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# Integer Solutions on Ternary Quadratic Diophantine Equation $3(x^2 + y^2) - 5xy = 36z^2$

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**Abstract:** The non-zero distinct integer solution to the Quadratic Diophantine equation with three unknowns are examined. We obtain the various patterns of integer points that satisfy the ternary Quadratic equation. A few interesting relationship between the solution and the unique pattern.

**Keywords:** Integral solutions, Ternary Quadratic equation, Polygonal number

## I. INTRODUCTION

Number theory is a type of mathematics concerned with the properties and relationship of number, specifically integers. Number theory has a long history and has made significant contributions to many fields of mathematics. A Diophantine equation with three variables is called a Ternary Diophantine equation. Find an integer solution for x, y, z such that the given equation is satisfied in order to solve Ternary Quadratic Diophantine Equation. In mathematics a Diophantine equation is an equation, typically a polynomial equation in two or more unknown with integer coefficients for which only integer solution are of interest. The Diophantine equations provide an infinite fields for study [1-3]. In particular, when solving Quadratic equation involving three unknowns, reference [4-10]. This correspondence belongs to an addition exciting equation, which is  $3(x^2 + y^2) - 5xy = 36z^2$ , which is a homogeneous quadratic equation with three unknown. The Objective is to determine the infinite number of non-zero integral points of this equation. A few amazing relationship between the solution are additionally shown.

## II. NOTATION

$$T_{m,n} = n \left[ 1 + \frac{(n-1)(m-2)}{2} \right] = \text{Polygonal number with rank } n \text{ and side } m.$$

$$O_n = \frac{(2n^3 + n)}{3} = \text{Octahedral number of rank } n.$$

$$4DF_n = \frac{n^2(n^2 - 1)}{12} = \text{Four dimensional figurate whose generating polygonal is a square.}$$

$$SO_n = n(2n^2 - 1) = \text{Stella octangula number of rank } n.$$

## III. METHOD OF ANALYSIS

To find the non-zero integral solution of the ternary quadratic Diophantine equation

$$3(x^2 + y^2) - 5xy = 36z^2 \tag{1}$$

The replacement of linear transformations

$$x = p + q \quad \text{and} \quad y = p - q \tag{2}$$

As (1) result in

$$p^2 + 11q^2 = 36z^2 \tag{3}$$

**A. Pattern 1**

consider,  $z = z(a, b) = a^2 + 11b^2$  (4)

When the integer a and b are non-zero

$$36 = (5 + i\sqrt{11})(5 - i\sqrt{11})$$
 (5)

By applying factorization techniques and the equation (4) and (5) in (3)

$$(p + i\sqrt{11}q)(p - i\sqrt{11}q) = (5 + i\sqrt{11})(5 - i\sqrt{11})(a + i\sqrt{11}b)^2(a - i\sqrt{11}b)^2$$
 (6)

By comparing the real and imaginary components of the equation and equating the like term, we obtain

$$p = 5a^2 - 55b^2 - 22b$$

$$q = a^2 - 11b^2 + 10ab$$

Equation (2) can be solved as an integer by substituting the above values for p and q

$$x = x(a, b) = 6a^2 - 66b^2 - 12ab$$

$$y = y(a, b) = 4a^2 - 44b^2 - 32ab$$

$$z = z(a, b) = a^2 + 11b^2$$

*Observation*

1.  $y(a, a) - 4x(2a, 2a) = 216a^2$  Nasty number.
2.  $x(1, 1) + y(1, 1) = -144$  Perfect square.
3.  $x(a, a) + y(a, a) + 12T_{26,a} + 66Gno_a \equiv 0 \pmod{66}$
4.  $y(a, a) + 8T_{20,a} + 32Gno_a \equiv 0 \pmod{32}$
5.  $2x(a, a) + 12T_{24,a} + 60Gno_a \equiv 0 \pmod{60}$
6.  $x(2a, 1) + y(a, 1) - 4T_{16,a} + 16Gno_a \equiv 0 \pmod{126}$

**B. Pattern 2**

Equation (3) is written as,

$$p^2 - 36z^2 = -11q^2$$
 (7)

compute equation (7)

$$(p + 6z)(p - 6z) = q \times -11q$$
 (8)

