



IJRASET

International Journal For Research in
Applied Science and Engineering Technology



INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 11 Issue: III Month of publication: March 2023

DOI: <https://doi.org/10.22214/ijraset.2023.49282>

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Generalized Investigated in Topological Space

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Abstract: In this paper, some characterizations and proportion of notion a investigated. Throughout this paper (X, τ) and (Y, σ) (simply, X and Y) represent topological spaces on which separation axioms are assumed unless otherwise mentioned. We introduce a new class of sets called regular generalized open sets which is properly placed in between the class of open sets and the class of β -open sets. Throughout this paper (X, τ) represents a topological space on which no separation axiom is assumed unless otherwise mentioned. For a subset A of a topological space X , $cl(A)$ and $int(A)$ denote the closure of A and the interior of A respectively. X/A or A^c denotes the complement of A in X . introduced and investigated semi open sets, generalized closed sets, regular semi open sets, weakly closed sets, semi generalized closed sets, weakly generalized closed sets, strongly generalized closed sets, generalized pre - regular closed sets, regular generalized closed sets, and generalized α -generalized closed sets respectively.

Keywords: Topological space, Cluster Point, Open and Closed set, β^* - Continuous, Subset, .Regular open closed set, Separation axioms

I. INTRODUCTION

A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called super-continuous (resp. α -continuous α -continuous pre-continuous δ - smi - continuous Z - continuous γ - continuous continuous Z^* - continuous, β - continuous e^* - continuous) if $f^{-1}(V)$ is δ - open (resp. α -open, α -open, per open, δ -semiopen, Z -open, γ -open, e - open, Z^* - open, β -open, e^* -open) in X , for each $V \in \sigma$. the notion of β -open sets and β -continuity in topological space. The concepts of Z^* - open set and Z^* - continuity introduced by Mubarki. The purpose of this paper introduce and study the notions of β^* - open sets, β^* - continuous functions and (β^* - open sets. For a subset A of a (X, τ) , $cl(A)$, $int(A)$ and $X \setminus A$ denote the closure of A , the interior of A and the complement of A , respectively. A subset A of a topological space (X, τ) is called regular open (resp. regular closed) if $A = int(cl(A))$ (resp. $A = cl(int(A))$). A point x of X is called δ - cluster point of A if $int(cl(U)) \cap A \neq \emptyset$, for every open set U of X containing x . The set of all δ -cluster points of A is called δ -closure of A and is denoted $cl \delta(A)$. A set A is δ -closed if and if $A = cl \delta(A)$. The complement of a α -open (resp. α -open, δ -semiopen, δ -preopen, Z -open, γ -open, e -open, Z^* -open, β -open, e^* -open) sets is called α -close (resp. α -closed δ -semi-closed, δ -pre-closed Z -closed γ -closed, e - closed Z^* -closed β -closed, e^* - closed The intersection of all δ - preclosed (resp. β -closed) set containing A is called the δ - preclosure (resp. β -closure) of A and is denoted by $\delta - pcl(A)$ (resp. $\beta-cl(A)$). The union of all δ -preopen (resp. β -open) sets contained in A is called the δ - pre - interior (resp. β - interior) of A and is denoted by $\delta - pint(A)$ (resp. $\beta-int(A)$). The family of all δ -open (resp. δ -semiopen, δ - preopen, Z^* - open, β - open, e^* - open) sets is denoted by $\delta O(X)$ (resp. $\delta SO(X)$, $\delta PO(X)$, $Z^* O(X)$, $\beta O(X)$, $e^* O(X)$).

Lemma 1.1

Let A be a subset of a space (X, τ) . Then:

- (1) $\delta - pint(A) = A \cap int(cl \delta(A))$ and $\delta - pcl(A) = A \cup cl(int \delta(A))$,
- (2) $\beta - Int(A) = A \cap cl(int(cl(A)))$ and $\beta - cl(A) = A \cup int(cl(int(A)))$.

