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International Journal For Research in  
Applied Science and Engineering Technology



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# **INTERNATIONAL JOURNAL FOR RESEARCH**

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

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**Volume: 13    Issue: III    Month of publication: March 2025**

**DOI: <https://doi.org/10.22214/ijraset.2025.67379>**

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# Intrinsic Solution on Ternary Quadratic Diophantine Equation

$$6x^2 + 6y^2 - 11xy = 72z^2$$

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## I. INTRODUCTION

Numerous ternary quadratic equations exist. For a more complete understanding, check [1-7]. The And form of the ternary quadratic Diophantine problem  $kxy + m(x + y) = z^2$  has been studied for solving non-trivial integrals. In [9-15], non-zero integral solutions to the various Diophantine equations are investigated. These findings have motivated us to hunt for an infinite number of non-zero integral solutions to the ternary quadratic equation given by, which is another fascinating equation. There are also a few intriguing correlations between the answers, as well as some unique numbers such as triangular, cantered, gnomon, and star. In addition, a Python program is used to code in the quadratic diophantine equation in the five patterns to determine the program's output.

## II. NOTATIONS

- $T_{m,n}$  = Triangular Number of rank n.
- $Gno_n$  = Gnomonic Number of rank n.
- $Star_n$  = Star Number of rank n.
- $CH_n$  = Cantered Hexagonal number of rank n.

## III. METHODOLOGY

The ternary equation of quadratic Diophantine condition is for its non-zero necessary arrangement

$$6x^2 + 6y^2 - 11xy = 72z^2 \quad \longrightarrow \quad (1)$$

Swapping out linear transformation

$$x = u + v \quad \text{and} \quad y = u - v \quad \longrightarrow \quad (2)$$

in(1) leads to.

$$u^2 + 23v^2 = 72z^2 \quad \longrightarrow \quad (3)$$

1) Template: 1

Suppose

$$z = z(a,b) = a^2 + 23b^2 \quad \longrightarrow \quad (4)$$

where a and b are non-zero integers.

$$72 = (7 + i\sqrt{23})(7 - i\sqrt{23}) \quad \longrightarrow \quad (5)$$

Operating (4) and (5) and using factorization Method ,

$$(u + i\sqrt{23}v)(u - i\sqrt{23}v) = (7 + i\sqrt{23})(7 - i\sqrt{23})(a + i\sqrt{23}b)^2(a - i\sqrt{23}b)^2 \quad \longrightarrow \quad (6)$$

Equating like terms and corresponding real and imaginary parts

$$u(a, b) = 7a^2 - 161b^2 + 46ab$$

$$v(a, b) = a^2 - 23b^2 + 14ab$$

Equating (2) can be solved as an integer by substituting the above values for u and v,

$$x = 8a^2 - 184b^2 - 32ab - 208$$

$$y = 6a^2 - 138b^2 - 60ab - 152$$

$$z = a^2 + 23b^2$$

Observation:

$$x(a,1) + y(a,1) - 2T_{16,a} + 40Gno_a \equiv (\text{mod} - 722)$$

$$x(a,1) + y(a,1) + z(a,1) - 5T_{8,a} - 41Gno_a \equiv (\text{mod} - 700)$$

$$x(a,a) - y(a,a) - 2z(a,a) + 16T_{10,a} + 24Gno_a \equiv (\text{mod} - 80)$$

$$x(a,1) + y(a,1) - 2z(a,1) - 2Star_a + 46Gno_a \equiv (\text{mod} - 776)$$

$$y(a,1) + x(a,1) + 92T_{14,a} + 230Gno_a \equiv (\text{mod} - 438)$$

$$3x(a,a) - 5y(a,a) - 14z(a,a) + 151T_{20,a} + 640Gno_a \equiv (\text{mod} - 1988)$$