Equation (8) is expressed as a ratio as

$$\frac{(p + 6z)}{-11q} = \frac{q}{(p - 6z)} = \frac{a}{b}$$

The two equations that follow are identical to this

$$bp + 11aq + 6bz = 0$$
 (9)

$$ap - bq - 6az = 0$$
 (10)

When we apply cross-multiplication, we obtain

$$p = 66a^2 - 6b^2$$

$$q = 11a^2 + b^2$$

$$z = -6ab - 6ab$$

The equivalent integer answer (1) can be obtained by substituting the two variables of p and q which appear previous them

$$x = x(a, a) = 77a^2 - 5a^2$$

$$y = y(a, b) = 55a^2 - 7b^2$$

$$z = z(a, b) = -6ab - 6ab$$

**Observation**

1.  $y(1,1) + z(1,1) = 36$  Perfect square
2.  $2x(a, a) + 2y(a, a) + 2z(a, a) = 216$  Cubical integer
3.  $2x(a, a) - y(a, a) = 96$  Nasty number
4.  $x(a, a) - y(a, a) + z(1,1) - 2T_{26,a} - 11Gno_a \equiv 0 \pmod{1}$
5.  $x(a, a) - z(a, a) - 12T_{16,a} - 36Gno_a \equiv 0 \pmod{36}$
6.  $x(a^2, a) + y(a^2, a) - 3360DE_a - 20T_{14,a} - 50Gno_a \equiv 0 \pmod{50}$

**C. Pattern 3**

Equation (3) as, 
$$p^2 = 36z^2 - 11q^2$$

Consider, 
$$p = p(a, b) = 36a^2 + 11b^2 \tag{11}$$

$$(6a + \sqrt{11}b)^2 (6a - \sqrt{11}b)^2 = (6z + \sqrt{11}q)(6z - \sqrt{11}q) \tag{12}$$

When we compare rational and irrational we get,

$$z = \frac{36a^2 + 11b^2}{6}$$

$$q = 12ab$$

Substituting the above mentioned p and q value into equation (2)

$$x = x(a, b) = 36a^2 - 11b^2 + 12ab$$

$$y = y(a, b) = 36a^2 - 11b^2 - 12ab$$

$$z = z(a, b) = \frac{36a^2 + 11b^2}{6}$$

When we substitute  $a=6A$  and  $b=6B$  in x, y and z we get

$$x = x(A, B) = 1296A^2 - 396B^2 + 432AB$$

$$y = y(A, B) = 1296A^2 - 396B^2 - 432AB$$

$$z = z(A, B) = 216A^2 + 66B^2$$

**Observation**

1.  $x(a, a) - z(a, a) - 105T_{22,a} - 472Gno_a - a \equiv 0 \pmod{472}$
2.  $x(a, 1) + z(a, 1) - 108T_{30,a} - 918Gno_a \equiv 0 \pmod{588}$
3.  $x(a, a) + y(a, a) - 66a^2$  Nasty number
4.  $x(a, a) + z(1, 1) - 222T_{14,a} - 555Gno_a \equiv 0 \pmod{837}$
5.  $z(a, a) - 94T_{8,a} - 94Gno_a \equiv 0 \pmod{94}$
6.  $y(a^2, a) - z(a^2, a) - 5180DE_a + 216SO_a - 412T_{5,a} + 5Gno_a \equiv 0 \pmod{5}$

**D. Pattern 4**

Equation (3) can be expressed as follows

$$p^2 + 11q^2 = 36z^2 \times 1 \tag{13}$$

‘1’ Should be written as

$$1 = \frac{(5 + i\sqrt{11})(5 - i\sqrt{11})}{36} \tag{14}$$

$$36 = (5 + i\sqrt{11})(5 - i\sqrt{11})$$

we can write using equation(14),(15) and the value of z

$$(p + i\sqrt{11}q)(p - i\sqrt{11}q) = (5 + i\sqrt{11})(5 - i\sqrt{11})(a^2 + \sqrt{11}b^2)^2 \frac{(5 + i\sqrt{11})(5 - i\sqrt{11})}{36}$$

Equating positive factor

$$(p + i\sqrt{11}q) = \frac{1}{6}(5 + i\sqrt{11})(5 + i\sqrt{11})(a + i\sqrt{11}b)^2$$

$$(p + i\sqrt{11}q) = \frac{(14a^2 - 154b^2 - 220ab) + i\sqrt{11}(10a^2 - 110b^2 + 28ab)}{6}$$

