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# Inelastic Behaviour and Strength of Steel Beam-Columns with Applied Torsion

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**Abstract:** *This dissertation presents the outcome of an experimental and theoretical study of the inelastic behavior and strength of steel beam-columns with applied torsion. Although the international steel design specifications contain interaction relations for biaxially loaded beam-columns, the influence of applied torsion on such members has been completely ignored in the past. A series of hollow square section steel members are tested using an apparatus specially designed to apply torsion in the presence of an axial load and biaxial bending. The theoretical analysis involves formulation of a system of materially nonlinear differential equations and their solution based on a finite integral formulation. The predicted member response agreed quite well with the experiments. A set of new yield limit and strength interaction expressions are then developed which include the influence of applied torsion. In addition, beam-column interaction relations used in Australia, Canada, China, Great Britain, Japan, Russia, the U.S. and Eurocode are modified to account for applied torsion. Finally, this dissertation presents new load-moment-torsion interaction relations for possible adoption in the international steel design specifications.*

## I. INTRODUCTION

### A. Introduction

This dissertation presents the outcome of a theoretical and experimental study into the inelastic behavior of steel beam-columns subjected to nonproportional biaxial bending and torsion. The study is focused on nonsway thin-walled steel members. The study of such members is complicated due to the interaction amongst biaxial bending, axial load, and torsion in the materially nonlinear or inelastic range. Symmetric steel buildings subjected to asymmetric loading or unsymmetric steel buildings under various practical loading conditions have their members subjected to such combined loads. Combined eccentric traffic loading and lateral loads due to wind and earthquake effects on bridges can also produce biaxial bending and torsion effects on bridge structural members.

The inelastic behavior of biaxially loaded beam-columns with applied torsion is governed by a system of four coupled materially nonlinear or inelastic ordinary differential equations of equilibrium. Closed-form solutions for these equations are not possible. The four equations can, however, be reduced to three by utilizing an expression for inelastic axial strain based on the differential equation for axial load equilibrium. In this dissertation, an iterative finite integral solution to the resulting three nonlinear differential equations is developed and programmed. To verify the validity of the theoretical predictions, laboratory tests are conducted on hollow rectangular section beam-columns under applied torsion. For the experimental study, special arrangements are made to the apparatus used by Zhao (13) allowing freedom of torsional rotation about the longitudinal axis at the test member bottom end. Although hollow rectangular steel sections are torsionally quite stiff, they can still be expected to carry torsional moments in the presence of axial load and uniaxial or biaxial bending moments. After validating the analysis, predictions are then made for the inelastic behavior and strength of steel members with larger or practical sizes under the combined action of axial compressive load, biaxial bending moments and torsional moment. The results generated in the study are first used to formulate a set of new yield limit interaction expressions. Finally, a new dimensionless torsional moment term is added to the ultimate strength interaction expressions present in the design specifications from Australia, Canada, China, Japan, Great Britain, and Russia as well as those in the American Institute of Steel Construction Manual and Eurocode

### B. Literature Review

#### 1) Beam-Column Studies with Nonproportional Loading

A number of researchers have previously studied the behavior of steel beam-columns without any applied torsion. Ketter (1) conducted a theoretical study beyond the elastic limit of structural members subjected to a constant load with increasing moment. The author used the finite integral approach intertwined with a cross-sectional elasto-plastic tangent stiffness procedure in the analysis.

Galambos and Ketter (2) formulated Newmark's procedure to determine load-deflection curves of wide-flange columns under combined bending and thrust. The theoretical solutions given in these investigations are also compared to tests conducted by Mason, Fisher, and Winter (3).

Milner (4) conducted both theoretical and experimental studies on biaxial columns under nonproportional loading. The author used iterative techniques such as the finite difference approach and a numerical integration procedure to solve the governing differential equations. Razzaq (5) and Marshall and Ellis (6) conducted an experimental and theoretical study on elastic-plastic behavior of thin-walled members with box sections subjected to biaxial bending and compression. Test results were in good agreement with the theoretical predictions.

Sakda (8) conducted an analysis of biaxially loaded columns. The limit theorems and tangent stiffness method were used to analyze the elastic-plastic behavior of the crosssection in order to obtain interaction equations. Razzaq and McVinnie (9, 10) conducted an experimental and theoretical study on hollow rectangular section nonsway beamcolumns subjected to nonproportional loading. Test results revealed that twisting may be neglected for columns of hollow square or rectangular sections.

