_{w_2}+\mathcal{G}_{q_2}+\mathcal{T}_{o_2}+\mathcal{K}_{e_2}+\mathcal{W}_{m_2}+\mathcal{Z}_{h_2}+\mathcal{S}_{x_2}+\mathcal{N}_{r_2} \end{pmatrix}\right]}{3\ f_4\ (1-g_4)}}$$

$$\mathcal{C}_{b_5} = \sqrt{\frac{2\ s_k \begin{pmatrix} \mathcal{H}_1+\mathcal{I}_{d_1}+\mathcal{M}_{i_1}+\mathcal{L}_{y_1}+\mathcal{R}_{i_1}+\mathcal{J}_{y_1}+\mathcal{B}_{i_1}+\mathcal{Q}_{y_1}+\mathcal{F}_{i_1}+\mathcal{N}_{y_1}+\mathcal{W}_{i_1}+\mathcal{V}_{y_1} \\ +\mathcal{S}_{z_1}+\mathcal{P}_{\ell_1}+\mathcal{I}_{\mathcal{C}_1}+\mathcal{M}_{u_1}+\mathcal{D}_{w_1}+\mathcal{G}_{q_1}+\mathcal{T}_{o_1}+\mathcal{K}_{e_1}+\mathcal{W}_{m_1}+\mathcal{Z}_{h_1}+\mathcal{S}_{x_1}+\mathcal{N}_{r_1} \end{pmatrix}}{f_5\ (1-g_5)}}$$

As the above results shows that $\mathcal{C}_{b_1} > \mathcal{C}_{b_2}$, $\mathcal{C}_{b_2} > \mathcal{C}_{b_3}$, $\mathcal{C}_{b_3} > \mathcal{C}_{b_4}$ and $\mathcal{C}_{b_4} > \mathcal{C}_{b_5}$, it does not satisfy the constraints 0< $\mathcal{C}_{b_1} \le \mathcal{C}_{b_2} \le \mathcal{C}_{b_3} \le \mathcal{C}_{b_4} \le \mathcal{C}_{b_5}$. Hence it is not a local optimum.

2) Step 2

Convert the inequality constraint $\mathcal{C}_{b_2} - \mathcal{C}_{b_1} \ge 0$ into equality constraint $\mathcal{C}_{b_2} - \mathcal{C}_{b_1} = 0$.

Then by the Lagrangean method, we have the Lagrangean function as $L\left(\mathcal{C}_{b_1}, \mathcal{C}_{b_2}, \mathcal{C}_{b_3}, \mathcal{C}_{b_4}, \mathcal{C}_{b_5}, \lambda\right) = \wp[\widetilde{\mathrm{TC}(\mathcal{C}_b)}] - \lambda\left(\mathcal{C}_{b_2} - \mathcal{C}_{b_1}\right)$

Hence, $\wp[\widetilde{\mathrm{TC}(\mathcal{C}_b)}] - \lambda\left(\mathcal{C}_{b_2} - \mathcal{C}_{b_1}\right) =$

$$\frac{1}{12}\left\{\begin{array}{l} \frac{f_1\, \mathcal{C}_{b_1} (1-g_1)}{2} + \frac{s_k}{\mathcal{C}_{b_5}} \begin{pmatrix} \mathcal{H}_1+\mathcal{I}_{d_1}+\mathcal{M}_{i_1}+\mathcal{L}_{y_1}+\mathcal{R}_{i_1}+\mathcal{J}_{y_1}+\mathcal{B}_{i_1}+\mathcal{Q}_{y_1}+\mathcal{F}_{i_1}+\mathcal{N}_{y_1}+\mathcal{W}_{i_1}+\mathcal{V}_{y_1} \\ +\mathcal{S}_{z_1}+\mathcal{P}_{\ell_1}+\mathcal{I}_{\mathcal{C}_1}+\mathcal{M}_{u_1}+\mathcal{D}_{w_1}+\mathcal{G}_{q_1}+\mathcal{T}_{o_1}+\mathcal{K}_{e_1}+\mathcal{W}_{m_1}+\mathcal{Z}_{h_1}+\mathcal{S}_{x_1}+\mathcal{N}_{r_1} \end{pmatrix} + \\ 3\left[\frac{f_2\, \mathcal{C}_{b_2} (1-g_2)}{2} + \frac{s_k}{\mathcal{C}_{b_4}} \begin{pmatrix} \mathcal{H}_2+\mathcal{I}_{d_2}+\mathcal{M}_{i_2}+\mathcal{L}_{y_2}+\mathcal{R}_{i_2}+\mathcal{J}_{y_2}+\mathcal{B}_{i_2}+\mathcal{Q}_{y_2}+\mathcal{F}_{i_2}+\mathcal{N}_{y_2}+\mathcal{W}_{i_2}+\mathcal{V}_{y_2} \\ +\mathcal{S}_{z_2}+\mathcal{P}_{\ell_2}+\mathcal{I}_{\mathcal{C}_2}+\mathcal{M}_{u_2}+\mathcal{D}_{w_2}+\mathcal{G}_{q_2}+\mathcal{T}_{o_2}+\mathcal{K}_{e_2}+\mathcal{W}_{m_2}+\mathcal{Z}_{h_2}+\mathcal{S}_{x_2}+\mathcal{N}_{r_2} \end{pmatrix}\right] + \\ 4\left[\frac{f_3\, \mathcal{C}_{b_3} (1-g_3)}{2} + \frac{s_k}{\mathcal{C}_{b_3}} \begin{pmatrix} \mathcal{H}_3+\mathcal{I}_{d_3}+\mathcal{M}_{i_3}+\mathcal{L}_{y_3}+\mathcal{R}_{i_3}+\mathcal{J}_{y_3}+\mathcal{B}_{i_3}+\mathcal{Q}_{y_3}+\mathcal{F}_{i_3}+\mathcal{N}_{y_3}+\mathcal{W}_{i_3}+\mathcal{V}_{y_3} \\ +\mathcal{S}_{z_3}+\mathcal{P}_{\ell_3}+\mathcal{I}_{\mathcal{C}_3}+\mathcal{M}_{u_3}+\mathcal{D}_{w_3}+\mathcal{G}_{q_3}+\mathcal{T}_{o_3}+\mathcal{K}_{e_3}+\mathcal{W}_{m_3}+\mathcal{Z}_{h_3}+\mathcal{S}_{x_3}+\mathcal{N}_{r_3} \end{pmatrix}\right] + \\ 3\left[\frac{f_4\, \mathcal{C}_{b_4} (1-g_4)}{2} + \frac{s_k}{\mathcal{C}_{b_2}} \begin{pmatrix} \mathcal{H}_4+\mathcal{I}_{d_4}+\mathcal{M}_{i_4}+\mathcal{L}_{y_4}+\mathcal{R}_{i_4}+\mathcal{J}_{y_4}+\mathcal{B}_{i_4}+\mathcal{Q}_{y_4}+\mathcal{F}_{i_4}+\mathcal{N}_{y_4}+\mathcal{W}_{i_4}+\mathcal{V}_{y_4} \\ +\mathcal{S}_{z_4}+\mathcal{P}_{\ell_4}+\mathcal{I}_{\mathcal{C}_4}+\mathcal{M}_{u_4}+\mathcal{D}_{w_4}+\mathcal{G}_{q_4}+\mathcal{T}_{o_4}+\mathcal{K}_{e_4}+\mathcal{W}_{m_4}+\mathcal{Z}_{h_4}+\mathcal{S}_{x_4}+\mathcal{N}_{r_4} \end{pmatrix}\right] + \\ \frac{f_5\, \mathcal{C}_{b_5} (1-g_5)}{2} + \frac{s_k}{\mathcal{C}_{b_1}} \begin{pmatrix} \mathcal{H}_5+\mathcal{I}_{d_5}+\mathcal{M}_{i_5}+\mathcal{L}_{y_5}+\mathcal{R}_{i_5}+\mathcal{J}_{y_5}+\mathcal{B}_{i_5}+\mathcal{Q}_{y_5}+\mathcal{F}_{i_5}+\mathcal{N}_{y_5}+\mathcal{W}_{i_5}+\mathcal{V}_{y_5} \\ +\mathcal{S}_{z_5}+\mathcal{P}_{\ell_5}+\mathcal{I}_{\mathcal{C}_5}+\mathcal{M}_{u_5}+\mathcal{D}_{w_5}+\mathcal{G}_{q_5}+\mathcal{T}_{o_5}+\mathcal{K}_{e_5}+\mathcal{W}_{m_5}+\mathcal{Z}_{h_5}+\mathcal{S}_{x_5}+\mathcal{N}_{r_5} \end{pmatrix} \end{array}\right\}$$

