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# Intrinsic Solution on Quadratic Diophantine Equation $z^2 = 170x^2 + y^2$

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**Abstract:** *The non-zero unique integer solutions of the Quadratic Diophantine Equation with three unknowns  $z^2 = 170x^2 + y^2$  are examined. There are several absorbing relationships between the answers and a few unique numbers of rank n that are special Polygonal numbers, Star number and Gnomonic numbers.*

**Keywords:** *Three-variable quadratic equation, integral solutions, Polygonal numbers, Star number and Gnomonic number.*

## I. INTRODUCTION

There are various distinct kinds of quadratic Diophantine equations. Anyone can look at [1,7] for an in-depth review of a broad spectrum of concerns and a substantial survey of the body of literature. The non-trivial integral solutions to the quadratic Diophantine problem of the form  $lxy + m(x, y) = z^2$  have been examined in [8]. The integral solutions of an distinct Pythagorean triangular problem have been investigated in [9,10]. Two parametric non-trivial integral solutions to the quadratic homogeneity Diophantine problem  $X^2 + PXY + Y^2 = Z^2$ , for which P constitutes a non-zero constant, are given in [11]. The non-trivial integral solutions of the quadratic homogeneity equation,  $l\alpha(x^2 + y^2) + bxy = 4l\alpha^2 z^2$  have been analysed in [12]. This form of quadratic Diophantine equation,  $(x - y)(x - z) + y^2 = 0$  is examined in [13] for its integral solutions at various angles, and their parameterized representations are identified.

we examine an additional fascinating quadratic equation,  $z^2 = 170x^2 + y^2$ , and derive distinct patterns of integral solutions that are non-trivial. Furthermore, a few fascinating relationships between the solutions special Polygonal number and Gnomonic numbers are illustrated.

## II. NOTATIONS

$$T_{m,n} = \left[ 1 + \frac{(n-1)(m-2)}{2} \right] = \text{Polygonal Number with rank n and sides m.}$$

$$Star_n = 6n(n-1) + 1 = \text{Star number of rank n.}$$

$$Gno_n = (2n-1) = \text{Gnomonic number of rank n.}$$

## III. METHOD OF ANALYSIS

The Diophantine equation of quadratic has to be computed for its non-zero integral solution is

$$z^2 = 170x^2 + y^2 \quad (1)$$

Assuming,

$$z = z(a,b) = a^2 + 170b^2 \quad (2)$$

where a and b are non-zero integers.

### A. Pattern: 1

Equation (1) can be written as

$$z^2 - 170x^2 = y^2 \quad (3)$$

Assuming  $y = y(a, b) = a^2 - 170b^2$  (4)

We get,

$$(a^2 + 170b^2)^2 = (170x^2 + y^2)$$

Using the factorization method, we have

$$z + \sqrt{170}x = (a + \sqrt{170}b)^2$$
 (5)

$$z - \sqrt{170}x = (a - \sqrt{170}b)^2$$
 (6)

Comparing the rational and irrational factors,

$$z = z(a, b) = a^2 + 170b^2$$

$$x = x(a, b) = 2ab$$

The corresponding non-zero distinct solutions are

$$x = x(a, b) = 2ab$$

$$y = y(a, b) = a^2 - 170b^2$$

$$z = z(a, b) = a^2 + 170b^2$$

Observations:

1. If  $a = b$  and  $a$  is even, then  $a$  is divisible by 2.
2. If  $a$  is odd and  $b$  is even, then  $z$  is divisible by 3.
3.  $x(b, b) + y(b, b) + z(b, b)I_{10,b} \equiv 0 \pmod{3}$
4.  $x(1, a) - y(1, a) + z(1, a) - 68T_{12,a} - 137GnO_a \equiv 0 \pmod{137}$
5.  $y(1, 1) + z(1, 1) + x(1, 1)$  is a perfect square.
6.  $y(2, 2) + z(2, 2)$  is a Deficient number.
7.  $11x(b, b) + y(b, b) + z(b, b)$  which represents a Nasty number.
8. For any values of  $a$  and  $b$ ,  $z - x - y$  is divisible by 2.

*B. Pattern: 2*

Equation (1) can be written as

$$170x^2 + y^2 = z^2 * 1$$
 (7)

Assuming

$$z = z(a, b) = a^2 + 170b^2$$

and write

$$1 = \frac{(13 + i12\sqrt{170})(13 - i12\sqrt{170})}{24649}$$
 (8)

Using factorization method, equation (7) can be written as

$$(y + i\sqrt{170}x)(y - i\sqrt{170}x) = \left[ \frac{13 + i12\sqrt{170}}{157} \right] \left[ \frac{13 - i12\sqrt{170}}{157} \right] [(a + i\sqrt{170}b)^2 (a - i\sqrt{170}b)^2]$$

we get

$$(y + i\sqrt{170}x) = \frac{1}{157} (13 + i12\sqrt{170})(a - i\sqrt{170}b)^2$$
 (9)

$$(y - i\sqrt{170}x) = \frac{1}{157}(13 + i12\sqrt{170})(a + i\sqrt{170}b)^2 \quad (10)$$

Comparing real and imaginary parts, we get

$$x = \frac{1}{157}[12a^2 - 2040b^2 + 26ab]$$

$$y = \frac{1}{157}[13a^2 - 2210b^2 - 4080ab]$$

Since finding integer solutions is what we specialize in, we have chosen and appropriately such that and are integers.

