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Integral Solutions of the Ternary Cubic Equation $6(x^2 + y^2) - 11xy = 288z^3$

C. Saranya[1], P. Kayathri [2]
[1]Assistant Professor, [2]PG student, PG & Research Department of Mathematics, Cauvery College for Women, (Autonomous) (Affiliated to Bharathidasan University), Trichy-18

Abstract: ***The Ternary cubic Diophantine Equation represented by*** $6(x^2 + y^2) - 11xy = 288z^3$ ***is analyzed for its infinite number of non-zero integral solutions. A few interesting among the solutions are also discussed.***
Keywords: Diophantine equation, Integral solutions, cubic equation with three unknowns, Ternary equation.

I. INTRODUCTION

Mathematics is the language of patterns and relationships and is used to describe anything that can be quantified.Diophantine equations have stimulated the interest of various mathematicians. Diophantine equations with higher degree greater than three can be reduced in to equations of degree 2 or 3 and it can be easily solved.In [1-3], theory of numbers is discussed. In [4-5], quadratic Diophantine equations are discussed. In [6-11], cubic, biquadratic and higher order equations are considered for its integral solutions. In this communication the non-homogeneous cubic equation with three unknowns represented by the equation $6(x^2 + y^2) - 11xy = 288z^3$is considered and in particular a few interesting relations among the solutions are presented.

A. *Notations*

$T_{6,n} = n(2n - 1)$ = Hexagonal number of rank n
$T_{8,n} = n(3n - 2)$ = Octagonal number of rank n
$T_{10,n} = n(4n - 3)$ = Decagonal number of rank n
$T_{12,n} = n(5n - 4)$ = Dodecagonal number of rank n
$T_{14,n} = n(6n - 5)$ = Tetradecagonal number of rank n
$T_{16,n} = n(7n - 6)$ = Hexadecagonal number of rank n
$T_{18,n} = n(8n - 7)$ = Octadecagonal number of rank n
$T_{20,n} = n(9n - 8)$ = Icosagonal number of rank n
$T_{22,n} = n(10n - 9)$ = Icosidigonal number of rank n
$T_{24,n} = n(11n - 10)$ = Icositetragonal number of rank n
$T_{26,n} = n(12n - 11)$ = Icosihexagonal number of rank n
$T_{28,n} = n(13n - 12)$ = Icosioctagonal number of rank n
$T_{30,n} = n(14n - 13)$ = Triacontagonal number of rank n
$T_{m,n} = n\left[1 + \frac{(n-1)(m-2)}{2}\right]$ = polygonal number of rank n
$O_n = \frac{1}{3}(2n^3 + n)$ = Octahedral number of rank n
$Gno_n = (2n - 1)$ =Gnomonic number of rank n
$P_n^m = \frac{n(n+1)}{6}[(m - 2)n + (5 - n)]$ =Pyramidal number of rank n

II. METHOD OF ANALYSIS

The ternary cubic Diophantine equation to be solved for its non-zero integral solutions is

$$6(x^2 + y^2) - 11xy = 288z^3 \quad (1)$$

The substitution of linear transformations

Let $x = u + v$ and $y = u - v$ (2)

In (1) leads to,

$$u^2 + 23v^2 = 288z^3 \quad (3)$$

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A. Pattern: 1

Assume,

$$z = z(a,b) = a^2 + 23b^2 \tag{4}$$

where a and b are non-zero integers.

$$288 = (14 + 2i\sqrt{23})(14 - 2i\sqrt{23}) \tag{5}$$

Using (4) and (5) in (3), and employing the method of factorization

$$\left(u + i\sqrt{23}\,v\right)\left(u - i\sqrt{23}\,v\right) = (14 + 2i\sqrt{23})(14 - 2i\sqrt{23})\,(a + i\sqrt{23}\,b)^3(a - i\sqrt{23}b)^3 \tag{6}$$