$$- \lambda\left(\mathcal{C}_{b_2} - \mathcal{C}_{b_1}\right) \text{------------------------}(4)$$

Differentiating (4) partially with respect to $\mathcal{C}_{b_1}, \mathcal{C}_{b_2}, \mathcal{C}_{b_3}, \mathcal{C}_{b_4}, \mathcal{C}_{b_5}, \lambda$ and equating to zero, we get

$$\mathcal{C}_{b_1} = \mathcal{C}_{b_2} = \sqrt{\frac{2\left[\begin{array}{l} s_k \begin{pmatrix} \mathcal{H}_5+\mathcal{I}_{d_5}+\mathcal{M}_{i_5}+\mathcal{L}_{y_5}+\mathcal{R}_{i_5}+\mathcal{J}_{y_5}+\mathcal{B}_{i_5}+\mathcal{Q}_{y_5}+\mathcal{F}_{i_5}+\mathcal{N}_{y_5}+\mathcal{W}_{i_5}+\mathcal{V}_{y_5} \\ +\mathcal{S}_{z_5}+\mathcal{P}_{\ell_5}+\mathcal{I}_{\mathcal{C}_5}+\mathcal{M}_{u_5}+\mathcal{D}_{w_5}+\mathcal{G}_{q_5}+\mathcal{T}_{o_5}+\mathcal{K}_{e_5}+\mathcal{W}_{m_5}+\mathcal{Z}_{h_5}+\mathcal{S}_{x_5}+\mathcal{N}_{r_5} \end{pmatrix} \\ +3\ s_k \begin{pmatrix} \mathcal{H}_4+\mathcal{I}_{d_4}+\mathcal{M}_{i_4}+\mathcal{L}_{y_4}+\mathcal{R}_{i_4}+\mathcal{J}_{y_4}+\mathcal{B}_{i_4}+\mathcal{Q}_{y_4}+\mathcal{F}_{i_4}+\mathcal{N}_{y_4}+\mathcal{W}_{i_4}+\mathcal{V}_{y_4} \\ +\mathcal{S}_{z_4}+\mathcal{P}_{\ell_4}+\mathcal{I}_{\mathcal{C}_4}+\mathcal{M}_{u_4}+\mathcal{D}_{w_4}+\mathcal{G}_{q_4}+\mathcal{T}_{o_4}+\mathcal{K}_{e_4}+\mathcal{W}_{m_4}+\mathcal{Z}_{h_4}+\mathcal{S}_{x_4}+\mathcal{N}_{r_4} \end{pmatrix} \end{array}\right]}{f_1\ (1-g_1)+3\ f_2\ (1-g_2)}}$$

$$\mathcal{C}_{b_3} = \sqrt{\frac{2\ \left[4\ s_k \begin{pmatrix} \mathcal{H}_3+\mathcal{I}_{d_3}+\mathcal{M}_{i_3}+\mathcal{L}_{y_3}+\mathcal{R}_{i_3}+\mathcal{J}_{y_3}+\mathcal{B}_{i_3}+\mathcal{Q}_{y_3}+\mathcal{F}_{i_3}+\mathcal{N}_{y_3}+\mathcal{W}_{i_3}+\mathcal{V}_{y_3} \\ +\mathcal{S}_{z_3}+\mathcal{P}_{\ell_3}+\mathcal{I}_{\mathcal{C}_3}+\mathcal{M}_{u_3}+\mathcal{D}_{w_3}+\mathcal{G}_{q_3}+\mathcal{T}_{o_3}+\mathcal{K}_{e_3}+\mathcal{W}_{m_3}+\mathcal{Z}_{h_3}+\mathcal{S}_{x_3}+\mathcal{N}_{r_3} \end{pmatrix}\right]}{4\ f_3\ (1-g_3)}}$$

$$\mathcal{C}_{\mathscr{b}_4} = \sqrt{\frac{2\left[3\, s_{\mathscr{k}} \begin{pmatrix} \mathcal{H}_2+\mathcal{I}_{d_2}+\mathcal{M}_{i_2}+\mathcal{L}_{y_2}+\mathcal{R}_{i_2}+\mathcal{J}_{y_2}+\mathcal{B}_{i_2}+\mathcal{Q}_{y_2}+\mathcal{F}_{i_2}+\mathcal{N}_{y_2}+\mathcal{W}_{i_2}+\mathcal{V}_{y_2} \\ +\mathcal{S}_{z_2}+\mathcal{P}_{\ell_2}+\mathcal{I}_{c_2}+\mathcal{M}_{u_2}+\mathcal{D}_{w_2}+\mathcal{G}_{a_2}+\mathcal{T}_{\sigma_2}+\mathcal{K}_{e_2}+\mathcal{W}_{m_2}+\mathcal{Z}_{h_2}+\mathcal{S}_{x_2}+\mathcal{N}_{r_2} \end{pmatrix}\right]}{3\, \mathscr{f}_4\ (1-g_4)}}$$

$$\mathcal{C}_{\mathscr{b}_5} = \sqrt{\frac{2\ s_{\mathscr{k}} \begin{pmatrix} \mathcal{H}_1+\mathcal{I}_{d_1}+\mathcal{M}_{i_1}+\mathcal{L}_{y_1}+\mathcal{R}_{i_1}+\mathcal{J}_{y_1}+\mathcal{B}_{i_1}+\mathcal{Q}_{y_1}+\mathcal{F}_{i_1}+\mathcal{N}_{y_1}+\mathcal{W}_{i_1}+\mathcal{V}_{y_1} \\ +\mathcal{S}_{z_1}+\mathcal{P}_{\ell_1}+\mathcal{I}_{c_1}+\mathcal{M}_{u_1}+\mathcal{D}_{w_1}+\mathcal{G}_{a_1}+\mathcal{T}_{\sigma_1}+\mathcal{K}_{e_1}+\mathcal{W}_{m_1}+\mathcal{Z}_{h_1}+\mathcal{S}_{x_1}+\mathcal{N}_{r_1} \end{pmatrix}}{\mathscr{f}_5\ (1-g_5)}}$$

Here, $\mathcal{C}_{\mathscr{b}_3} > \mathcal{C}_{\mathscr{b}_4}$, $\mathcal{C}_{\mathscr{b}_4} > \mathcal{C}_{\mathscr{b}_5}$. It does not satisfy the constraint $0< \mathcal{C}_{\mathscr{b}_1} \le \mathcal{C}_{\mathscr{b}_2} \le \mathcal{C}_{\mathscr{b}_3} \le \mathcal{C}_{\mathscr{b}_4} \le \mathcal{C}_{\mathscr{b}_5}$, therefore it is not a local optimum.