β^* - Open sets

Definition 2.1

A subset A of a topological space (X, τ) is said to be:

- (1) a β^* - open set if $A \subseteq cl(int(cl(A))) \cup int(cl \delta(A))$,
- (2) a β^* - closed set if $int(cl(int(A))) \cap cl(int \delta c(A)) \subseteq A$.
- (3) The family of all β^* - open (resp. β^* - closed) subsets of a space (X, τ) will be as always denoted by $\beta^* O(X)$ (resp. $\beta^* C(X)$).

Remark 3.1

The following diagram holds for each a subset A of X.

None of these implications are reversible as shown in the following examples

Example 4.1

Let $X = \{a, b, c, d\}$, with topology $\tau = \{\phi, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, c, d\}, X\}$. Then:

- (1) A subset $\{b, c\}$ of X is β^* - open but it is not β -open,
- (2) A subset $\{b, d\}$ of X is e^* - open but it is not β^* - open,

Example 4.2

Let $X = \{a, b, c, d, e\}$ and $\tau = \{\phi, \{a, b\}, \{c, d\}, \{a, b, c, d\}, X\}$. Then $\{a, e\}$ is β^* - open but it is not Z^* - open.

Remark 3.3

By the following example we show that the intersection of any two β^* -open sets is not β^* - open.

Example 4.4

Let $X = \{a, b, c\}$ with topology $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$. Then $A = \{a, c\}$ and $B = \{b, c\}$ are β^* -open sets. But, $A \cap B = \{c\}$ is not β^* - open

Definition 2.2

Let (X, τ) be a topological space. Then:

- (1) The union of all β^* - open sets of contained in A is called the β^* -interior of A and is denoted by $\beta^* - int(A)$,
- (2) The intersection of all β^* - closed sets of X containing A is called
- (3) The β^* - closure of A and is denoted by $\beta^* - cl(A)$.

Theorem 5.1

Let A, B be two subsets of a topological space (X, τ) . Then the following are hold:

- (1) $\beta^* - int(X) = X$ and $\beta^* - int(\phi) = \phi$,
- (2) $\beta^* - int(A) \subseteq A$,
- (3) If $A \subseteq B$, then $\beta^* - int(A) \subseteq \beta^* - int(B)$,
- (4) $x \in \beta^* - int(A)$ if and only if there exist β^* - open W such that $x \in W \subseteq A$,
- (5) A is β^* - open set if and only if $A = \beta^* - int(A)$,
- (6) $\beta^* - int(\beta^* - int(A)) = \beta^* - int(A)$,
- (7) $\beta^* - int(A \cap B) \subseteq \beta^* - int(A) \cap \beta^* - int(B)$,
- (8) $\beta^* - int(A) \cup \beta^* - int(B) \subseteq \beta^* - int(A \cup B)$.

Example 4.5

Let $X = \{a, b, c, d\}$ with topology $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$.

- (1) If $A = \{a, c\}, B = \{b, c\}$, then $\beta^* - cl(A) = A, \beta^* - cl(B) = B$ and $\beta^* - cl(A \cup B) = X$. Thus $\beta^* - cl(A \cup B) \not\subseteq \beta^* - cl(A) \cup \beta^* - cl(B)$,
- (2) If $A = \{a, c\}, C = \{a, b\}$, then $\beta^* - cl(C) = X, \beta^* - cl(A) = A$ and $\beta^* - cl(A \cap C) = \{a\}$. Thus $\beta^* - cl(A) \cap \beta^* - cl(C) \not\subseteq \beta^* - cl(A \cap C)$,
- (3) If $E = \{c, d\}, F = \{b, d\}$, then $E \cup F = \{b, c, d\}$ and hence $\beta^* - int(E) = \phi, \beta^* - int(F) = F$ and $\beta^* - int(E \cup F) = \{b, c, d\}$. Thus $\beta^* - int(E \cup F) \not\subseteq \beta^* - int(E) \cup \beta^* - int(F)$.

Theorem 5.2

For a subset A in a topological space (X, τ) , the following statements are true:

- (1) $B^* - cl(X/A) = X \setminus \beta^* - int(A)$,
- (2) $\beta^* - int(X \setminus A) = X / \beta^* - cl(A)$.