2) *Template: 2*

Consider (3) as ,

$$u^2 - 49v^2 + 72v^2 = 72z^2 \quad \longrightarrow \quad (7)$$

Compute equation (7)

$$u^2 - 49v^2 = 72(z^2 - v^2) \quad \longrightarrow \quad (8)$$

Equation (8) in the form of ratio as

$$\frac{u + 7v}{z + v} = \frac{72(z - v)}{u - 7v} = \frac{A}{B}$$

The two equations that follow are identical to this,

$$Bu + (7B - A)v - Az = 0 \quad \longrightarrow \quad (9)$$

$$-Au - (72B - 7A)v + 72Bz = 0 \quad \longrightarrow \quad (10)$$

Apply the cross multiplication,

$$u = 7A^2 + 504B^2 - 144AB$$

$$v = A^2 - 72B^2$$

$$z = A^2 - 14AB + 72B^2$$

After, Operating the value of u ,v in eqn (2) and the corresponding integer solution (1),

$$x = 8A^2 - 144AB + 432B^2$$

$$y = 6A^2 - 144AB + 567B^2$$

$$z = A^2 - 14AB + 72B^2$$

Observation:

$$1. y(A,1) + x(A,1) - 2T_{16,A} + 138 Gno_A \equiv (\text{mod } 870)$$

$$2. x(1,1) + y(A,1) - 2T_{8,A} + 70 Gno_A \equiv (\text{mod } 820)$$

$$3. x(A,A) + y(1,1) - 2T_{10,A} - 69 Gno_A \equiv (\text{mod } - 75)$$

$$4. x(A,A) + z(1,1) - 148 T_{6,A} + 74 Gno_A \equiv (\text{mod } 133)$$

$$5. x(A,A) - y(A,A) - 2z(A,1) + 12 T_{26,A} + 52 Gno_A \equiv (\text{mod } - 196)$$

3) *Template: 3*

Equation (3) as,

$$u^2 = 72z^2 - 23v^2$$

By consider the linear transformation

$$z = x + 72T \text{ \& } v = x + 23T \longrightarrow (11)$$

Substituting (11) in (3)

$$u^2 = 49(X^2 + 1656T^2) \longrightarrow (12)$$

write  $u = 7U$   $\longrightarrow (13)$

$$\text{As, } U^2 = X^2 + 1656T^2 \longrightarrow (14)$$

The corresponding solution of (14) is

$$T = 2ab$$

$$U = 1656a^2 - b^2 \longrightarrow (15)$$

$$X = 1658a^2 + b^2$$

Substituting (15) in (11) and (13) we get,

$$u = 7(1656a^2 - b^2)$$

$$v = 1656a^2 + b^2 + 144ab$$

$$z = 1656a^2 + b^2 + 46ab$$

After, Operating the value of u, v in eqn (2) and the corresponding integer solution (1),

$$x = 13248a^2 - 6b^2 + 144ab$$

$$y = 9936a^2 - 8b^2 - 144ab$$

$$z = 1656a^2 + b^2 + 46ab$$

Observation

$$1. x(a, a) - y(a, a) + 4z(a, a) - 38132 T_{3,a} - 9533 Gno_a \equiv (\text{mod } -9533)$$

$$2. x(a, 1) - y(a, 1) - 400 T_{20,a} - 1600 Gno_a \equiv (\text{mod } 1602)$$

$$3. 3y(a, 1) - 4x(a, 1) + 12z(a, 1) + 314 T_{26,a} + 1727 Gno_a \equiv (\text{mod } 1715)$$

$$4. x(a, a) - y(a, a) - 2z(a, a) - 1598 T_{6,a} - 799 Gno_a \equiv (\text{mod } 799)$$