3) Step 3

Convert the inequality constraints $\mathcal{C}_{\mathscr{b}_2} - \mathcal{C}_{\mathscr{b}_1} \ge 0$ and $\mathcal{C}_{\mathscr{b}_3} - \mathcal{C}_{\mathscr{b}_2} \ge 0$ into equality constraints $\mathcal{C}_{\mathscr{b}_2} - \mathcal{C}_{\mathscr{b}_1} = 0$ and $\mathcal{C}_{\mathscr{b}_3} - \mathcal{C}_{\mathscr{b}_2} = 0$ .Optimize $\mathscr{p}[\widetilde{\mathrm{TC}(\mathcal{C}_{\mathscr{b}})}]$ subject to $\mathcal{C}_{\mathscr{b}_2} - \mathcal{C}_{\mathscr{b}_1} = 0$ and $\mathcal{C}_{\mathscr{b}_3} - \mathcal{C}_{\mathscr{b}_2} = 0$ by the Lagrangean method. Then the Lagrangean function is $L\left(\mathcal{C}_{\mathscr{b}_1}, \mathcal{C}_{\mathscr{b}_2}, \mathcal{C}_{\mathscr{b}_3}, \mathcal{C}_{\mathscr{b}_4}, \mathcal{C}_{\mathscr{b}_5}, \lambda_1, \lambda_2\right) = \mathscr{p}[\widetilde{\mathrm{TC}(\mathcal{C}_{\mathscr{b}})}] - \lambda_1\left(\mathcal{C}_{\mathscr{b}_2} - \mathcal{C}_{\mathscr{b}_1}\right) - \lambda_2\left(\mathcal{C}_{\mathscr{b}_3} - \mathcal{C}_{\mathscr{b}_2}\right)$.

Hence $\mathscr{p}[\widetilde{\mathrm{TC}(\mathcal{C}_{\mathscr{b}})}] - \lambda_1\left(\mathcal{C}_{\mathscr{b}_2} - \mathcal{C}_{\mathscr{b}_1}\right) - \lambda_2\left(\mathcal{C}_{\mathscr{b}_3} - \mathcal{C}_{\mathscr{b}_2}\right) =$

$$\frac{1}{12}\left\{\begin{array}{l} \frac{\mathscr{f}_1\, \mathcal{C}_{\mathscr{b}_1}(1-g_1)}{2} + \frac{s_{\mathscr{k}}}{\mathcal{C}_{\mathscr{b}_5}}\begin{pmatrix} \mathcal{H}_1+\mathcal{I}_{d_1}+\mathcal{M}_{i_1}+\mathcal{L}_{y_1}+\mathcal{R}_{i_1}+\mathcal{J}_{y_1}+\mathcal{B}_{i_1}+\mathcal{Q}_{y_1}+\mathcal{F}_{i_1}+\mathcal{N}_{y_1}+\mathcal{W}_{i_1}+\mathcal{V}_{y_1} \\ +\mathcal{S}_{z_1}+\mathcal{P}_{\ell_1}+\mathcal{I}_{c_1}+\mathcal{M}_{u_1}+\mathcal{D}_{w_1}+\mathcal{G}_{a_1}+\mathcal{T}_{\sigma_1}+\mathcal{K}_{e_1}+\mathcal{W}_{m_1}+\mathcal{Z}_{h_1}+\mathcal{S}_{x_1}+\mathcal{N}_{r_1} \end{pmatrix} + \\ 3\left[\frac{\mathscr{f}_2\, \mathcal{C}_{\mathscr{b}_2}(1-g_2)}{2} + \frac{s_{\mathscr{k}}}{\mathcal{C}_{\mathscr{b}_4}}\begin{pmatrix} \mathcal{H}_2+\mathcal{I}_{d_2}+\mathcal{M}_{i_2}+\mathcal{L}_{y_2}+\mathcal{R}_{i_2}+\mathcal{J}_{y_2}+\mathcal{B}_{i_2}+\mathcal{Q}_{y_2}+\mathcal{F}_{i_2}+\mathcal{N}_{y_2}+\mathcal{W}_{i_2}+\mathcal{V}_{y_2} \\ +\mathcal{S}_{z_2}+\mathcal{P}_{\ell_2}+\mathcal{I}_{c_2}+\mathcal{M}_{u_2}+\mathcal{D}_{w_2}+\mathcal{G}_{a_2}+\mathcal{T}_{\sigma_2}+\mathcal{K}_{e_2}+\mathcal{W}_{m_2}+\mathcal{Z}_{h_2}+\mathcal{S}_{x_2}+\mathcal{N}_{r_2} \end{pmatrix}\right] + \\ 4\left[\frac{\mathscr{f}_3\, \mathcal{C}_{\mathscr{b}_3}(1-g_3)}{2} + \frac{s_{\mathscr{k}}}{\mathcal{C}_{\mathscr{b}_3}}\begin{pmatrix} \mathcal{H}_3+\mathcal{I}_{d_3}+\mathcal{M}_{i_3}+\mathcal{L}_{y_3}+\mathcal{R}_{i_3}+\mathcal{J}_{y_3}+\mathcal{B}_{i_3}+\mathcal{Q}_{y_3}+\mathcal{F}_{i_3}+\mathcal{N}_{y_3}+\mathcal{W}_{i_3}+\mathcal{V}_{y_3} \\ +\mathcal{S}_{z_3}+\mathcal{P}_{\ell_3}+\mathcal{I}_{c_3}+\mathcal{M}_{u_3}+\mathcal{D}_{w_3}+\mathcal{G}_{a_3}+\mathcal{T}_{\sigma_3}+\mathcal{K}_{e_3}+\mathcal{W}_{m_3}+\mathcal{Z}_{h_3}+\mathcal{S}_{x_3}+\mathcal{N}_{r_3} \end{pmatrix}\right] + \\ 3\left[\frac{\mathscr{f}_4\, \mathcal{C}_{\mathscr{b}_4}(1-g_4)}{2} + \frac{s_{\mathscr{k}}}{\mathcal{C}_{\mathscr{b}_2}}\begin{pmatrix} \mathcal{H}_4+\mathcal{I}_{d_4}+\mathcal{M}_{i_4}+\mathcal{L}_{y_4}+\mathcal{R}_{i_4}+\mathcal{J}_{y_4}+\mathcal{B}_{i_4}+\mathcal{Q}_{y_4}+\mathcal{F}_{i_4}+\mathcal{N}_{y_4}+\mathcal{W}_{i_4}+\mathcal{V}_{y_4} \\ +\mathcal{S}_{z_4}+\mathcal{P}_{\ell_4}+\mathcal{I}_{c_4}+\mathcal{M}_{u_4}+\mathcal{D}_{w_4}+\mathcal{G}_{a_4}+\mathcal{T}_{\sigma_4}+\mathcal{K}_{e_4}+\mathcal{W}_{m_4}+\mathcal{Z}_{h_4}+\mathcal{S}_{x_4}+\mathcal{N}_{r_4} \end{pmatrix}\right] + \\ \frac{\mathscr{f}_5\, \mathcal{C}_{\mathscr{b}_5}(1-g_5)}{2} + \frac{s_{\mathscr{k}}}{\mathcal{C}_{\mathscr{b}_1}}\begin{pmatrix} \mathcal{H}_5+\mathcal{I}_{d_5}+\mathcal{M}_{i_5}+\mathcal{L}_{y_5}+\mathcal{R}_{i_5}+\mathcal{J}_{y_5}+\mathcal{B}_{i_5}+\mathcal{Q}_{y_5}+\mathcal{F}_{i_5}+\mathcal{N}_{y_5}+\mathcal{W}_{i_5}+\mathcal{V}_{y_5} \\ +\mathcal{S}_{z_5}+\mathcal{P}_{\ell_5}+\mathcal{I}_{c_5}+\mathcal{M}_{u_5}+\mathcal{D}_{w_5}+\mathcal{G}_{a_5}+\mathcal{T}_{\sigma_5}+\mathcal{K}_{e_5}+\mathcal{W}_{m_5}+\mathcal{Z}_{h_5}+\mathcal{S}_{x_5}+\mathcal{N}_{r_5} \end{pmatrix} \end{array}\right\}$$

