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Intrinsic solutions of Diophantine Equation Involving Centered Square Number

$$E^4 - H^4 = (n^2 + (n-1)^2)(k-l)R^2$$

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Abstract: The bi-quadratic Diophantine equation with five unknown parameters $E^4 - H^4 = (n^2 + (n-1)^2)(k-l)R^2$ ***is researched for its quasi complex arithmetic values. A few correlations between the solutions and plethora of other figures notably the triangular, pronic, stella octagula and gnomonic notation are effectively portrayed.***

Keywords: Bi-quadratic, Non-homogeneous, Integer solution, Diophantine equation, Centered square number.

I. INTRODUCTION

In introductory number theory, a centered square value is a way to portray the guesstimated number of dots together in square which would have perhaps one dot in the centre and every additional dot facing something in preliminary square strands. The range of centers in each centered square multitude is equal to the number of markings on a conventional square pattern with in a particular demographic block altitude of the centre dot. Centered square numbers, like figurate numbers in terms of appearance, have few if any applications in the real world, however they are sporadically studied in entertainment mathematics for their spectacular architectural and mathematically wonderful aspects. While isolated equations have indeed been explored throughout history as a kind of dilemma, the modernization of rigorous conceptions of Diophantine equations is a massive achievement of the twentieth century.[1-3] gives a detailed and self-explanatory study of Diophantine equations. For different techniques towards solving various Diophantine as well as exponential Diophantine equations. [4-19] have been referred. The purpose of research is to delineate non-trivial integral solutions to the five unknowns in the bi-qudratic Diophantine equation facilitated by $E^4 - H^4 = (n^2 + (n-1)^2)(k-l)R^2$. Numerous incredibly interesting correlations between specific solutions and the numbers notably the triangular number, conjointly gnomonic number, pronic number, stella octangula number are proposed.

II. NOTATIONS

1) $Gno_n = (2n-1)$ = Gnomonic number of rank n.
2) $So_n = n(2n^2 - 1)$ =Stella Octangula number of rank n.
3) $C_{m,n} = 1 + \frac{mn(n-1)}{2}$ =Centered m-goal number of rank n.
4) $CS_n = n^2 + (n-1)^2$ =Centered square number of rank n.
5) $\Pr_n = n(n+1)$ =Pronic number of rank n.
6) $T_{m,n} = n(1 + \frac{(n-1)(m-2)}{2})$ =Triangular number of rank n.
7) $W_n = n2^n - 1$ =Woodall number.
8) $(2^n + 1)^2 - 2$ =Kynea number.
9) $M_{Mn} = 2^{2^n - 1} - 1$ =Double Mersenne number.

III. TECHNIQUE FOR ANALYSIS

The five unknown bi-quadratic equation is predicted to be,

$$E^4 - H^4 = (n^2 + (n-1)^2)(k-l)R^2 \qquad (1)$$

The linear transformations,

$$\left.\begin{aligned} E &= c+d \\ H &= c-d \\ k &= 4(1+cd) \\ l &= 4(1-cd) \end{aligned}\right\} \qquad (2)$$

Now that (1) has been simplified to,

$$(c+d)^4 - (c-d)^4 = (n^2 + (n-1)^2)(4+4cd-4+4cd)R^2$$

$$8cd(c^2+d^2) = (n^2 + (n-1)^2)(8cd)R^2$$

$$c^2 + d^2 = (n^2 + (n-1)^2)R^2 \qquad (3)$$

As we compute (3), we observe an indefinite range of choices for various patterns.

A. *Pattern: I*

Let $R = s^2 + t^2$ (4)

Utilizing (4) in (3) and using the method of factorization define,

$$c + id = (n + i(n-1))(s+it)^2$$

$$= (n + i(n-1))(s^2 + i2st - t^2)$$

$$c + id = ns^2 + i2nst - nt^2 + i(n-1)s^2 - 2(n-1)st - i(n-1)t^2$$

Equalizing the real and the imaginary, we get

$$c = ns^2 - 2(n-1)st - nt^2$$

$$d = (n-1)s^2 + 2nst - (n-1)t^2$$

Hence the non-zero integral solutions to (1) are,

$$E = (2n-1)s^2 + 2st - (2n-1)t^2$$

$$H = s^2 - 2(2n-1)st - t^2$$

$$k = 4 + 4n(n-1)s^4 + 8(n^2 - (n-1)^2)s^3t$$
$$- 24n(n-1)s^2t^2 - 8\left(n^2 - (n-1)^2\right)st^3 + 4n(n-1)t^4$$

$$l = 4 - 4n(n-1)s^4 - 8(n^2 - (n-1)^2)s^3t + 24n(n-1)s^2t^2$$
$$+ 8(n^2 - (n-1)^2)st^3 - 4n(n-1)t^4$$

$$R = s^2 + t^2$$

Observations

1) $E(s,s,1) - H(s,s,1) = 4s^2$ is a square integer.
2) R(2,2)-1=7 is a double mersenne number.
3) $5k(1,1,1) + R(1,1) + 1 = 23$ is a Woodall number.

4) $k(1,s,1)-4So_s+2Gno_s\equiv 0(\bmod 2)$
5) $E(s,s,1)-H(s,s,1)-2761S+Gno_S\equiv 0(\bmod 1)$

Some numerical examples are listed below.