Proof.

It follows from the fact the complement of β^* - open set is a β^* - closed

And $\bigcap_i (X / A_i) = X / \bigcup_i A_i$.

Theorem 5.3

Let A be a subset of a topological space (X, τ) . Then the following are

Equivalent to :

- (1) A is a β^* - open set,
- (2) $A = \beta - int(A) \cup pint\delta(A)$

.Proof

(1) \Rightarrow (2). Let A be a β^* - open set. Then $A \subseteq cl(int(cl(A))) \cup int(cl\delta(A))$ and hence by (Lemma 1.1) $A \subseteq (A \cap cl(int(cl(A)))) \cup (A \cap int(cl\delta(A))) = \beta - int(A) \cup pint\delta(A) \subseteq A$, (2) \Rightarrow (1).

Theorem 2.4

For a subset A of space (X, τ) . Then the following are equivalent:

- (1) A is a β^* - closed set,
- (2) $A = \beta - cl(A) \cap pcl\delta(A)$,

Proof

Theorem 5.5

β^* - Continuous function

Example 4.6

Let $X = \{a, b, c, d\}$ with topology $\tau = \{\emptyset, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, c, d\}, X\}$. Then, the function $f: (X, \tau) \rightarrow (X, \tau)$ defined by $f(a) = a, f(b) = f(c) = c$ and $f(d) = d$ is β^* - continuous but it is not β^* - continuous. The function $f: (X, \tau) \rightarrow (X, \tau)$ defined by $f(a) = d, f(b) = a, f(c) = c$ and $f(d) = b$ is e^* - continuous but it is not β^* - continuous .

Example 4.7

Let $X = \{a, b, c, d, e\}$ with topology $\tau = \{\emptyset, \{a, b\}, \{c, d\}, \{a, b, c, d\}, X\}$. Then function $f: (X, \tau) \rightarrow (X, \tau)$ which defined by $f(a) = a, f(b) = e, f(c) = c, f(d) = d$ and $f(e) = b$ is β^* - continuous but it is not Z^* - continuous.

Theorem 5.5

Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a function. Then the following statements are

Equivalent :

- (1) f is β^* - continuous,
- (2) For each $x \in X$ and $V \in \sigma$ containing $f(x)$, there exists $U \in \beta^* - O(X)$
- (3) containing x such that $f(U) \subseteq V$,
The inverse image of each closed set in Y is β^* - closed in X ,
- (4) $int(cl(int(f^{-1}(B)))) \cap cl(int\delta(f^{-1}(B))) \subseteq f^{-1}(cl(B))$, for each $B \subseteq Y$,
- (5) $f^{-1}(int(B)) \subseteq cl(int(cl(f^{-1}(B)))) \cup int(cl\delta(f^{-1}(B)))$, for each $B \subseteq Y$,

- (6) $\beta^* - cl(f - I(B)) \subseteq f - I(cl(B))$, for each $B \subseteq Y$,
- (7) $f(\beta^* - cl(A)) \subseteq cl(f(A))$, for each $A \subseteq X$,
- (8) $f - I(int(B)) \subseteq \beta^* - int(f - I(B))$, for each $B \subseteq Y$.

Proof

(1) \Leftrightarrow (2) and (1) \Leftrightarrow (3) are obvious,

(3) \Rightarrow (4). Let $B \subseteq Y$. Then by (3) $f - I(cl(B))$ is β^* -closed. This

means $f - I(cl(B)) \supseteq int(cl(int(f - I(cl(B)))) \cap cl(int(\delta(f - I(cl(B)))) \supseteq int(cl(int(f - I(B)))) \cap cl(int(\delta(f - I(B))))$,

(4) \Rightarrow (5). By replacing Y/B instead of B in (4), we have

$Int(cl(int(f - I(Y/B)))) \cap cl(int(\delta(f - I(Y/B)))) \subseteq f - I(cl(Y/B))$, and

therefore $f - I(int(B)) \subseteq cl(int(cl(f - I(B)))) \cup int(cl(\delta(f - I(B))))$, for each $B \subseteq Y$,