Equating the like terms and comparing the rational and irrational parts, we get

$$u = u(a,b) = 14a^3 - 966ab^2 - 138a^2b + 1058b^3$$

$$v = v(a,b) = 2a^3 - 138ab^2 + 42a^2b - 322b^3$$

Substituting the above values of u & v in equation (2), the corresponding integer solutions of (1) are given by

$$x = x(a,b) = 16a^3 - 1104ab^2 - 96a^2b + 736b^3$$

$$y = y(a,b) = 12a^3 - 828ab^2 - 180a^2b + 1380b^3$$

$$z = z(a,b) = a^2 + 23b^2$$

Observations

1) $6z(a,a)$ is perfect square.
2) $9z(a,a)$ is a cubical integer
3) $6z(a,a)$ is nasty number.
4) $x(a,a) - y(a,a) + az(a,a) + 80p_a^5 - 4T_{22,a} \equiv 0(mod\ 9)$
5) $y(a,a) - 6z(a,a) + 15O_a + 55T_{6,a} - 50Gno_a \equiv 0(mod\ 260)$
6) $x(a,a) + y(a,a) + 4z(a,a) - 48O_a + 16Gno_a \equiv 0(mod\ 16)$
7) $y(a,a) + x(a,a) + z(a,a) + 80p_a^5 - 4T_{22,a} \equiv 0(mod\ 36)$
8) $x(a,a) + y(a,a) + 11z(a,a) - 100T_{6,a} \equiv 0(mod\ 100)$
9) $y(a,a) + z(a,a) - 51T_{18,a} \equiv 0(mod\ 357)$
10) $x(a,a) + y(a,a) + 11z(a,a) - 20T_{12,a} - 26T_{6,a} - 12T_{10,a} \equiv 0(mod\ 142)$

B. Pattern: 2

Instead of (5), we write 288 as

$$288 = (9 + 3i\sqrt{23})(9 - 3i\sqrt{23}) \tag{7}$$

Using (4) and (7) in (3), and employing the method of factorization,

$$\left(u + i\sqrt{23}\,v\right)\left(u - i\sqrt{23}\,v\right) = (9 + 3i\sqrt{23})(9 - 3i\sqrt{23})\,(a + i\sqrt{23}\,b)^3(a - i\sqrt{23}b)^3 \tag{8}$$

Equating the like terms and comparing the rational and irrational parts, we get

$$u = u(a,b) = 9a^3 - 621ab^2 - 207a^2b + 1587b^3$$
$$v = v(a,b) = 3a^3 - 207ab^2 + 27a^2b - 207b^3$$

Substituting the above values of u & v in equation (2), the corresponding integer solutions of (1) are given by

$$x = x(a,b) = 12a^3 - 828ab^2 - 180a^2b + 1380b^3$$
$$y = y(a,b) = 6a^3 - 414ab^2 - 234a^2b + 1794b^3$$
$$z = z(a,b) = a^2 + 23b^2$$

Observations

1) $6z(a,a)$ is a perfect square.
2) $6z(a,a)$ is a nasty number.
3) $9z(a,a)$ is a cubical integer.
4) $x(a,a) - y(a,a) - 58T_{10,a} \equiv 0(mod\ 174)$
5) $y(a,a) - z(a,a) - 64T_{6,a} \equiv 0(mod\ 1)$
6) $x(b,b) + y(b,b) + z(b,b) - 280T_{6,b} \equiv 0(mod\ 1)$
7) $x(a,a) + y(a,a) + z(a,a) - 840O_a + 140Gno_a \equiv 0(mod\ 140)$
8) $48x(a,a) - 19y(a,a) + 24z(a,a) - 36O_a + 6Gno_a \equiv 0(mod\ 6)$
9) $x(a,1) - 6z(a,1) + 15O_a + 55T_{6,a} - 50Gno_a \equiv 0(mod\ 260)$
10) $y(a,a) + x(a,a) + az(a,a) + 416p_a^5 - 26T_{18,a} \equiv 0(mod\ 182)$

C. Pattern: 3

288 in (3) can be written as

$$288 = \frac{(50+2i\sqrt{23})(50-2i\sqrt{23})}{9} \quad (9)$$

Using (4) & (9) in (3),

$$(u + i\sqrt{23}v)(u - i\sqrt{23}v) = \frac{1}{9}(50 + 2i\sqrt{23})(50 - 2i\sqrt{23})\ (a + i\sqrt{23}\,b)^3\ (a + i\sqrt{23}\,b)^3 \quad (10)$$

