



# IJRASET

International Journal For Research in  
Applied Science and Engineering Technology



# INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

**Volume:** 11    **Issue:** X    **Month of publication:** October 2023

**DOI:** <https://doi.org/10.22214/ijraset.2023.55962>

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# Identical Product of Graphs

Parvathy Haridas

N S S Hindu College Changanacherry

**Abstract:** A new graph product called identical product is introduced in this paper.

**Keywords:** Graph products, Identical Product

## I. INTRODUCTION

A graph [1] is an ordered triple  $G = (V(G), E(G), I_G)$  where  $V(G)$  is a nonempty set  $E(G)$  is a set disjoint from  $V(G)$  and  $I_G$  is an "incidence" relation that associates with each element of  $E(G)$  an unordered pair of elements (same or distinct) of  $V(G)$ . Elements of  $V(G)$  are called the vertices (or nodes or points) of  $G$ ; and elements of  $E(G)$  are called the edges (or lines) of  $G$ :  $V(G)$  and  $E(G)$  are the vertex set and edge set of  $G$ , respectively. If, for the edge  $e$  of  $G$ ,  $I_G(e) = \{u, v\}$  Number of vertices and the number of edges in a graph  $G$  is called the order  $n(G)$  and the size  $m(G)$  of  $G$  respectively. Number of edges incident on a vertex  $v$  of a graph  $G$  is called degree of  $v$  in  $G$  and is denoted by  $d_G(v)$ . A graph  $G$  is regular if degree of all vertices in  $G$  are equal. Let  $G_1 = (V_1, E_1)$  and  $G_2 = (V_2, E_2)$  be two simple graphs. Any product [1]  $G_1 * G_2$  has its vertex set  $V_1 \times V_2$ . For any two vertices  $(u_1, v_1)$  and  $(u_2, v_2)$  are adjacent in  $G_1 * G_2$ , there are various possibilities:

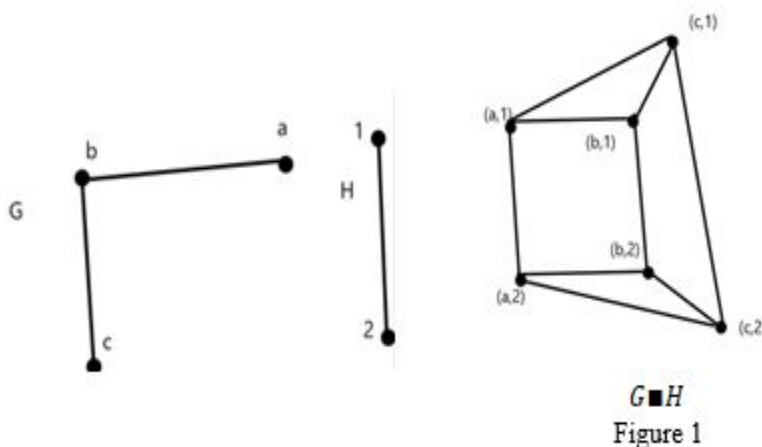
$u_1$  adjacent to  $v_1$  in  $G_1$  or  $u_1$  non-adjacent to  $v_1$  in  $G_1$ ;  $u_2$  adjacent to  $v_2$  in  $G_2$  or  $u_2$  non-adjacent to  $v_2$  in  $G_2$  and  $u_1 = u_2$  and/or  $v_1 = v_2$ . Two graph products

## II. IDENTICAL PRODUCT

### 1) Definition

Let  $G_1 = (V_1, E_1)$  and  $G_2 = (V_2, E_2)$  be two simple graphs. The identical product  $G_1 \blacksquare G_2$  has its vertex set  $V_1 \times V_2$ . Any two vertices  $(u_1, v_1)$  and  $(u_2, v_2)$  are adjacent in  $G_1 \blacksquare G_2$  if and only if  $u_1 = u_2$  or  $v_1 = v_2$ .

Example:



### 2) Theorem

The identical product of any two graphs  $G$  and  $H$  with  $n_1$  and  $n_2$  vertices respectively

**Proof:** From the definition of the identical product, it is clear that the adjacency of two vertices in  $G \blacksquare H$  will not depend on the adjacency of vertices in  $G$  or  $H$  (since  $(u_1, v_1)$  and  $(u_2, v_2)$  are adjacent in  $G_1 \blacksquare G_2$  if and only if  $u_1 = u_2$  or  $v_1 = v_2$ ). Hence the theorem.

3) *Theorem*

The number of edges in the identical product of any two graphs G and H with  $n_1$  and  $n_2$  vertices respectively is  $\frac{n_1 n_2 (n_1 + n_2 - 2)}{2}$

*Proof:* Let u be any vertex in graph G. Then there are  $n_2$  vertices in  $G \blacksquare H$  in the form (u,x) where x is any vertex in H and these vertices are adjacent to each other. Therefore, there are  $\frac{n_1 n_2 (n_2 - 1)}{2}$  edges in this case. Also if v be any vertex in graph H, there are  $n_1$  of the form (x,v) where y be any vertex in G and these vertices are adjacent to each other. Therefore, there are  $\frac{n_1 n_2 (n_1 - 1)}{2}$  edges in this case.

Hence the total number of edges in  $G \blacksquare H = \frac{n_1 n_2 (n_2 - 1)}{2} + \frac{n_1 n_2 (n_1 - 1)}{2} = \frac{n_1 n_2 (n_1 + n_2 - 2)}{2}$ .

4) *Theorem*

Identical product of any two graphs is regular

*Proof:* Let G and H be two graphs with  $n_1$  and  $n_2$  vertices respectively. Let (u,v) be any vertex in  $G \blacksquare H$

$d(u, v) = n_2 - 1 + n_1 - 1 = n_2 + n_1 - 2$

Hence identical product is regular.

**III. CONCLUSIONS**

In this paper the identical product of two graphs is defined and proved some results relating to this

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