Consider  $a = 157A$  and  $b = 157B$

The integer solutions are

$$x = 1884A^2 - 320280B^2 + 4082AB$$

$$y = 2041A^2 - 346970B^2 - 640560AB$$

$$z = 24649A^2 + 4190330B^2$$

Observations

1. If  $A=B$ , then  $x-y-z$  is divisible by 2.
2.  $x(A, A) - y(A, A) - 134235T_{12,A} - 268470GnO_A \equiv 0 \pmod{268470}$
3.  $z(B, B) - y(B, B) - x(B, B) - 393913T_{30,A} \equiv 0 \pmod{5120869}$
4.  $z(A, 1) + x(A, 1) - 2041T_{28,A} - 14287GnO_A \equiv 0 \pmod{3884337}$
5.  $y(B, B) + z(B, B) - 293590T_{24,A} \equiv 0 \pmod{2935900}$

C. Pattern: 3

Equation (1) can be written as

$$z^2 - y^2 = 170x^2$$

and we get

$$(z + y)(z - y) = 170x \cdot x \quad (11)$$

a) Case 1

Equation (1) can be written as

$$\frac{z + y}{170x} = \frac{x}{z - y} = \frac{P}{Q} \quad (12)$$

From equation (12), we get two equations

$$-170Px + Qy + Qz = 0$$

$$Qx + Py - Pz = 0$$

Applying cross ratio method, we get the integer solutions are

$$x = x(P, Q) = -2PQ$$

$$y = y(P, Q) = Q^2 - 170P^2$$

$$z = z(P, Q) = -Q^2 - 170P^2$$

Observations

1. For all values of P and Q,  $x + y - z$  is divisible by 2.

2.  $2y(1, P) - x(1, P) - 2z(1, P) - 2T_{6,P} - 2GnO_P \equiv 0 \pmod{2}$
3.  $x(P, 1) - 4y(P, 1) - 4z(P, 1) - 170T_{18,P} - 594GnO_P \equiv 0 \pmod{594}$
4. Each of the following expressions
  - (i)  $z(Q, Q) + 2x(Q, Q) - y(Q, Q)$
  - (ii)  $T_{30,P} - 3x(Q, 1) + T_{18,P}$

represents a nasty number.

*b) Case 2*

Equation (11) can be written as

$$\frac{z + y}{x} = \frac{170x}{z - y} = \frac{P}{Q} \quad (13)$$

From equation (13), we have two equations

$$-Px + Qy + Qz = 0$$

$$170Qx + Py - Pz = 0$$

Applying cross ratio method, we get the integer solutions are

$$x = x(P, Q) = -2QP$$

$$y = y(P, Q) = 170Q^2 - P^2$$

$$z = z(P, Q) = -P^2 - 170Q^2$$

**Observations**

1.  $y(1, Q) - x(1, Q) - 9z(1, Q) - 680T_{7,Q} - 511GnO_Q \equiv 0 \pmod{519}$
2.  $10x(1, Q) + 3y(1, Q) - 85Star_Q - 245GnO_Q \equiv 0 \pmod{257}$
3.  $y(Q, Q) - 6x(Q, Q) - z(Q, Q) - 44T_{18,Q} - 154GnO_Q \equiv 0 \pmod{154}$
4.  $3T_{16,P} - 3x(P, 1)$  represents a Nasty number.
5.  $y(1, 1) - x(1, 1)$  represents a Palindromic number.
6.  $z(1, 1) + y(1, 1) - 5x(1, 1)$  is a Dudeney number.

*D. Pattern: 4*

Equation (1) can be written as

$$z^2 - 170x^2 = y^2 * 1 \quad (14)$$

and write

$$1 = (\sqrt{170} + 13)(\sqrt{170} - 13)$$

Assuming,

$$y = y(a, b) = a^2 - 170b^2 \quad (15)$$

We have

$$z^2 - 170x^2 = (a^2 - 170b^2) \left[ (\sqrt{170} + 13)(\sqrt{170} - 13) \right]$$

Using factorization method, we get

$$z + \sqrt{170}x = (a + \sqrt{170}b)^2 \left[ (\sqrt{170} + 13) \right] \quad (16)$$



$$z - \sqrt{170}x = (a + \sqrt{170}b)^2 \left[ (\sqrt{170} + 13) \right] \quad (17)$$

From equation (16), we get

Thus, the corresponding non-zero distinct integer solutions are

$$x = x(a, b) = a^2 + 170b^2 + 26ab$$

$$y = y(a, b) = a^2 - 170b^2$$

$$z = z(a, b) = 13a^2 + 2210b^2 + 340ab$$

Observations

1.  $z(b, b) - x(b, b) - 169T_{30, b} \equiv 0 \pmod{2197}$
2.  $z(a, 1) - y(a, 1) + x(a, 1) - T_{28, a} - 189GnO_a \equiv 0 \pmod{2739}$
3.  $z(b, b) - y(b, b) - x(b, b) - 507T_{12, b} - 1014GnO_b \equiv 0 \pmod{1014}$
4.  $x(1, 1) + y(1, 1)$  represents a perfect number.
5.  $x(1, 1) - y(1, 1)$  represents an even composite number.

#### IV. CONCLUSION

The Diophantine quadratic equation using special polygonal numbers has been presented. Furthermore, one can look for integer solutions of the quadratic diophantine equations with other special numbers.

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