Darbhamulla (11) performed an experimental and theoretical study of the inelastic stability of nonproportionally loaded imperfect rectangular tubular nonsway steel beam-columns. The authors presented a new set of inelastic slope-deflection equations which were derived from a system of nonlinear ordinary differential equations. Eidan (12) performed an experimental and theoretical comprehensive study of the inelastic stability of sway steel beam-columns subjected to nonproportional loads. The authors formulated a set of materially nonlinear differential equations of equilibrium for planar and biaxially loaded beam-columns including sidesway. Zhao (13) conducted an experimental and theoretical investigation on thermo-elasto-plastic behavior of biaxially loaded steel beamcolumns including those from the World Trade Center towers. Thermo-elasto-plastic stiffness degradation and load-moment interaction curves were formulated for typical WTC beam-columns that were in the impacted area during the 9/11 attacks.

## 2) Structural Member Studies Including Torque

Many studies have been conducted on steel members under pure torsion. Donnell (18) conducted a theoretical and experimental study on the stability of thin-walled sections under torsion. The author has developed a theoretical solution for round, thin-walled tubes for which the walls become unstable under torsion. The results of the theory were compared to the experimental results. The experimental failure torque was found to be lower than the theoretical predicted buckling torque. Burgoyne and Brown (19) performed a theoretical study of nonlinear uniform torsion of prismatic members. The authors have introduced two methods for solving the governing differential equations. Chen (20) presented a simple approximate and experimental method to evaluate the increase of torsional rigidity of thin prismatic bars and tubular members due to the presence of initial twist. Test data observed were used to verify the analysis.

Ellis, Jury and Davies (21) conducted theoretical and experimental studies on torsional behavior of rectangular hollow sections. Marshall's simplified thick wall torsion theory (22) and finite element (FE) analysis were used to predict the observed torque-twist behavior in the investigation. Test results observed agreed with the predicted values in the elastic range. However, in the inelastic range test results appeared to be significantly lower. Carey (23) conducted an experimental and theoretical study of plastic strength in torsion of prismatic bars with a concentric or eccentric hole using the sand-heap analogy. The respective circular, square, and rectangular cross sections were tested. It was concluded that the circular shaft with an interior hole had the maximum strength.

Previous studies have been published on structural steel members subjected to combined uniaxial bending moment and torsion. Abramyan (26) conducted a theory for prismatic rods of hollow rectangular sections subjected to bending and torsion. Numerical technique was used to solve the linear differential equations. Estabrooks and Grondin (27) performed an experimental and theoretical study of steel I-Shaped beams subjected to combined bending and torsion. It was concluded that many design methods did not consider the destabilizing effects of torsion, warping stresses, nor biaxial bending in the inelastic range. Nuttall and Gaydon (28) conducted a theoretical study on combined bending and twisting of beams of various sections. Upper and lower approximations were used to obtain the interaction curves for cylinders, rectangles, I-shaped and box sections.

Ishikawa (29) conducted an elasto-plastic stress analysis of a prismatic bar under combined bending and torsion. In the study, an analytical method was formulated for a compressible isotropic work hardening material exhibiting a nonlinear stress strain law. Wiener and Bathe (30) conducted a study on elastic-plastic analysis of I-beams in bending and torsion. It was concluded that the I-beam model studied in this paper could be used for general analysis. Kyungsik and Chai (31) studied the ultimate strengths of steel rectangular box beams subjected to the combined action of bending and torsion. A commercial finite element analysis program (ABAQUS) was used to verify the proposed predictor ultimate strength interaction equations.



Hill and Siebel (32) conducted a theoretical study on the combined bending and twisting of thin tubes in the plastic range, von Mises yield criterion and Reuss stress-strain relations were used in the analysis. Equations were formulated to determine the stresses and warping in a partially plastic thin tube of arbitrary cross-section. Hill and Siebel (33) conducted an experimental and theoretical investigation of plastic distortion of solid bars by combined bending and twisting. Upper and lower approximate values were used to obtain the fully-plastic state of a plastic-rigid material. However, test results were not sufficient enough to show which approximation was better.