(5) \Rightarrow (1). Obvious,

(3) \Rightarrow (6). Let $B \subseteq Y$ and $f - I(cl(B))$ be β^* -closed in X . Then

$\beta^* - cl(f - I(B)) \subseteq \beta^* - cl(f - I(cl(B))) = f - I(cl(B))$,

(7). Let $A \subseteq X$. Then $f(A) \subseteq Y$. By (6), we have $f - I(cl(f(A))) \supseteq \beta^* -$

$cl(f - I(f(A))) \supseteq \beta^* - cl(A)$. Therefore, $cl(f(A)) \supseteq f - I(cl(f(A))) \supseteq f(\beta^* - cl(A))$,

(7) \Rightarrow (3). Let $F \subseteq Y$ be a closed set. Then, $f - I(F) = f - I(cl(F))$. Hence by

(7), $f(\beta^* - cl(f - I(F))) \subseteq cl(f - I(F)) \subseteq (F) = F$, thus, $\beta^* - cl(f - I(F)) \subseteq f - I(F)$,

so, $f - I(F) = \beta^* - cl(f - I(F))$. Therefore, $f - I(F) \in \beta^*C(X)$,

$Int(f - I(int(B))) \subseteq \beta^* - int(f - I(B))$. Therefore, $f - I(int(B)) \subseteq \beta^* - int(f - I(B))$,

(8) \Rightarrow (1). Let $U \subseteq Y$ be an open set. Then $f - I(U) = f - I(int(U)) \subseteq \beta^* - int(f - I(U))$.

Hence, $f - I(U)$ is β^* -open in X . Therefore, f is β^* -continuous.

Remarks 3.3

The composition of two β^* -continuous functions need not be β^* -continuous as show by the following example.

Example 4.7

Let $X = Y = Z = \{a, b, c, d\}$ with topologies $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$

. Let $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ be functions defined

by $f(a) = b, f(b) = b, f(c) = c, f(d) = d$ and $g(a) = a, g(b) = c, g(c) = a, g(d) = d$,

respectively. Then f and g are β^* -continuous but $g \circ f$ is not β^* -continuous.

Theorem 5.6

If $f: (X, \tau) \rightarrow (Y, \sigma)$ is a β^* -continuous function and A is δ -open in X , then the restriction $f|_A: (A, \tau) \rightarrow (Y, \sigma)$ is β^* -continuous.

Proof.

Let V be an open set of Y . Then by hypothesis $f^{-1}(V)$ is β^* -open in X . we have $(f|_A)^{-1}(V) = f^{-1}(V) \cap A$
 $\beta^* \in O(A)$. Thus, it follows that $f|_A$ is β^* -continuous.

Lemma 1.2

Let A and B be two subsets of a space (X, τ) . If $A \in \delta O(X)$ and $B \in \beta^* O(A)$, the $A \cap B \in \beta^* O(X)$.

Theorem 5.7

Let $(X, \tau) \rightarrow (Y, \sigma)$ be a function and $\{G_i: i \in I\}$ be a cover of X by δ -open sets of (X, τ) . If $f|_{G_i}: (G_i, \tau|_{G_i}) \rightarrow (Y, \sigma)$ is β^* -continuous for each $i \in I$, then f is β^* -continuous.

Proof.

Let V be an open set of (Y, σ) . Then by $(f|_V)^{-1}(V) = X \cap (f|_V)^{-1}(V) = \cup \{G_i \cap f^{-1}(V): i \in I\} = \cup \{(f|_{G_i})^{-1}(V): i \in I\}$. Since $f|_{G_i}$ is β^* -

Continuous for each $i \in I$, then $(f|_{G_i})^{-1}(V) \in \beta^* O(G_i)$ for each $i \in I$. we see $(f|_{G_i})^{-1}(V)$ is β^* -continuous in X . Therefore, f is β^* -continuous in (X, τ) .