$$- \lambda_1\left(\mathcal{C}_{\mathscr{b}_2} - \mathcal{C}_{\mathscr{b}_1}\right) - \lambda_2\left(\mathcal{C}_{\mathscr{b}_3} - \mathcal{C}_{\mathscr{b}_2}\right) \quad \text{------------------------ (5)}$$

Differentiating (5) partially with respect to $\mathcal{C}_{\mathscr{b}_1}, \mathcal{C}_{\mathscr{b}_2}, \mathcal{C}_{\mathscr{b}_3}, \mathcal{C}_{\mathscr{b}_4}, \mathcal{C}_{\mathscr{b}_5}, \lambda_1, \lambda_2$ and equating to zero, we get

$$\mathcal{C}_{\mathscr{b}_1} = \mathcal{C}_{\mathscr{b}_2} = \mathcal{C}_{\mathscr{b}_3}$$

$$\sqrt{\frac{2\left[\begin{array}{l} s_{\mathscr{k}}\begin{pmatrix} \mathcal{H}_5+\mathcal{I}_{d_5}+\mathcal{M}_{i_5}+\mathcal{L}_{y_5}+\mathcal{R}_{i_5}+\mathcal{J}_{y_5}+\mathcal{B}_{i_5}+\mathcal{Q}_{y_5}+\mathcal{F}_{i_5}+\mathcal{N}_{y_5}+\mathcal{W}_{i_5}+\mathcal{V}_{y_5} \\ +\mathcal{S}_{z_5}+\mathcal{P}_{\ell_5}+\mathcal{I}_{c_5}+\mathcal{M}_{u_5}+\mathcal{D}_{w_5}+\mathcal{G}_{a_5}+\mathcal{T}_{\sigma_5}+\mathcal{K}_{e_5}+\mathcal{W}_{m_5}+\mathcal{Z}_{h_5}+\mathcal{S}_{x_5}+\mathcal{N}_{r_5} \end{pmatrix} \\ +3\, s_{\mathscr{k}}\begin{pmatrix} \mathcal{H}_4+\mathcal{I}_{d_4}+\mathcal{M}_{i_4}+\mathcal{L}_{y_4}+\mathcal{R}_{i_4}+\mathcal{J}_{y_4}+\mathcal{B}_{i_4}+\mathcal{Q}_{y_4}+\mathcal{F}_{i_4}+\mathcal{N}_{y_4}+\mathcal{W}_{i_4}+\mathcal{V}_{y_4} \\ +\mathcal{S}_{z_4}+\mathcal{P}_{\ell_4}+\mathcal{I}_{c_4}+\mathcal{M}_{u_4}+\mathcal{D}_{w_4}+\mathcal{G}_{a_4}+\mathcal{T}_{\sigma_4}+\mathcal{K}_{e_4}+\mathcal{W}_{m_4}+\mathcal{Z}_{h_4}+\mathcal{S}_{x_4}+\mathcal{N}_{r_4} \end{pmatrix} \\ +4\, s_{\mathscr{k}}\begin{pmatrix} \mathcal{H}_3+\mathcal{I}_{d_3}+\mathcal{M}_{i_3}+\mathcal{L}_{y_3}+\mathcal{R}_{i_3}+\mathcal{J}_{y_3}+\mathcal{B}_{i_3}+\mathcal{Q}_{y_3}+\mathcal{F}_{i_3}+\mathcal{N}_{y_3}+\mathcal{W}_{i_3}+\mathcal{V}_{y_3} \\ +\mathcal{S}_{z_3}+\mathcal{P}_{\ell_3}+\mathcal{I}_{c_3}+\mathcal{M}_{u_3}+\mathcal{D}_{w_3}+\mathcal{G}_{a_3}+\mathcal{T}_{\sigma_3}+\mathcal{K}_{e_3}+\mathcal{W}_{m_3}+\mathcal{Z}_{h_3}+\mathcal{S}_{x_3}+\mathcal{N}_{r_3} \end{pmatrix} \end{array}\right]}{\mathscr{f}_1\ (1-g_1)+3\, \mathscr{f}_2\ (1-g_2)+4\, \mathscr{f}_3\ (1-g_3)}}$$

$$\mathcal{C}_{\mathscr{b}_4} = \sqrt{\frac{2\left[3\, s_{\mathscr{k}} \begin{pmatrix} \mathcal{H}_2+\mathcal{I}_{d_2}+\mathcal{M}_{i_2}+\mathcal{L}_{y_2}+\mathcal{R}_{i_2}+\mathcal{J}_{y_2}+\mathcal{B}_{i_2}+\mathcal{Q}_{y_2}+\mathcal{F}_{i_2}+\mathcal{N}_{y_2}+\mathcal{W}_{i_2}+\mathcal{V}_{y_2} \\ +\mathcal{S}_{z_2}+\mathcal{P}_{\ell_2}+\mathcal{I}_{c_2}+\mathcal{M}_{u_2}+\mathcal{D}_{w_2}+\mathcal{G}_{a_2}+\mathcal{T}_{\sigma_2}+\mathcal{K}_{e_2}+\mathcal{W}_{m_2}+\mathcal{Z}_{h_2}+\mathcal{S}_{x_2}+\mathcal{N}_{r_2} \end{pmatrix}\right]}{3\, \mathscr{f}_4\ (1-g_4)}}$$

$$\mathcal{C}_{\mathscr{b}_5} = \sqrt{\frac{2\ s_{\mathscr{k}} \begin{pmatrix} \mathcal{H}_1+\mathcal{I}_{d_1}+\mathcal{M}_{i_1}+\mathcal{L}_{y_1}+\mathcal{R}_{i_1}+\mathcal{J}_{y_1}+\mathcal{B}_{i_1}+\mathcal{Q}_{y_1}+\mathcal{F}_{i_1}+\mathcal{N}_{y_1}+\mathcal{W}_{i_1}+\mathcal{V}_{y_1} \\ +\mathcal{S}_{z_1}+\mathcal{P}_{\ell_1}+\mathcal{I}_{c_1}+\mathcal{M}_{u_1}+\mathcal{D}_{w_1}+\mathcal{G}_{a_1}+\mathcal{T}_{\sigma_1}+\mathcal{K}_{e_1}+\mathcal{W}_{m_1}+\mathcal{Z}_{h_1}+\mathcal{S}_{x_1}+\mathcal{N}_{r_1} \end{pmatrix}}{\mathscr{f}_5\ (1-g_5)}}$$

Here, $\mathcal{C}_{\mathscr{b}_4} > \mathcal{C}_{\mathscr{b}_5}$. It does not satisfy the constraints $0< \mathcal{C}_{\mathscr{b}_1} \le \mathcal{C}_{\mathscr{b}_2} \le \mathcal{C}_{\mathscr{b}_3} \le \mathcal{C}_{\mathscr{b}_4} \le \mathcal{C}_{\mathscr{b}_5}$. Therefore, it not a local optimum.