Imegwu (34) studied, theoretically, the plastic flexure and torsion of prismatic beams loaded by terminal bending and twisting moments. The interaction curve was obtained and compared with the lower bound curve given by Hill and Siebel, and the difference between the two curves was outlined. Onat and Shield (35) investigated the influence of the loading program on the agreement between the predictions of the Hencky and the Prandtl-Reuss stress-strain relations for a perfectly plastic material in the case of the combined bending and twisting of thin-walled tubes. Good agreement was found in the results between the developed approximate method "composite" beam and those obtained from the incremental approach.

Boulton (36) conducted an experimental and theoretical study of plastic twisting and bending of an I-section in which the warp is restricted. The lower bound theory and Tresca failure criterion were used to obtain the full-plastic twisting and bending of an I-section in which warping of the cross-section is prevented at the ends. However, close agreement could not be found between the predicted and test values. Dinno and Merchant (37) outlined a procedure for calculating the plastic collapse of I-sections under bending and torsion. Experiments showed the introduced method to be safe and suitable for the design of I-section members under torsional strength with warping restraint and on the combined action of bending and torsion.

A number of studies on the behavior of structural members subjected to combined action of biaxial bending moments and torsional moment have been published. Razzaq and Galambos (38) presented a theoretical and experimental study of biaxial bending of a beam with or without torsion. The semi-analytic method was found to be the most efficient one among the six techniques considered for solving the three simultaneous governing differential equations for bending and torsion. Trahair and Bild (39) conducted a theoretical investigation of elastic biaxial bending and torsion of thin-walled open section members. It was concluded that the classical equations developed by Timoshenko, Vlasov and others were too complex for solution by hand; therefore, numerical methods such as a finite integral were used. Strelbytska and Evseyenko (40) conducted both theoretically and experimentally an investigation of cantilever bars in biaxial bending with torsion of thinwalled bars beyond the elastic limit. Test results were in good agreement with the theoretical predictions.

Shugyo and Li (41) presented an inelastic and stability analysis of linearly tapered box columns with a hollow square section under biaxial bending and torsion. Using extended Home's stability criterion, the ultimate strength of the columns was obtained. The numerical results were presented in the form of interaction diagrams. Hodge and Sankaranarayanan (42) presented a theoretical study for the determination of safe loads of beams subjected to combined twisting and biaxial bending moments. The lower-bound theorem of limit analysis was used to obtain the yield criterion in terms of the stress resultants for a beam under combined twisting and biaxial moments. Iwegwu (43) conducted a theoretical study of a uniform prismatic beam of square section under combined plastic biaxial bending and torsion. Numerical solution of the second-order nonlinear differential equation derived by Hill for a Levy-Mises material was used. Results showed that the moments from the interaction of bending moments and a twisting moment were found to deviate by a small amount.

The behavior of structural members under combined axial and torsional loading have been previously published. Avitzur and Pan (44) provided an upper bound technique concept to analyze a cylindrical bar under combined axial force and torsion actions. The solution from this analysis coincided with the older existing classic solution for combined loading. Mahendran and Murray (45) conducted a theoretical and experimental study on the ultimate load behavior of box-columns under combined axial and torsional loads. The theoretical results were found to be in good agreement with test results. Wang, Nie and Fan (46) analyzed the mechanical behavior of CFST columns subjected to combined axial force and torsion. A simplified formula was proposed based on the regression method. Theoretical results were found to be in good agreement with the proposed method.

Sved and Brooks (47) investigated, theoretically, the elasto-plastic behavior of a round bar subjected to axial force and torque. Numerical results showed that the effect of Poisson's ratio is relatively small. Feign (48) conducted experimental and theoretical studies on thin-walled tubes under combined tension-torsion. Test results fall between the predictions of the flow and deformation theory. Mii (49) conducted a theoretical investigation of plastic deformation of light-metal bars strained with combined tension and torsion. The deformation of the material in the strain-hardening region was assumed to obey the maximum shear stress theory in the study. Ono (50) analyzed the stress and strain in metals undergoing a plastic flow-tube wall subjected to axial pull and torque. The general procedure of analysis was outlined and demonstrated through numerical examples.