$$5. y(a, 1) - 6z(a, 1) + 70 Star_a \equiv (\text{mod } -14)$$

$$6. x(a, a) - y(a, a) - 21942152 Star_a \equiv (\text{mod } 0)$$

4) *Template: 4*

(3) can be written as  $u^2 + 23v^2 = 72z^2$

$$u^2 + 23v^2 = 72z^2 \longrightarrow (16)$$

“72” can be written as

$$72 = \frac{(5 + i7\sqrt{23})(5 - i7\sqrt{23})}{16} \longrightarrow (17)$$

Employing (17) in (16) and proceeding as in like template 1. We get the non-zero distinct integer solution as

$$u + i\sqrt{23}v(u - i\sqrt{23}v) = \frac{(5 + i7\sqrt{23})(5 - i7\sqrt{23})}{16} (a + i\sqrt{23}b)^2 \longrightarrow (18)$$

Equating like terms and corresponding real and imaginary parts,

$$u = 5a^2 - 115b^2 - 322ab$$

$$v = 7a^2 - 161b^2 + 10ab$$

Since finding integer answer is what we are interested in, pick  $a$  and  $b$  choose wisely as  $a = 4A$  &  $b = 4B$  Considering (2), the whole number arrangement of (1) are give by

$$X = 48A^2 - 1104B^2 - 1248AB$$

$$Y = 8A^2 - 184B^2 + 1328AB$$

$$z = 16A^2 + 368B^2$$

Observation

$$1.z(1,0) = 16 = 4^2 = \text{perfectsquare}$$

$$2.y(1,0) \text{ is a cubic root}$$

$$3.x(1, B) + 184T_{14,B} - 164Gno_B \equiv (\text{mod } 212)$$

$$4.2[y(1,1)] = 2304(\text{Ducknumber})$$

$$5.5[z(A, A) - x(1,1)] - 160T_{26,A} - 880Gno_A \equiv (\text{mod } 12400)$$

5) Template: 5

$$(3) \text{ can be written as } u^2 + 23v^2 = 72z^2$$

$$u^2 + 23v^2 = 72z^2 \cdot 1 \longrightarrow (19)$$

'1' can be written as

$$1 = \frac{(11 + i\sqrt{23})(11 - i\sqrt{23})}{144}$$

Adapting it in equation (19) and proceeding as in "template1", we get

$$\begin{aligned} (u + i\sqrt{23}v)(u - i\sqrt{23}v) &= (7 + i\sqrt{23})(7 - i\sqrt{23})(a + i\sqrt{23}b)^2 \frac{(11 + i\sqrt{23})(11 - i\sqrt{23})}{144} \\ &= (7 + i\sqrt{23})(a + i\sqrt{23}b)^2 \frac{(11 + i\sqrt{23})(11 - i\sqrt{23})}{144} \longrightarrow (20) \end{aligned}$$

Equating the eqn (20) like terms and corresponding real and imaginary parts,

$$u = \frac{1}{12}(54a^2 - 1242b^2 - 828ab)$$

$$v = \frac{1}{12}(18a^2 - 414b^2 + 108ab)$$

Since finding integer answer is what we are interested in, pick  $a$  and  $b$  choose wisely as  $a = 12A$  &  $b = 12B$ .