Definition 2.3 The β^* -s frontier of a subset A of X , denoted by $\beta^* - Fr(A)$, is defined by $\beta^* - Fr(A) = \beta^* - cl(A) \cap \beta^* - Cl(X/A)$ equivalently $\beta^* - Fr(A) = \beta^* - cl(A) / \beta^* - int(A)$

Theorem 5.8

The set of all points x of X at which a function $f: (X, \tau) \rightarrow (Y, \sigma)$ is not β^* -continuous is identical with the union of the β^* -frontiers of the inverse images of open sets containing $f(x)$.

Proof. Necessity. Let x be a point of X at which f is not β^* -continuous. Then, there is an open set V of Y containing $f(x)$ such that $U \cap (Xf^{-1}(V)) \neq \phi$, for every $U \in \beta^*O(X)$ containing x . Thus, we have $x \in \beta^* - cl(Xf^{-1}(V)) = X / \beta^* - int(f^{-1}(V))$ and $x \in f^{-1}(V)$. Therefore, we have $x \in \beta^* - Fr(f^{-1}(V))$. Sufficiency. Suppose that $x \in \beta^* - Fr(f^{-1}(V))$, for some V is open set containing $f(x)$. Now, we assume that f is β^* -continuous at $x \in X$. Then there exists $U \in \beta^*O(X)$ containing x such that $f(U) \subseteq V$. Therefore, we have $x \in U \subseteq f^{-1}(V)$ and hence $x \in \beta^* - int(f^{-1}(V)) \subseteq X / \beta^* - Fr(f^{-1}(V))$. This is a contradiction. This means that f is not β^* -continuous at $x \in V$.

Theorem 5.9

If $f: (X, \tau) \rightarrow (Y, \sigma)$ is a β^* -continuous injection and (Y, σ) is T_i , then (X, τ) is $\beta^* - T_i$, where $i = 0, 1, 2$.

Proof.

We prove that the theorem for $i = 1$. Let Y be T_1 and x, y be distinct points in X . There exist open subsets U, V in Y such that $f(x) \in U, f(y) \notin U, f(x) \notin V$ and $f(y) \in V$. Since f is β^* -continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are β^* -open subsets of X such that $x \in f^{-1}(U), y \notin f^{-1}(U), x \notin f^{-1}(V)$ and $y \in f^{-1}(V)$. Hence, X is $\beta^* - T_1$.

Theorem 5.10

If $f: (X, \tau) \rightarrow (Y, \sigma)$ is β^* -continuous, $g: (X, \tau) \rightarrow (Y, \sigma)$ is super-continuous and Y is Hausdorff, then the set $\{x \in X: f(x) = g(x)\}$ is β^* -closed in X . f is β^* -continuous in (X, τ) .

Proof :

Let $A = \{x \in X: f(x) = g(x)\}$ and $x \notin A$. Then $f(x) \neq g(x)$. Since Y is Hausdorff, there exist open sets U and V of Y such that $f(x) \in U, g(x) \in V$ and $U \cap V = \phi$. Since f is β^* -continuous, there exists a β^* -open set G containing x such that $f(G) \subseteq U$. Since g is super-continuous, there exist an δ -open set H of X containing x such that $g(H) \subseteq V$. Now, put $W = G \cap H$, we have W is a β^* -open set containing x and $f(W) \cap g(W) \subseteq U \cap V = \phi$. Therefore, we obtain $W \cap A = \phi$ and hence $x \in \beta^* - cl(A)$. This shows that A is β^* -closed in X . Let $f: X \rightarrow Y$ be a function. The subset $\{(x, f(x)): x \in X\}$ of the product space $X \times Y$ is called the graph of f and is denoted by $G(f)$.

Explain : Definition 2.4 A function $f: X \rightarrow Y$ has a (β^*, τ) -graph if for each $(x, y) \in (X \times Y) / G(f)$, there exist a β^* -open U of X containing x and an open set V of Y containing y such that $(U \times V) \cap G(f) = \phi$.

Proof: It follows readily from the above definition.

Theorem 5.11

If $f: X \rightarrow Y$ is a β^* -continuous function and Y is Hausdorff, then f has a (β^*, τ) -s-graph.

Proof.