A very few studies have been conducted on steel members subjected to combined action of an axial compressive load, uniaxial or biaxial bending and torsion. Kitada and Nakai (51) conducted an experimental study on ultimate strength of thin-walled box stub or short columns subjected to the combined action of torsion in addition to axial load and uniaxial bending. Test results were compared with a computer program developed in another study but without including second-order effects. It was found that the global axial, flexural and torsional stiffnesses of the box column specimen up to the ultimate state were not affected when local buckling in a plate element occurred. Timoshenko (52) developed a theory for combined axial load, bending, and torsion for a thin-walled member; however, it is applicable only in the elastic range for open cross sections. Trahair and Pi (53) studied the behavior, analysis and design of compressed members under the combined actions of torsion and bending, but their research was also limited to the elastic range. Interaction equations were formulated and could be used for design purposes with this type of loading, assuming that the member will remain elastic.

Bleich and Bleich (54) conducted a theoretical study of bending, torsion and buckling of bars composed of thin walls in the elastic range only. Goodier (55) conducted an elastic theoretical investigation of flexural-torsional buckling of bars of open sections, under bending, eccentric thrust and induced torsional effects. Galambos (56) conducted a theoretical and experimental study on failure of steel wide-flange columns subjected to an axial force, bending moments in the plane of their strong axis, and induced torsional effects. It was concluded that at a certain critical load, these columns deflect in their weak direction accompanied by twisting about the shear center.

To the best of the author's knowledge, no experimental and theoretical study has previously been published on the inelastic behavior of steel beam-columns subjected to nonproportional biaxial bending in the presence of applied torsion.

### C. Problem Definition

The main problem addressed in this dissertation is to develop an inelastic theoretical behavior prediction model for biaxially loaded nonsway beam-columns with applied torsion and verify it experimentally. Figure 1 shows an imperfect steel member BT with  $z$  as its longitudinal axis, subjected to a concentrated torsional moment  $M_z$  applied at the bottom end, an axial compressive or thrust load  $P$ , and bending moments  $M_{Bx}$ ,  $M_{By}$ ,  $M_{Tx}$ , and  $M_{Ty}$ . Subscripts B and T refer to the column bottom and top ends. The boundaries of the member have partial rotational end restraints about the  $x$  and  $y$  axes. The initial geometric imperfection is taken in the form of half-sine wave functions  $i_q$  and  $v_L$  in the  $xz$  and  $yz$  planes as shown in Figure 1 and Figure 2. The total displacement  $U$  shown in Figure 2 is the sum of  $u$  and  $u_t$  where  $u$  is the displacement due to the applied loads. Similarly,  $V$  is the sum of  $v$  and  $v_t$ , where  $v$  is the displacement owing to the applied loads. Figure 2 also shows the angle of twist,  $v'/L$ , at any location  $z$  along the member's length.

Two types of cross sections are considered in the theoretical analysis, namely, a hollow rectangular section and an I-shaped section. The cross sections possess initial residual stresses due to manufacturing processes. The theoretical challenge is to predict the behavior and strength of the member as governed by a system of four simultaneous nonlinear ordinary differential equations based on an iterative finite integral solution scheme. The experimental part of the problem involves the development of a new torsional testing method in the presence of both biaxial bending and axial compression and comparing hollow square section member behavior with that predicted theoretically. The last portion of the problem is to develop new yield limit interaction relations and to modify ultimate strength interaction expressions from eight major regions or countries of the world so that the effect of applied torsion is accounted for.

### D. Objectives and Scope

The principal objectives of the research embodied in this dissertation are as follows:

- 1) Incorporate the influence of applied torsion into the governing nonlinear differential equations for the problem including both biaxial bending and axial thrust.
- 2) Develop and program an iterative nonlinear algorithm based on a finite integral solution for the governing differential equations.
- 3) Develop a torsional testing method and test assembly that can be incorporated into a previously developed biaxial bending apparatus.
- 4) Conduct a series of tests on hollow square section members with an axial thrust, uniaxial or biaxial bending, and torsion to understand the basic behavior as well as to validate the theoretical solutions.
- 5) Develop new yield limit interaction relations for members with both hollow rectangular and I-shaped cross sections. 6. Incorporate a new dimensionless torsional moment term into biaxial bending and axial thrust interaction expressions from the following eight major regions of the world:

- a) Australia (AS4100 Specification)
- b) Japan (AU Specification)
- c) U.S. (AISC-LFRD Specification)
- d) Great Britain (BS 5950 Specification)
- e) Canada (CSA Specification)
- f) Europe (Eurocode 3)
- g) China (GBJ Specification)
- h) Russia (SNiP Specification)

A total of 60 steel member tests are conducted on hollow square sections with 111 3 dimensions 1 -x 1-x -x 3 7 - in. The majority of experiments are conducted with 2 2 8 4 J J r pinned boundary conditions. However, some tests included a combination of pinned and fixed boundary conditions. Nevertheless, the theory utilizes partial rotational end restraints about the x and y axes.