When we solve the preceding equation for the real and imaginary part, we obtain

$$p = \frac{(14a^2 - 154b^2 - 220ab)}{6}$$

$$q = \frac{(10a^2 - 110b^2 + 28ab)}{6}$$

Substituting the following values of p and q into equation (2)

$$x = \frac{(24a^2 - 264b^2 - 192ab)}{6}$$

$$y = \frac{(4a^2 - 44b^2 - 248ab)}{6}$$

$$z = a^2 + 11b^2$$

Now if replace a=6A and b=6B in x, y and z we get

$$x = x(A,B) = 144A^2 - 1584B^2 - 1152AB$$

$$y = y(A,B) = 24A^2 - 264B^2 - 1488AB$$

$$z = z(A,B) = 36A^2 - 396B^2$$

*Observation*

1.  $x(a,1) + y(a,1) - 28T_{14,a} + 1250Gno_a \equiv (\text{mod } 3098)$
2.  $y(a,a) + z(a,a) + 288T_{11,a} + 504Gno_a \equiv 0(\text{mod } 504)$
3.  $y(a^2, a) - x(a^2, a) + 5760DF_a + 168SO_a - 100T_{26,a} - 466Gno_a \equiv 0(\text{mod } 466)$
4.  $z(a, a) - 54T_{14,a} - 189Gno_a \equiv 0(\text{mod } 189)$
5.  $y(a, a) - x(a, a) = 864a^2$  Nasty number
6.  $x(a^2, 1) - y(a, a) - 6912DF_a + 576SO_a - 64T_{11,a} + 176Gno_a \equiv 0(\text{mod } 176)$

#### IV. CONCLUSION

This paper presents an unlimited number of different integer solutions, all non-zero, to the ternary quadratic Diophantine problem  $3(x^2 + y^2) - 5xy = 36z^2$ . Because quadratic equations are various, one can search for other quadratic equation with variable higher than properties using special number.

#### REFERENCES

- [1] Dickson, L. E. (1919). "History of the Theory of Numbers". (Vol. 1). Carnegie Institution of Washington.
- [2] Mordell, L. J. (1969). "Diophantine equations". Academic press.
- [3] Niven, Ivan, Zuckerman, S. Herbert and Montgomery, L. Hugh, An introduction to the theory of Numbers, John Wiley and sons, Inc, New York, 2004
- [4] Janaki, G., & Saranya, C. (2016). Observations on the Ternary Quadratic Diophantine Equation  $6(x^2 + y^2) - 11xy + 3x + 3y + 9 = 72z^2$ . International Journal of Innovative Research in Science, Engineering and Technology, 5, 2060-2065.



- [5] Saranya, C., & Kayathri, P.(2023). Integral Solutions of the Ternary Cubic Equation  $3x^2 + 2y^2 = 275z^2$  . Advances and Applications in mathematical Science, Vol21(3)
- [6] Sangeetha, P., & Divyapriya, T.(2023). On The Ternary Quadratic Diophantine Equation  $x^2 + 14xy + y^2 = z^2$  . International Journal for Research in Applied Science & Engineering Technology, vol 11(30)
- [7] Janaki, G., & Sangeetha, P. (2023). Integral Solutions of the Ternary cubic equation  $6(x^2 + y^2) - 11xy + x + y + 1 = 552z^3$  .Asian Journal of science and technology, Vol 14(85)
- [8] Janaki, G.,& Saranya, C. (2020). Integral Solutions of the Non-Homogeneous Sextic Equation with three Unknowns  $3(x^2 + y^2) - 2xy = 972z^6$  . Int. J. Sci. Res. in Multidisciplinary Studies Vol, 6(3).
- [9] Janaki, G., & Vidhya, S. (2016). On the Integer Solutions of the Pell Equation  $x^2 - 79y^2 = 9^k$  . International Journal of Scientific Research in Science, Engineering and Technology, Vol2(2), 1195-1197.
- [10] Saranya, P., & Poorani, K. (2023) Intrinsic solutions of Diophantine Equation Involving Centered Square Number  $E^4 - H^4 = (n^2 + (n-1)^2)(k-1)R^2$  . International Journal for research in Applied science and Engineering Technology, Vol 11(3)



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