Considering (2), the whole number arrangement of (1) are give by

$$X = 864A^2 - 19872B^2 - 8640AB$$

$$Y = 432A^2 - 9936B^2 - 11232AB$$

$$Z = 144A^2 + 3312B^2$$

Observation

$$1.x(1,1) + y(A,1) - 72Star_A + 5616Gno_A \equiv (\text{mod} - 43200)$$

$$2.2z(A,1) - 3x(1,1) - 24T_{26,A} - 132Gno_A \equiv (\text{mod} - 81372)$$

$$3.2[y(A,A) - x(A,A)] - 1536T_{20,A} - 6144Gno_A \equiv (\text{mod} 6144)$$

$$4.y(A,A) - x(A,A) - 1152T_{14,A} - 2880Gno_A \equiv (\text{mod} 2880)$$

$$5.x(1,B) - 6y(1,B) - 4416T_{20,B} - 47040Gno_B \equiv 0(\text{mod} 45312)$$

$$6.z(A,1) - 24T_{14,A} - 60Gno_A \equiv 0(\text{mod} 3372)$$

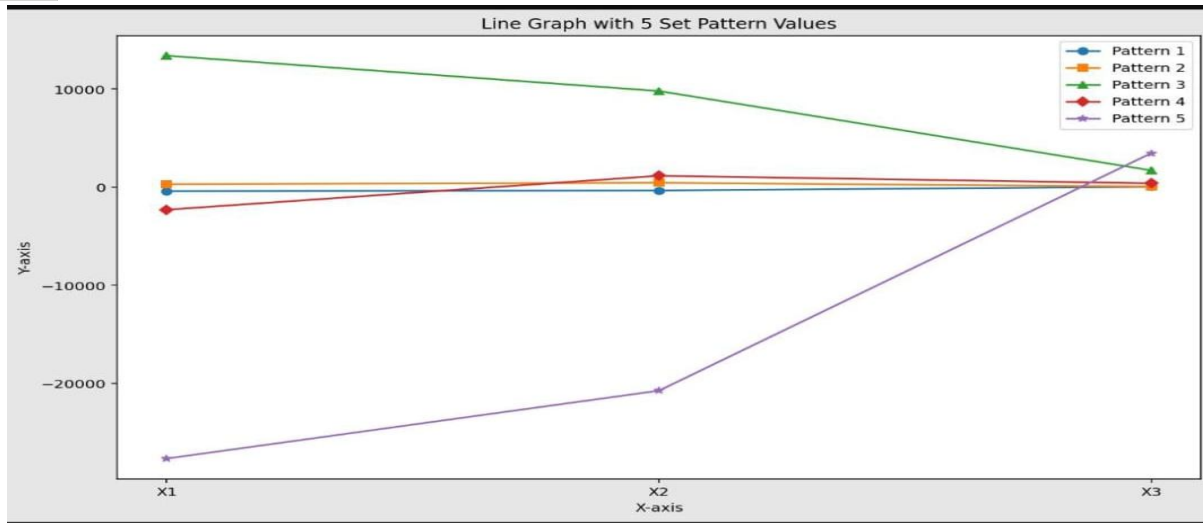
$$6x^2 + 6y^2 - 11xy = 72z^2$$

Python program for Solving Quadratic Diophantine Equation:

Program coding for line graph.

```
import matplotlib.pyplot as plt
import numpy as np
#Define the data
x = np.arange(3)
y1 = [-416, -344, 24]
y2 = [296, 438, 59]
y3 = [13386, 9784, 1703]
y4 = [-2304, 1152, 384]
y5 = [-27648, -20736, 3456]
#Create the figure and axis
fig, ax = plt.subplots()
#Create the line graph
ax.plot(x, y1, marker='o', label='Pattern 1')
ax.plot(x, y2, marker='s', label='Pattern 2')
ax.plot(x, y3, marker='^', label='Pattern 3')
ax.plot(x, y4, marker='D', label='Pattern 4')
ax.plot(x, y5, marker='*', label='Pattern 5')
#Set the title and labels
ax.set_title('Line Graph with 5 Set Pattern Values')
ax.set_xlabel('X-axis')
ax.set_ylabel('Y-axis')
ax.set_xticks(x)
ax.set_xticklabels(['x1', 'x2', 'x3'])
#Add a legend
ax.legend()
#Show the plot
plt.show()
```





#### IV. CONCLUSION

This work presents infinitely many non-zero unique integer solutions to the ternary quadratic diophantine problem  $6(x^2 + y^2) - 11xy = 72z^2$ , along with some observations about the solutions. For alternative options of ternary quadratic diophantine equations, one could look for additional examples of non-zero whole number remarkable arrangements and their corresponding highlights, or increase the quadratic diophantine equation of five patterns using Python program coding of line graph to find a diagram output.

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