Let $(x, y) \in X \times Y$ such that $y \neq f(x)$. Then there exist open set that $y \in U, f(x) \in V$ and $V \cap U = \phi$. Since f is β^* -continuous, there exists β^* -open W containing x such that $f(W) \subseteq V$. This $W \cap U \subseteq V \cap U = \phi$.

Definition 2.5

A space X is said to be β^* -compact if every β^* -open cover of X has a finite subcover.

Theorem 5.12

If $f: (X, \tau) \rightarrow (Y, \sigma)$ has a (β^*, τ) -graph, then $f(K)$ is closed in (Y, σ) for each subset K which is β^* -compact relative to (X, τ) .

Proof.

Suppose that $y \in f(K)$. Then $(x, y) \in G(f)$ for each $x \in K$. Since $G(f)$ is (β^*, τ) -graph, there exist a β^* -open set U containing x and an open set V of Y containing y such that $f(U) \cap V = \emptyset$. The family $\{U_x : x \in K\}$ is a cover of K by β^* -open sets. Since K is β^* -compact relative to (X, τ) , there exists a finite subset K_0 that $K \subseteq \cup \{U_x : x \in K_0\}$. Let $V = \cap \{V_x : x \in K_0\}$. have $f(K) \cap V \subseteq (\cup_{x \in K_0} f(U_x)) \cap V \subseteq \cup_{x \in K_0} (f(U_x) \cap V) = \emptyset$. It follows that, $y \in cl(f(K))$. Therefore, $f(K)$ is closed in (Y, σ) .

Corollary 6.1

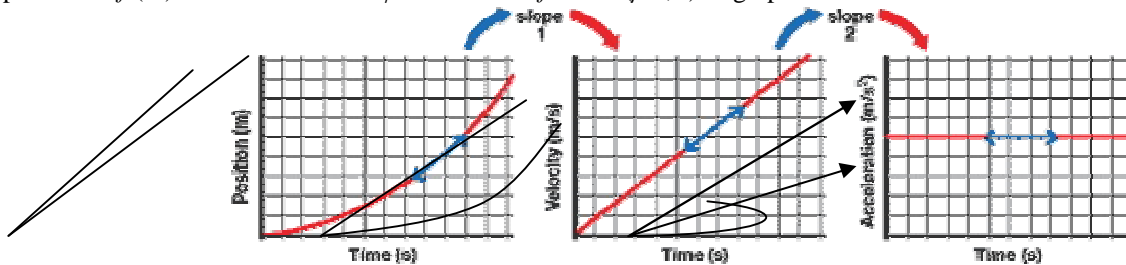
If $f: (X, \tau) \rightarrow (Y, \sigma)$ is β^* -continuous function and Y is Hausdorff, then $f(K)$ is closed in (Y, σ) for each subset K which is β^* -compact relative to (X, τ) .

Theorem 5.13

If $f: X \rightarrow Y$ is a β^* -continuous function and Y is a Hausdorff space, then f has a (β^*, τ) -graph.

Proof

Let $(x, y) \in X \times Y$ such that $y \neq f(x)$ and Y be a Hausdorff space. Then there exist two open sets U and V such that $y \in U, f(x) \in V$ and $V \cap U = \emptyset$. Since f is β^* -continuous, there exists a β^* -open set W containing x such that $f(W) \subseteq V$. This implies that $f(W) \cap U \subseteq V \cap U = \emptyset$. Therefore f has a (β^*, τ) -graph.



Corollary 6.2

If $f: X \rightarrow Y$ is β^* -continuous and Y is Hausdorff, then $G(f)$ is β^* -closed in $X \times Y$.

Theorem 5.14

If $f: X \rightarrow Y$ has a (β^*, τ) -graph and $g: Y \rightarrow Z$ is a β^* -continuous function, then the set $\{(x, y): f(x) = g(y)\}$ is β^* -closed in $X \times Y$.