4) Step 4

Convert the inequality constraints $\mathcal{C}_{b_2} - \mathcal{C}_{b_1} \geq 0, \mathcal{C}_{b_3} - \mathcal{C}_{b_2} \geq 0$ and $\mathcal{C}_{b_4} - \mathcal{C}_{b_3} \geq 0$ into equality constraints $\mathcal{C}_{b_2} - \mathcal{C}_{b_1} = 0, \mathcal{C}_{b_3} - \mathcal{C}_{b_2} = 0$ and $\mathcal{C}_{b_4} - \mathcal{C}_{b_3} = 0$. Optimize $\wp[\widetilde{TC}(\mathcal{C}_b)]$ subject to $\mathcal{C}_{b_2} - \mathcal{C}_{b_1} = 0, \mathcal{C}_{b_3} - \mathcal{C}_{b_2} = 0$ and $\mathcal{C}_{b_4} - \mathcal{C}_{b_3} = 0$ by the Lagrangean method. Hence the Lagrangean function is given by $L\left(\mathcal{C}_{b_1}, \mathcal{C}_{b_2}, \mathcal{C}_{b_3}, \mathcal{C}_{b_4}, \mathcal{C}_{b_5}, \lambda_1, \lambda_2, \lambda_3\right) = \wp[\widetilde{TC}(\mathcal{C}_b)] - \lambda_1(\mathcal{C}_{b_2} - \mathcal{C}_{b_1}) - \lambda_2(\mathcal{C}_{b_3} - \mathcal{C}_{b_2}) - \lambda_3(\mathcal{C}_{b_4} - \mathcal{C}_{b_3})$

Hence, $\wp[\widetilde{TC}(\mathcal{C}_b)] - \lambda_1(\mathcal{C}_{b_2} - \mathcal{C}_{b_1}) - \lambda_2(\mathcal{C}_{b_3} - \mathcal{C}_{b_2}) - \lambda_3(\mathcal{C}_{b_4} - \mathcal{C}_{b_3}) =$

$$\frac{1}{12}\left\{\begin{array}{l}
\frac{f_1 \mathcal{C}_{b_1}(1-g_1)}{2} + \frac{s_k}{\mathcal{C}_{b_5}}\begin{pmatrix}\mathcal{H}_1 + \mathcal{J}_{d_1} + \mathcal{M}_{i_1} + \mathcal{L}_{y_1} + \mathcal{R}_{i_1} + \mathcal{J}_{y_1} + \mathcal{B}_{i_1} + \mathcal{Q}_{y_1} + \mathcal{F}_{i_1} + \mathcal{N}_{y_1} + \mathcal{W}_{i_1} + \mathcal{V}_{y_1} \\ + \mathcal{S}_{z_1} + \mathcal{P}_{\ell_1} + \mathcal{J}_{C_1} + \mathcal{M}_{u_1} + \mathcal{D}_{w_1} + \mathcal{G}_{a_1} + \mathcal{T}_{o_1} + \mathcal{K}_{e_1} + \mathcal{W}_{m_1} + \mathcal{Z}_{h_1} + \mathcal{S}_{x_1} + \mathcal{N}_{r_1}\end{pmatrix} + \\
3\left[\frac{f_2 \mathcal{C}_{b_2}(1-g_2)}{2} + \frac{s_k}{\mathcal{C}_{b_4}}\begin{pmatrix}\mathcal{H}_2 + \mathcal{J}_{d_2} + \mathcal{M}_{i_2} + \mathcal{L}_{y_2} + \mathcal{R}_{i_2} + \mathcal{J}_{y_2} + \mathcal{B}_{i_2} + \mathcal{Q}_{y_2} + \mathcal{F}_{i_2} + \mathcal{N}_{y_2} + \mathcal{W}_{i_2} + \mathcal{V}_{y_2} \\ + \mathcal{S}_{z_2} + \mathcal{P}_{\ell_2} + \mathcal{J}_{C_2} + \mathcal{M}_{u_2} + \mathcal{D}_{w_2} + \mathcal{G}_{a_2} + \mathcal{T}_{o_2} + \mathcal{K}_{e_2} + \mathcal{W}_{m_2} + \mathcal{Z}_{h_2} + \mathcal{S}_{x_2} + \mathcal{N}_{r_2}\end{pmatrix}\right] + \\
4\left[\frac{f_3 \mathcal{C}_{b_3}(1-g_3)}{2} + \frac{s_k}{\mathcal{C}_{b_3}}\begin{pmatrix}\mathcal{H}_3 + \mathcal{J}_{d_3} + \mathcal{M}_{i_3} + \mathcal{L}_{y_3} + \mathcal{R}_{i_3} + \mathcal{J}_{y_3} + \mathcal{B}_{i_3} + \mathcal{Q}_{y_3} + \mathcal{F}_{i_3} + \mathcal{N}_{y_3} + \mathcal{W}_{i_3} + \mathcal{V}_{y_3} \\ + \mathcal{S}_{z_3} + \mathcal{P}_{\ell_3} + \mathcal{J}_{C_3} + \mathcal{M}_{u_3} + \mathcal{D}_{w_3} + \mathcal{G}_{a_3} + \mathcal{T}_{o_3} + \mathcal{K}_{e_3} + \mathcal{W}_{m_3} + \mathcal{Z}_{h_3} + \mathcal{S}_{x_3} + \mathcal{N}_{r_3}\end{pmatrix}\right] + \\
3\left[\frac{f_4 \mathcal{C}_{b_4}(1-g_4)}{2} + \frac{s_k}{\mathcal{C}_{b_2}}\begin{pmatrix}\mathcal{H}_4 + \mathcal{J}_{d_4} + \mathcal{M}_{i_4} + \mathcal{L}_{y_4} + \mathcal{R}_{i_4} + \mathcal{J}_{y_4} + \mathcal{B}_{i_4} + \mathcal{Q}_{y_4} + \mathcal{F}_{i_4} + \mathcal{N}_{y_4} + \mathcal{W}_{i_4} + \mathcal{V}_{y_4} \\ + \mathcal{S}_{z_4} + \mathcal{P}_{\ell_4} + \mathcal{J}_{C_4} + \mathcal{M}_{u_4} + \mathcal{D}_{w_4} + \mathcal{G}_{a_4} + \mathcal{T}_{o_4} + \mathcal{K}_{e_4} + \mathcal{W}_{m_4} + \mathcal{Z}_{h_4} + \mathcal{S}_{x_4} + \mathcal{N}_{r_4}\end{pmatrix}\right] + \\
\frac{f_5 \mathcal{C}_{b_5}(1-g_5)}{2} + \frac{s_k}{\mathcal{C}_{b_1}}\begin{pmatrix}\mathcal{H}_5 + \mathcal{J}_{d_5} + \mathcal{M}_{i_5} + \mathcal{L}_{y_5} + \mathcal{R}_{i_5} + \mathcal{J}_{y_5} + \mathcal{B}_{i_5} + \mathcal{Q}_{y_5} + \mathcal{F}_{i_5} + \mathcal{N}_{y_5} + \mathcal{W}_{i_5} + \mathcal{V}_{y_5} \\ + \mathcal{S}_{z_5} + \mathcal{P}_{\ell_5} + \mathcal{J}_{C_5} + \mathcal{M}_{u_5} + \mathcal{D}_{w_5} + \mathcal{G}_{a_5} + \mathcal{T}_{o_5} + \mathcal{K}_{e_5} + \mathcal{W}_{m_5} + \mathcal{Z}_{h_5} + \mathcal{S}_{x_5} + \mathcal{N}_{r_5}\end{pmatrix}
\end{array}\right\}$$

$$- \lambda_1(\mathcal{C}_{b_2} - \mathcal{C}_{b_1}) - \lambda_2(\mathcal{C}_{b_3} - \mathcal{C}_{b_2}) - \lambda_3(\mathcal{C}_{b_4} - \mathcal{C}_{b_3}) \text{------------------------ (6)}$$