Equating the like terms and comparing the real and imaginary parts, we get

$$u = u(a,a) = \frac{1}{3}(150a^3 - 3450ab^2 - 138a^2b + 1058b^3)$$

$$v = v(a,a) = \frac{1}{3}(2a^3 - 138ab^2 + 150a^2b - 1150b^3)$$

As our intension is to find integer solutions, we suitably choose $a = 3A$ and $b = 3B$,then the values of

$$x = x(a,a) = 450A^3 - 31050AB^2 - 1242A^2B + 9522B^3$$

$$y = y(a,a) = 18A^3 - 1242AB^2 + 1350A^2B - 10350B^3$$

$$z = z(a,a) = 9A^2 + 207B^2$$

Observations

1) $z(a,a)$ is a cubical integer.
2) $z(a,a)$ is a nasty number.
3) $x(a,a) - y(a,a) + az(a,a) + 23760p_a^5 - 990T_{26,a} \equiv 0(mod\ 10890)$
4) $155x(b,1) - 71y(b,1) - 216(b,1) - 5SO_b - 19T_{14,b} + 25Gno_b \equiv 0(mod\ 215)$
5) $60z(a,a) + y(a,a) + x(a,a) - 1296O_a + 216Gno_a \equiv 0(mod\ 216)$
6) $z(a,a) - x(a,a) - y(a,a) - 1026T_{26,a} \equiv 0(mod\ 11)$
7) $x(a,a) - y(a,a) + z(a,a) - 17820O_a - 2970Gno_a \equiv 0(mod\ 2970)$

III. CONCLUSION

In this paper we have presented three different patterns of non-zero distinct integers solutions of the non-homogeneous cone given by$6(x^2 + y^2) - 11xy = 288z^3$.To conclude one may search for other patterns of non-zero integer distinct solutions and their corresponding properties for other choices of cubic Diophantine equations.

REFERENCES

[1] Carmichael, R.D., The theory of numbers and Diophantine Analysis, Dover Publications,New York, 1959.
[2] Dickson L.E, History of Theory of Numbers, Vol.11, Chelsea Publishing company, New York,1952.
[3] Mordell. L.J, Diophantine equations, Academic Press,London,1969 Telang, S.G., Number theory, Tata McGraw Hill publishing company, New Delhi, 1996.
[4] Gopalan.M.A.,Vidhyalakshmi.S and Umarani.J., "On ternary Quadratic Diophantine equation $6(x^2 + y^2) - 8xy = 21z^2$", Sch.J. Eng. Tech. 2(2A); 108- 112,2014.

[5] Janaki.G and Saranya.C.,Observations on the Ternary Quadratic Diophantine Equation $6(x^2+y^2)-11xy+3x+3y+9=72z^2$, International Journal of Innovative Research in Science, Engineering and Technology, Vol-5, Issue-2, Pg.no: 2060-2065, Feb 2016.

[6] Janaki.G and Vidhya.S., On the integer solutions of thehomogeneous biquadratic Diophantine equation $x^4-y^4=82(z^2-w^2)p^2$, International Journal of Engineering Science and Computing, Vol. 6, Issue 6, pp.7275-7278, June, 2016.