#### E. Assumptions and Conditions

The following are the main assumptions and conditions adopted in the study presented in this dissertation:

- 1) All external loads are applied at the ends of the structural member in a quasi-static manner up to its ultimate strength.
- 2) Small deflection theory is adopted.
- 3) The axial force is applied to the centroid of the cross section and retains this position until the member load-carrying capacity is reached.
- 4) Local buckling is not included.
- 5) The member material follows elastic-perfectly-plastic normal and shear stress-strain relationships.
- 6) The stress-strain relationships have for compression and tension the same shape.
- 7) A concentrated torsional moment is applied only at the bottom end of the member.
- 8) The influence of material unloading is ignored.

## II. CONCLUSIONS

Based on the results from the experimental and theoretical study of the inelastic behavior and strength of steel beam-columns with applied torsion presented in this dissertation, the main conclusions are summarized as follows.

- 1) For the experimental study requiring the application of a torsional moment, a new torsion application apparatus is developed and worked well when integrated into a biaxial bending test setup.
- 2) The experimental behavior of steel beam-columns with applied torsion is in good agreement with the predicted behavior based on the finite integral solution of the materially nonlinear governing differential equations.
- 3) The presence of axial load and bending moments results in a substantial reduction of member torsional capacity at yield limit as well as the ultimate member strength.
- 4) The member initial crookedness and residual stresses significantly decrease the torsional moment capacity at yield limit as well as the ultimate strength in the presence of axial load and bending moments.
- 5) Flexural partial end restraints decrease the torsional moment capacity at yield limit for the beam-columns analyzed.
- 6) A member fully plastified in torsion can still carry a significant axial load as well as bending moment.
- 7) Based on both experimental and theoretical results, a new load-moment-torsion yield limit interaction expression is developed.
- 8) Using the materially nonlinear analysis, a new load-moment-torsion ultimate strength interaction is formulated.
- 9) Modifications are developed for the beam-column interaction expressions from Australia, Canada, China, Great Britain, Japan, Russia, U.S., and those in Eurocode to include the effect of applied torsion. Acknowledgment

## REFERENCES

- [1] Ketter, R. C., "Stability of Beam-Columns above the Elastic Limit", Proc. ASCE, Vol.81, No.692, October 1955.
- [2] Galambos, T.V. and R.L. Ketter., "Columns Under Combined Bending and Thrust", Proceedings. ASCE, Vol. 85, EM2, 1959.
- [3] Mason, R. E., G.P. Fisher and G. Winter., "Eccentrically Loaded Hinged Steel Columns", ASCE, Vol. 84, EM4, 1958.
- [4] Milner, H. R., "The Elastic Plastic Stability of Stanchions Bent about Two Axes", Ph.D. Dissertation, Imperial College, University of London, December 1965.
- [5] Razaq, Z., "Theoretical and Experimental Investigation of Biaxially Loaded Rectangular Tubular Columns", M.S. Thesis, University of Windsor, Windsor, Ontario, Canada, 1968.
- [6] Marshall, P. J. and Ellis, J. S., "The Ultimate capacity of Biaxially Loaded Box Steel Columns", The Royal Military College of Canada, Kingston, Ontario, Canada, February 1969.