Differentiating (6) partially with respect to $\mathcal{C}_{b_1}, \mathcal{C}_{b_2}, \mathcal{C}_{b_3}, \mathcal{C}_{b_4}, \mathcal{C}_{b_5}, \lambda_1, \lambda_2, \lambda_3$, and equating to zero, we get

$$\mathcal{C}_{b_1} = \mathcal{C}_{b_2} = \mathcal{C}_{b_3} = \mathcal{C}_{b_4} = \sqrt{\frac{2\left[\begin{array}{l}
s_k\begin{pmatrix}\mathcal{H}_5 + \mathcal{J}_{d_5} + \mathcal{M}_{i_5} + \mathcal{L}_{y_5} + \mathcal{R}_{i_5} + \mathcal{J}_{y_5} + \mathcal{B}_{i_5} + \mathcal{Q}_{y_5} + \mathcal{F}_{i_5} + \mathcal{N}_{y_5} + \mathcal{W}_{i_5} + \mathcal{V}_{y_5} \\ + \mathcal{S}_{z_5} + \mathcal{P}_{\ell_5} + \mathcal{J}_{C_5} + \mathcal{M}_{u_5} + \mathcal{D}_{w_5} + \mathcal{G}_{a_5} + \mathcal{T}_{o_5} + \mathcal{K}_{e_5} + \mathcal{W}_{m_5} + \mathcal{Z}_{h_5} + \mathcal{S}_{x_5} + \mathcal{N}_{r_5}\end{pmatrix} \\
+ 3\, s_k\begin{pmatrix}\mathcal{H}_4 + \mathcal{J}_{d_4} + \mathcal{M}_{i_4} + \mathcal{L}_{y_4} + \mathcal{R}_{i_4} + \mathcal{J}_{y_4} + \mathcal{B}_{i_4} + \mathcal{Q}_{y_4} + \mathcal{F}_{i_4} + \mathcal{N}_{y_4} + \mathcal{W}_{i_4} + \mathcal{V}_{y_4} \\ + \mathcal{S}_{z_4} + \mathcal{P}_{\ell_4} + \mathcal{J}_{C_4} + \mathcal{M}_{u_4} + \mathcal{D}_{w_4} + \mathcal{G}_{a_4} + \mathcal{T}_{o_4} + \mathcal{K}_{e_4} + \mathcal{W}_{m_4} + \mathcal{Z}_{h_4} + \mathcal{S}_{x_4} + \mathcal{N}_{r_4}\end{pmatrix} \\
+ 4\, s_k\begin{pmatrix}\mathcal{H}_3 + \mathcal{J}_{d_3} + \mathcal{M}_{i_3} + \mathcal{L}_{y_3} + \mathcal{R}_{i_3} + \mathcal{J}_{y_3} + \mathcal{B}_{i_3} + \mathcal{Q}_{y_3} + \mathcal{F}_{i_3} + \mathcal{N}_{y_3} + \mathcal{W}_{i_3} + \mathcal{V}_{y_3} \\ + \mathcal{S}_{z_3} + \mathcal{P}_{\ell_3} + \mathcal{J}_{C_3} + \mathcal{M}_{u_3} + \mathcal{D}_{w_3} + \mathcal{G}_{a_3} + \mathcal{T}_{o_3} + \mathcal{K}_{e_3} + \mathcal{W}_{m_3} + \mathcal{Z}_{h_3} + \mathcal{S}_{x_3} + \mathcal{N}_{r_3}\end{pmatrix} \\
+ 3\, s_k\begin{pmatrix}\mathcal{H}_2 + \mathcal{J}_{d_2} + \mathcal{M}_{i_2} + \mathcal{L}_{y_2} + \mathcal{R}_{i_2} + \mathcal{J}_{y_2} + \mathcal{B}_{i_2} + \mathcal{Q}_{y_2} + \mathcal{F}_{i_2} + \mathcal{N}_{y_2} + \mathcal{W}_{i_2} + \mathcal{V}_{y_2} \\ + \mathcal{S}_{z_2} + \mathcal{P}_{\ell_2} + \mathcal{J}_{C_2} + \mathcal{M}_{u_2} + \mathcal{D}_{w_2} + \mathcal{G}_{a_2} + \mathcal{T}_{o_2} + \mathcal{K}_{e_2} + \mathcal{W}_{m_2} + \mathcal{Z}_{h_2} + \mathcal{S}_{x_2} + \mathcal{N}_{r_2}\end{pmatrix}
\end{array}\right]}{f_1(1-g_1) + 3 f_2(1-g_2) + 4 f_3(1-g_3) + 3 f_4(1-g_4)}}$$

$$\mathcal{C}_{b_5} = \sqrt{\frac{2\, s_k\begin{pmatrix}\mathcal{H}_1 + \mathcal{J}_{d_1} + \mathcal{M}_{i_1} + \mathcal{L}_{y_1} + \mathcal{R}_{i_1} + \mathcal{J}_{y_1} + \mathcal{B}_{i_1} + \mathcal{Q}_{y_1} + \mathcal{F}_{i_1} + \mathcal{N}_{y_1} + \mathcal{W}_{i_1} + \mathcal{V}_{y_1} \\ + \mathcal{S}_{z_1} + \mathcal{P}_{\ell_1} + \mathcal{J}_{C_1} + \mathcal{M}_{u_1} + \mathcal{D}_{w_1} + \mathcal{G}_{a_1} + \mathcal{T}_{o_1} + \mathcal{K}_{e_1} + \mathcal{W}_{m_1} + \mathcal{Z}_{h_1} + \mathcal{S}_{x_1} + \mathcal{N}_{r_1}\end{pmatrix}}{f_5(1-g_5)}}$$

It does not satisfy the constraints $0 < \mathcal{C}_{b_1} \leq \mathcal{C}_{b_2} \leq \mathcal{C}_{b_3} \leq \mathcal{C}_{b_4} \leq \mathcal{C}_{b_5}$. Therefore, it is not a local optimum.

5) Step 5

Now convert the inequality constraints $\mathcal{C}_{b_2} - \mathcal{C}_{b_1} \geq 0, \mathcal{C}_{b_3} - \mathcal{C}_{b_2} \geq 0, \mathcal{C}_{b_4} - \mathcal{C}_{b_3} \geq 0$ and $\mathcal{C}_{b_5} - \mathcal{C}_{b_4} \geq 0$ into equality constraints $\mathcal{C}_{b_2} - \mathcal{C}_{b_1} = 0, \mathcal{C}_{b_3} - \mathcal{C}_{b_2} = 0, \mathcal{C}_{b_4} - \mathcal{C}_{b_3} = 0$ and $\mathcal{C}_{b_5} - \mathcal{C}_{b_4} = 0$. Now the Lagrangean function is given by $L\left(\mathcal{C}_{b_1}, \mathcal{C}_{b_2}, \mathcal{C}_{b_3}, \mathcal{C}_{b_4}, \mathcal{C}_{b_5}, \lambda_1, \lambda_2, \lambda_3, \lambda_4\right) = \wp[\widetilde{TC}(\mathcal{C}_b)] - \lambda_1(\mathcal{C}_{b_2} - \mathcal{C}_{b_1}) - \lambda_2(\mathcal{C}_{b_3} - \mathcal{C}_{b_2}) - \lambda_3(\mathcal{C}_{b_4} - \mathcal{C}_{b_3}) - \lambda_4(\mathcal{C}_{b_5} - \mathcal{C}_{b_4})$.