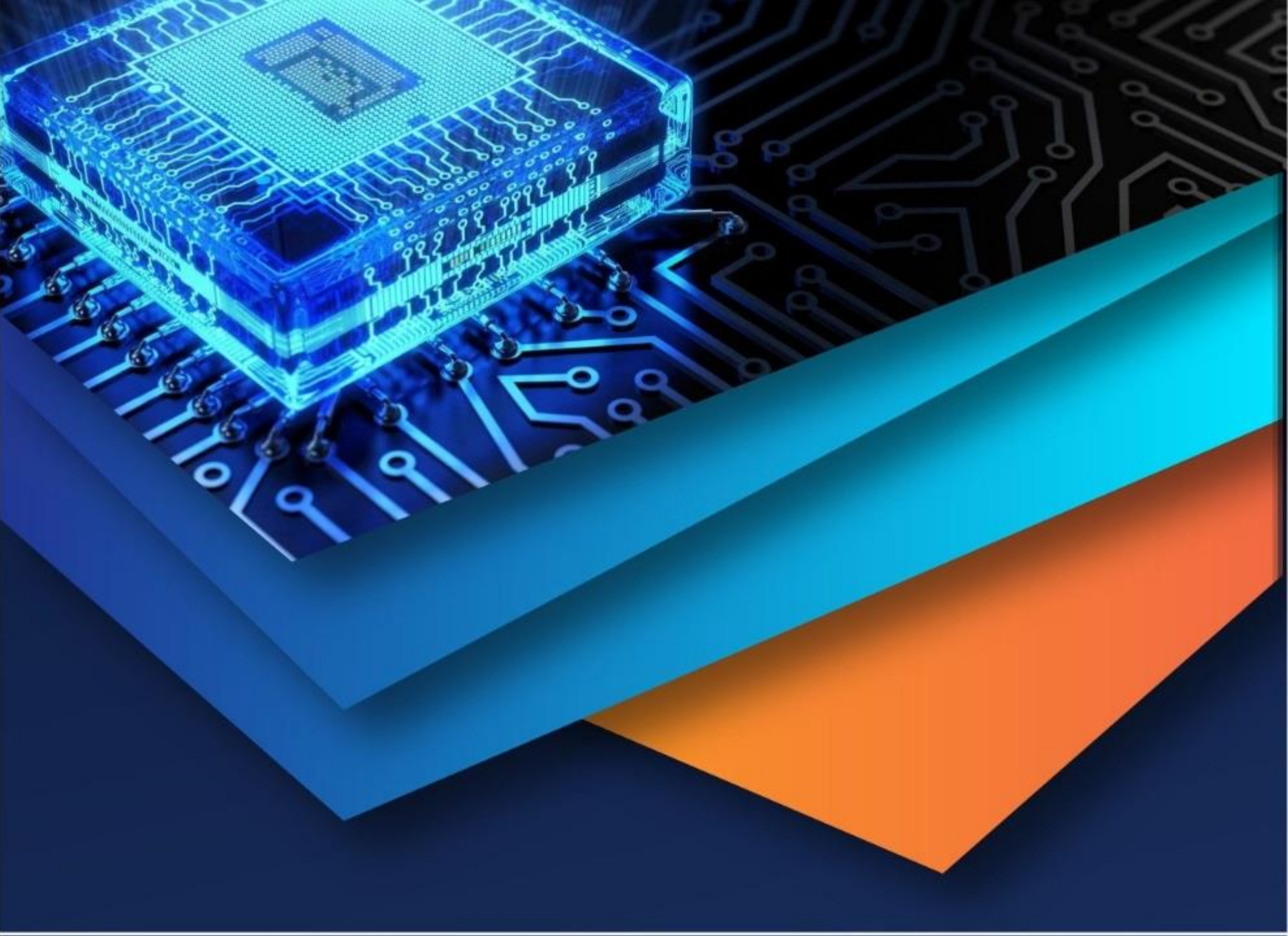
[7] Gopalan.M.A and Janaki.G, Integral solutions of $(x^2-y^2)(3x^2+3y^2-2xy=2(z^2-w^2)p^3$, Impact J.Sci.,Tech., 4(1), 97-102, 2010.

[8] Janaki.G and Saranya.P., On the ternary Cubic Diophantineequation $5(x^2+y^2)-6xy+4(x+y)+4=40z^3$, International Journal of Science and Research- online, Vol 5, Issue 3, Pg.No:227-229, March 2016.

[9] Janaki.G and Saranya.C., Integral Solutions of the non-homogeneous heptic equation with five unknowns $5(x^3-y^3)-7(x^2+y^2)+4(z^3-w^3+3wz-xy+1)=972p^7$, International Journal of Engineering Science and Computing, Vol. 6, Issue 5, pp.5347-5349, May, 2016.

[10] Janaki.G and Saranya.C., Integral Solutions of the ternary cubic equation $3(x^2+y^2)-4xy+2(x+y+1)=972\ z^3$, International Research Journal of Engineering and Technology, Vol. 4, Issue 3, pp.665-669, March, 2017.

[11] Janaki.G and Saranya.C., Integral Solutions of the homogeneous biquadratic Diophantine equation $3(x^4-y^4)-2xy(x^2-y^2)=972\,(z+w)p^3$, International Journal for Research in Applied Science and Engineering Technology, Vol. 5, Issue 8, pp.1123-1127, Aug 2017.



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