- [7] Syal, I. C. and Sharma, S. S., "Elastic Behavior of Biaxially Loaded Steel Columns", *Journal of Structural Division, ASCE*, Vol.96, No. ST3, Proc Paper 7143, March 1970, pp. 469-486
- [8] Sakda, S. and Chen W.F., "Analysis of Biaxially Loaded Columns", Fritz Engineering Laboratory Report No.331.12, September 1970.
- [9] Razzaq, Z., and McVinnie, W. W., "Rectangular Tubular Steel Columns Loaded Biaxially," *Journal of Structural Mechanics*, Vol, X, No. 4, 1982.
- [10] Razzaq, Z., and McVinnie, W. W., "Theoretical and Experimental Behavior of Biaxially Loaded Inelastic Columns," *Journal of Structural Mechanics*, Vol. 14, No. 3, March, 1986.
- [11] Darbhamalla, S. P., "Nonproportionally Loaded Steel Beam-Column and Flexibly Connected Nonsway Frames," Ph.D. Dissertation, Old Dominion University, May, 1990.
- [12] Eidan, S. A., "Inelastic Stability of Nonproportionally Loaded Steel Sway Beam Columns and Space Frames," Ph.D. Dissertation, Old Dominion University, 1992.
- [13] Zhao, Yanhong., "Thermo-Elasto-Plastic Behavior of Biaxially Loaded Steel Beam Columns Including Those From World Trade Center", Ph.D. Dissertation, Old Dominion University, 2013.
- [14] Saint-Venant, M. "Memoire sur la Torsion des prismes, *Memoires des Savants Etrangers*, XIV, 1855, pp. 233-560.
- [15] A. Nadai., "Z. Angew", *Math Mechanik* 3, 1923, pp. 442.
- [16] Timoshenko, S. P., "Einige Stabilitaetsprobleme der Elastizitaestheorie", In *Collected Papers of Stephen P. Timoshenko*, McGraw-Hill, New York, 1953, pp. 1-50.
- [17] Timoshenko, S. P., "Sur la Stabilité des Systemes Elastiques", In *Collected Papers of Stephen P. Timoshenko*, McGraw-Hill, New York, 1953, pp.92-224.
- [18] Donnell, L.H., "Stability of Thin-Walled Tubes under Torsion," *Naca-Report*, No.479, 1935.
- [18] Burgoyne, C.J. and Brown, E.H., "Nonuniform Elastic Torsion", *Int.J.Mech.Sci*, Vol.36, No. 1, 1994, pp. 23-38
- [19] Chen, Chu., "The effect of initial twist on the torsional rigidity of thin prismatical bars and tubular members", *US National Congress of Applied Mechanics*, Vol 1, 1951, pp.265- 269.
- [20] Ellis, J. S., Jury, E. J. and Kirk, D.W., "Torsional Behavior of Rectangular Hollow Sections," *Journal of Constructional Steel Research*, No.59, 2003, pp. 641-663.
- [21] Marshall, J., "Derivation of Torsional Formulas for Multiply Connected Thick-Walled Rectangular Sections", *Journal of Applied Mechanics*, Vol.2, No.37, 1970, pp. 399-402.
- [22] Carey, G.J.JR., "An investigation of plastic torsion in circular prismatic bars with a concentric or eccentric hole using the sand-heap analogy," Master. Thesis, Graduate School of the Agriculture and Mechanical College of Texas, May 1963.
- [23] Vlasov, V. Z., "Thin-Walled Elastic Beams", 2nd ed., Israel Program for Scientific Translation, Jerusalem, 1961.
- [24] Wagner, H., "Verdrehung und Knickung von offenen Profilen (Torsion and buckling of open sections)", *NACA TM 807*, 1936.
- [25] Abramyan, B.L., "Torsion and Bending of Prismatic Rods of Hollow Rectangular Sections", *National Advisory Committee for Aeronautics*, TM 1319, November 1951.
- [26] Estabrooks, B. G. and Grondin, G. Y., "Combined Bending and Torsion of Steel I Shaped Beams", *Structural Engineering Report No.276*, January 2008.
- [27] Nuttall, H., and Gaydon, F.A., "On the Combined Bending and Twisting of Beams of Various Sections", *Journal of the Mechanics and Physics of Solids*, Vol 6, 1957, pp. 17-26.
- [28] Ishikawa, H., "Elasto-Plastic Stress Analysis of Prismatic Bar under Combined Bending and Torsion", *ZAMM*, Vol.53, No. 1, 1973, pp. 17-30.
- [29] Wiener, P.M., and Bathe, K.J., "On Elastic-Plastic Analysis of I-Beams in Bending and Torsion," *Computers and Structures*, Vol. 17, No. 5-6, 1983, pp. 711-718.





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