$\wp[\widetilde{TC}(\mathcal{C}_b)] - \lambda_1(\mathcal{C}_{b_2} - \mathcal{C}_{b_1}) - \lambda_2(\mathcal{C}_{b_3} - \mathcal{C}_{b_2}) - \lambda_3(\mathcal{C}_{b_4} - \mathcal{C}_{b_3}) - \lambda_4(\mathcal{C}_{b_5} - \mathcal{C}_{b_4}) =$

$$\frac{1}{12}\left\{\begin{array}{l}
\frac{f_1 \mathcal{C}_{b_1}(1-g_1)}{2} + \frac{s_k}{\mathcal{C}_{b_5}}\begin{pmatrix}\mathcal{H}_1 + \mathcal{J}_{d_1} + \mathcal{M}_{i_1} + \mathcal{L}_{y_1} + \mathcal{R}_{i_1} + \mathcal{J}_{y_1} + \mathcal{B}_{i_1} + \mathcal{Q}_{y_1} + \mathcal{F}_{i_1} + \mathcal{N}_{y_1} + \mathcal{W}_{i_1} + \mathcal{V}_{y_1} \\ + \mathcal{S}_{z_1} + \mathcal{P}_{\ell_1} + \mathcal{J}_{C_1} + \mathcal{M}_{u_1} + \mathcal{D}_{w_1} + \mathcal{G}_{a_1} + \mathcal{T}_{o_1} + \mathcal{K}_{e_1} + \mathcal{W}_{m_1} + \mathcal{Z}_{h_1} + \mathcal{S}_{x_1} + \mathcal{N}_{r_1}\end{pmatrix} + \\
3\left[\frac{f_2 \mathcal{C}_{b_2}(1-g_2)}{2} + \frac{s_k}{\mathcal{C}_{b_4}}\begin{pmatrix}\mathcal{H}_2 + \mathcal{J}_{d_2} + \mathcal{M}_{i_2} + \mathcal{L}_{y_2} + \mathcal{R}_{i_2} + \mathcal{J}_{y_2} + \mathcal{B}_{i_2} + \mathcal{Q}_{y_2} + \mathcal{F}_{i_2} + \mathcal{N}_{y_2} + \mathcal{W}_{i_2} + \mathcal{V}_{y_2} \\ + \mathcal{S}_{z_2} + \mathcal{P}_{\ell_2} + \mathcal{J}_{C_2} + \mathcal{M}_{u_2} + \mathcal{D}_{w_2} + \mathcal{G}_{a_2} + \mathcal{T}_{o_2} + \mathcal{K}_{e_2} + \mathcal{W}_{m_2} + \mathcal{Z}_{h_2} + \mathcal{S}_{x_2} + \mathcal{N}_{r_2}\end{pmatrix}\right] + \\
4\left[\frac{f_3 \mathcal{C}_{b_3}(1-g_3)}{2} + \frac{s_k}{\mathcal{C}_{b_3}}\begin{pmatrix}\mathcal{H}_3 + \mathcal{J}_{d_3} + \mathcal{M}_{i_3} + \mathcal{L}_{y_3} + \mathcal{R}_{i_3} + \mathcal{J}_{y_3} + \mathcal{B}_{i_3} + \mathcal{Q}_{y_3} + \mathcal{F}_{i_3} + \mathcal{N}_{y_3} + \mathcal{W}_{i_3} + \mathcal{V}_{y_3} \\ + \mathcal{S}_{z_3} + \mathcal{P}_{\ell_3} + \mathcal{J}_{C_3} + \mathcal{M}_{u_3} + \mathcal{D}_{w_3} + \mathcal{G}_{a_3} + \mathcal{T}_{o_3} + \mathcal{K}_{e_3} + \mathcal{W}_{m_3} + \mathcal{Z}_{h_3} + \mathcal{S}_{x_3} + \mathcal{N}_{r_3}\end{pmatrix}\right] + \\
3\left[\frac{f_4 \mathcal{C}_{b_4}(1-g_4)}{2} + \frac{s_k}{\mathcal{C}_{b_2}}\begin{pmatrix}\mathcal{H}_4 + \mathcal{J}_{d_4} + \mathcal{M}_{i_4} + \mathcal{L}_{y_4} + \mathcal{R}_{i_4} + \mathcal{J}_{y_4} + \mathcal{B}_{i_4} + \mathcal{Q}_{y_4} + \mathcal{F}_{i_4} + \mathcal{N}_{y_4} + \mathcal{W}_{i_4} + \mathcal{V}_{y_4} \\ + \mathcal{S}_{z_4} + \mathcal{P}_{\ell_4} + \mathcal{J}_{C_4} + \mathcal{M}_{u_4} + \mathcal{D}_{w_4} + \mathcal{G}_{a_4} + \mathcal{T}_{o_4} + \mathcal{K}_{e_4} + \mathcal{W}_{m_4} + \mathcal{Z}_{h_4} + \mathcal{S}_{x_4} + \mathcal{N}_{r_4}\end{pmatrix}\right] + \\
\frac{f_5 \mathcal{C}_{b_5}(1-g_5)}{2} + \frac{s_k}{\mathcal{C}_{b_1}}\begin{pmatrix}\mathcal{H}_5 + \mathcal{J}_{d_5} + \mathcal{M}_{i_5} + \mathcal{L}_{y_5} + \mathcal{R}_{i_5} + \mathcal{J}_{y_5} + \mathcal{B}_{i_5} + \mathcal{Q}_{y_5} + \mathcal{F}_{i_5} + \mathcal{N}_{y_5} + \mathcal{W}_{i_5} + \mathcal{V}_{y_5} \\ + \mathcal{S}_{z_5} + \mathcal{P}_{\ell_5} + \mathcal{J}_{C_5} + \mathcal{M}_{u_5} + \mathcal{D}_{w_5} + \mathcal{G}_{a_5} + \mathcal{T}_{o_5} + \mathcal{K}_{e_5} + \mathcal{W}_{m_5} + \mathcal{Z}_{h_5} + \mathcal{S}_{x_5} + \mathcal{N}_{r_5}\end{pmatrix}
\end{array}\right.$$

$$- \lambda_1(\mathcal{C}_{b_2} - \mathcal{C}_{b_1}) - \lambda_2(\mathcal{C}_{b_3} - \mathcal{C}_{b_2}) - \lambda_3(\mathcal{C}_{b_4} - \mathcal{C}_{b_3}) - \lambda_4(\mathcal{C}_{b_5} - \mathcal{C}_{b_4})$$

------------------------ (7)