Proof:

Let $A = \{(x, y): f(x) = g(y)\}$ and $(x, y) \in A$. We have $f(x) = g(y)$ and then $(x, g(y)) \in G(f)$. Since f has a (β^*, τ) -graph, then there exist a β^* -open set U and an open set V containing x and $g(y)$, respectively such that $f(U) \cap V = \emptyset$. Since g is a β^* -continuous function, then there exist an β^* -open set G containing y such that $g(G) \subseteq V$. We have $f(U) \cap g(G) = \emptyset$. This implies that $(U \times G) \cap A = \emptyset$. Since $U \times G$ is β^* -open, then $(x, y) \in \beta^*cl(A)$. Therefore, A is β^* -closed in $X \times Y$.

Corollary 6.3

If $f: X \rightarrow Y, g: Y \rightarrow Z$ are β^* -continuous functions and Z is Hausdorff, then the Set $\{(x, y): f(x) = g(y)\}$ is β^* -closed in $X \times Y$.

Theorem 5.15

If $f: X \rightarrow Y$ is a β^* -continuous function and Y is Hausdorff, then the set $\{(x, y) \in X \times X: f(x) = f(y)\}$ is β^* -closed in $X \times X$.

Proof:

Let $A = \{(x, y): f(x) = f(y)\}$ and let $(x, y) \in (X \times X) \setminus A$. Then $f(x) \neq f(y)$. Since Y is Hausdorff, then there exist open sets U and V containing $f(x)$ and $f(y)$, respectively, such that $U \cap V = \emptyset$. But, f is β^* -continuous, then there exist β^* -open sets H and G in X containing x and y , respectively, such that $f(H) \subseteq U$ and $f(G) \subseteq V$. This implies $(H \times G) \cap A = \emptyset$ we have $H \times G$ is a β^* -open set in $X \times X$ containing (x, y) . Hence, A is β^* -closed in $X \times X$.

Theorem 5.16

If $f: (X, \tau) \rightarrow (Y, \sigma)$ is β^* -continuous and S is closed in $X \times Y$, then $v_x(S \cap G(f))$ is β^* -closed in X , where v_x represents the projection of $X \times Y$ onto X .

Proof.

Let S be a closed subset of $X \times Y$ and $x \in \beta^*\text{-cl}(S)$. Let $U \in \tau$ containing x and $V \in \sigma$ containing $f(x)$. Since f is β^* -continuous, $x \in f^{-1}(V) \subseteq \beta^*\text{-int}(f^{-1}(V))$. Then $U \cap \beta^*\text{-int}(f^{-1}(V)) \cap v_x(S \cap G(f))$ contains some point z of X . This implies that $(z, f(z)) \in S$ and $f(z) \in V$. Thus we have $(U \times V) \cap S \neq \emptyset$ and hence $x \in \beta^*\text{-int}(S)$. Since S is closed, then $x \in v_x(S \cap G(f))$ and $x \in v_x(S \cap G(f))$. Therefore $v_x(S \cap G(f))$ is β^* -closed in (X, τ) .

II. CONCLUSION

A topological space (X, τ) is said to be β^* -connected if it is not the union of two nonempty disjoint β^* -open sets. If (X, τ) is a β^* -connected space and $f: (X, \tau) \rightarrow (Y, \sigma)$ has a (β^*, τ) -graph and β^* -continuous function, the constant. Suppose that f is not constant. There exist disjoint points $x, y \in X$ such that $f(x) \neq f(y)$. Since $(x, f(x)) \in G(f)$, then $y \notin G(f)$, hence by, there exist open sets U and V containing x and $f(x)$ respectively such that $f(U) \cap V = \emptyset$. Since f is β^* -continuous, there exist β^* -open sets G containing y such that $f(G) \subseteq V$. Since U and V are disjoint β^* -open sets of (X, τ) , it follows that (X, τ) is not β^* -connected. Therefore, f is constant. Let $(X_1, \tau_1), (X_2, \tau_2)$ and (X, τ) be topological spaces. Define a function $f: (X, \tau) \rightarrow (X_1 \times X_2, \tau_1 \times \tau_2)$ by $f(x) = (f(x_1), f(x_2))$. Then $f_i: X \rightarrow (X_i, \tau_i)$, where $(i = 1, 2)$ is β^* -continuous if f is β^* -continuous.

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