Differentiating (7) partially with respect to $\mathcal{C}_{b_1}, \mathcal{C}_{b_2}, \mathcal{C}_{b_3}, \mathcal{C}_{b_4}, \mathcal{C}_{b_5}, \lambda_1, \lambda_2, \lambda_3, \lambda_4$ and equating to zero, we get

$$\widetilde{\mathcal{C}_{b}}^{*} = \sqrt{\frac{2\left[\begin{array}{l} s_k\begin{pmatrix}\mathcal{H}_5+\mathcal{I}_{d_5}+\mathcal{M}_{i_5}+\mathcal{L}_{y_5}+\mathcal{R}_{i_5}+\mathcal{J}_{y_5}+\mathcal{B}_{i_5}+\mathcal{Q}_{y_5}+\mathcal{F}_{i_5}+\mathcal{N}_{y_5}+\mathcal{W}_{i_5}+\mathcal{V}_{y_5}\\ +\mathcal{S}_{z_5}+\mathcal{P}_{\ell_5}+\mathcal{I}_{\mathcal{C}_5}+\mathcal{M}_{u_5}+\mathcal{D}_{w_5}+\mathcal{G}_{q_5}+\mathcal{T}_{o_5}+\mathcal{K}_{e_5}+\mathcal{W}_{m_5}+\mathcal{Z}_{h_5}+\mathcal{S}_{x_5}+\mathcal{N}_{r_5}\end{pmatrix}\\ +3\,s_k\begin{pmatrix}\mathcal{H}_4+\mathcal{I}_{d_4}+\mathcal{M}_{i_4}+\mathcal{L}_{y_4}+\mathcal{R}_{i_4}+\mathcal{J}_{y_4}+\mathcal{B}_{i_4}+\mathcal{Q}_{y_4}+\mathcal{F}_{i_4}+\mathcal{N}_{y_4}+\mathcal{W}_{i_4}+\mathcal{V}_{y_4}\\ +\mathcal{S}_{z_4}+\mathcal{P}_{\ell_4}+\mathcal{I}_{\mathcal{C}_4}+\mathcal{M}_{u_4}+\mathcal{D}_{w_4}+\mathcal{G}_{q_4}+\mathcal{T}_{o_4}+\mathcal{K}_{e_4}+\mathcal{W}_{m_4}+\mathcal{Z}_{h_4}+\mathcal{S}_{x_4}+\mathcal{N}_{r_4}\end{pmatrix}\\ +4\,s_k\begin{pmatrix}\mathcal{H}_3+\mathcal{I}_{d_3}+\mathcal{M}_{i_3}+\mathcal{L}_{y_3}+\mathcal{R}_{i_3}+\mathcal{J}_{y_3}+\mathcal{B}_{i_3}+\mathcal{Q}_{y_3}+\mathcal{F}_{i_3}+\mathcal{N}_{y_3}+\mathcal{W}_{i_3}+\mathcal{V}_{y_3}\\ +\mathcal{S}_{z_3}+\mathcal{P}_{\ell_3}+\mathcal{I}_{\mathcal{C}_3}+\mathcal{M}_{u_3}+\mathcal{D}_{w_3}+\mathcal{G}_{q_3}+\mathcal{T}_{o_3}+\mathcal{K}_{e_3}+\mathcal{W}_{m_3}+\mathcal{Z}_{h_3}+\mathcal{S}_{x_3}+\mathcal{N}_{r_3}\end{pmatrix}\\ +3\,s_k\begin{pmatrix}\mathcal{H}_2+\mathcal{I}_{d_2}+\mathcal{M}_{i_2}+\mathcal{L}_{y_2}+\mathcal{R}_{i_2}+\mathcal{J}_{y_2}+\mathcal{B}_{i_2}+\mathcal{Q}_{y_2}+\mathcal{F}_{i_2}+\mathcal{N}_{y_2}+\mathcal{W}_{i_2}+\mathcal{V}_{y_2}\\ +\mathcal{S}_{z_2}+\mathcal{P}_{\ell_2}+\mathcal{I}_{\mathcal{C}_2}+\mathcal{M}_{u_2}+\mathcal{D}_{w_2}+\mathcal{G}_{q_2}+\mathcal{T}_{o_2}+\mathcal{K}_{e_2}+\mathcal{W}_{m_2}+\mathcal{Z}_{h_2}+\mathcal{S}_{x_2}+\mathcal{N}_{r_2}\end{pmatrix}\\ s_k\begin{pmatrix}\mathcal{H}_1+\mathcal{I}_{d_1}+\mathcal{M}_{i_1}+\mathcal{L}_{y_1}+\mathcal{R}_{i_1}+\mathcal{J}_{y_1}+\mathcal{B}_{i_1}+\mathcal{Q}_{y_1}+\mathcal{F}_{i_1}+\mathcal{N}_{y_1}+\mathcal{W}_{i_1}+\mathcal{V}_{y_1}\\ +\mathcal{S}_{z_1}+\mathcal{P}_{\ell_1}+\mathcal{I}_{\mathcal{C}_1}+\mathcal{M}_{u_1}+\mathcal{D}_{w_1}+\mathcal{G}_{q_1}+\mathcal{T}_{o_1}+\mathcal{K}_{e_1}+\mathcal{W}_{m_1}+\mathcal{Z}_{h_1}+\mathcal{S}_{x_1}+\mathcal{N}_{r_1}\end{pmatrix}\end{array}\right]}{f_1\,(1-g_1)+3\,f_2\,(1-g_2)+4\,f_3\,(1-g_3)+3\,f_4\,(1-g_4)+f_5\,(1-g_5)}}$$

-------------------- (8)

The above equation (8) is the required fuzzy optimal production quantity of this model.

IV. NUMERICAL EXAMPLE

s_k = 5,00	$\mathcal{I}_d$= 15	$\mathcal{M}_i$= 25	$\mathcal{L}_y$= 20	$\mathcal{R}_i$= 30	$\mathcal{J}_y$= 25	$\mathcal{B}_i$= 15	$\mathcal{Q}_y$= 10	$\mathcal{F}_i$= 20	$\mathcal{N}_y$= 40
$\mathcal{W}_i$= 350	$\mathcal{V}_y$= 250	$\mathcal{S}_z$= 5	$\mathcal{P}_\ell$= 10	$\mathcal{I}_\mathcal{C}$= 40	$\mathcal{M}_u$= 25	$\mathcal{D}_w$= 20	$\mathcal{G}_q$= 40	$\mathcal{T}_o$= 35	$\mathcal{K}_e$= 20

$\mathcal{W}_m$= 100	$\mathcal{Z}_h$= 200	$\mathcal{S}_x$= 20	$\mathcal{N}_r$= 25	f= 5	g= 0.66	$\mathcal{H}$= 50	$\mathcal{A}$= 750

V. SOLUTION IN CRISP MODEL

By using the given values in the data, we obtain the optimal production quantity in the crisp sense as $\mathcal{C}_b$ = 917.8

By using the equation (1) in this model, we obtain the Total cost in crisp sense as

TC($\mathcal{C}_b$) = 1514.43

VI. SOLUTION IN FUZZY MODEL

By using the given values in the data, we obtain the optimal production quantity in the fuzzy sense as $\widetilde{\mathcal{C}_b}^{*}$ = 917.8

By using the equation (3) in this model, we obtain the Total cost in crisp sense as

$[\widetilde{TC(\mathcal{C}_b)}]^*$ = 1508.92.

VII. CONCLUSION

The EPQ model for green quality is incredibly practical and provides the best solution to the problem of generating green quality products. This model is quite realistic because it includes all conceivable production and quality control costs. This model differs from others in that it considers product switching costs and types. By reducing costs, this approach assists the industrial sectors in achieving the aim of consumer pleasure with green quality products. This device is also environmentally friendly because it helps to protect the environment from waste's harmful impacts. This model is a comprehensive model because it includes all the costs of producing green quality items.

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