TABLE 1

s	t	n	E	H	k	l	R	L.H.S= R.H.S
1	1	1	2	-2	4	4	2	0
2	1	1	7	-1	52	-44	5	2400
3	-2	1	-7	17	-236	244	13	-81120
-4	6	1	-68	28	3844	-3836	52	20766720

B. *Pattern: II*

Rewriting (3) as,

$$c^2+d^2=1*(n^2+(n-1)^2)R^2 \qquad (5)$$

Write 1 as,

$$1=\frac{15^2+20^2}{25^2} \qquad (6)$$

And take, ‘t’ as in (4)

Using (6) in (5) and proceeding as in pattern 1 we get,

$$c+id=(n+i(n-1))(s+it)^2(15+i20)$$

$$c+id=15ns^2+i30nst-15nt^2+i15(n-1)s^2-30(n-1)st$$
$$-i15(n-1)t^2+i20ns^2-40nst-i20nt^2-20(n-1)s^2$$
$$-i40(n-1)st+20(n-1)t^2$$

Equalizing the real and imaginary part, we get

$$c=-5ns^2+20s^2+5nt^2-20t^2-70nst+30st$$

$$d=35ns^2-15s^2-35nt^2+15t^2-10nst+40st$$

Realizing that the goal is to determine the integer result, we get from taking s=25S, t=25T

$$c=-125nS^2+500S^2+125nT^2-500T^2-17500nST+750ST$$

$$d=875nS^2-375S^2-875nT^2+375T^2-250nST+1000ST$$

Consequently, as we compute for the quasi integral solutions of (1) we get,

$$E=750nS^2+125S^2-750nT^2-125T^2-2000nST+1750ST$$
$$H=-1000nS^2+875S^2+1000T^2-875T^2-1500nST-250ST$$
$$k=4(1-109375n^2S^4+656250n^2S^2T^2-1500000n^2S^3T+1500000n^2ST^3$$
$$-109375n^2T^4+484375nS^4-2906250nS^2T^2-1062500nST^3$$
$$+1062500nS^3T+484375nT^4-187500S^4+112500S^2T^2$$
$$+218750S^3T-218750ST^3-187500T^4)$$

$$l=4(1+109375n^2S^4-656250n^2S^2T^2+1500000n^2S^3T-1500000n^2ST^3$$
$$+109375n^2T^4-484375nS^4+2906250nS^2T^2+1062500nST^3$$

$$-1062500nS^3T - 484375nT^4 + 187500S^4 - 1125000S^2T^2$$
$$-218750S^3T + 218750ST^3 + 187500T^4)$$
$$R = 625S^2 + 625T^2$$

Observations

1) $k(s,1,1) + l(s,1,1) = 8s^3$ is a cubical integer.
2) $E(s,1,1) - 125T_{16,s} - 250Gno_s + 625 = 0$
3) $k(s,1,1) + 3000s^2E(s,s,1) - 500sH(s,s,1) + 22500000CS_s + 22062500Gno_s \equiv 0(\mathrm{mod}\,1187504)$
4) $-[E(1,1,1) + H(1,1,1)] - R(1,1) + 145 = 895$ is a Woodall number.
5) $-[H(1,1,1)] - 663 = 1087$ is a Kynea number.
6) $R(s,1) - 125T_{12,s} - 250Gno_s \equiv 0(\mathrm{mod}\,875)$

Some numerical examples are listed below.

TABLE 2								
s	t	n	E	H	k	L	R	L.H.S=R.H.S
1	1	1	-250	-1750	-2999996	3000004	1250	-9375000000000.00
4	-3	3	67625	42125	2798625004	-2798624996	15625	177647094726563000000
6	4	2	-21500	-100500	-9637999996	9638000004	32500	-1018013750000000000000
-2	3	5	30125	67125	-3598249996	3598250004	8125	-194783392578125000000.00

C. Pattern: III

'1' in (6) can also be written as,

$$1 = \frac{8^2 + 15^2}{17^2} \qquad (7)$$

Utilizing (7) in (5) and proceeding as in pattern-II we get

$$c + id = 1*(n + i(n-1))(8 + i15)(x + iy)^2$$

Equating the real and imaginary part, we get

$$c = -7ns^2 + 15s^2 + 7nt^2 - 15t^2 - 46nst + 16st$$
$$d = 23ns^2 - 8s^2 - 23nt^2 + 8t^2 - 14nst + 30st$$

Realizing that the goal is to determine the integer result, we get from taking s=17S, t=17T

$$c = -119nS^2 + 225S^2 + 119nT^2 - 225T^2 - 782nST + 275ST$$
$$d = 391nS^2 - 136S^2 - 391nT^2 + 136T^2 - 238nST + 510ST$$

Consequently, as we compute for the quasi integral of (1), we get

$$E = 272nS^2 + 89S^2 - 272nT^2 - 89T^2 - 1020nST + 782ST$$
$$H = -510nS^2 + 361S^2 + 510nT^2 - 361T^2 - 544nST - 233ST$$

$$k = 4(1-46529n^2S^4 + 279174n^2S^2T^2 - 277440n^2S^3T + 277440n^2ST^3$$
$$-46529n^2T^4 + 104159nS^4 - 671874nS^2T^2 + 98912nS^3T$$
$$-98912nST^3 + 104159nT^4 - 30600S^4 + 199920S^2T^2$$
$$-77758S^3T + 77758ST^3 - 30600T^4 \qquad)$$
$$l = 4(1+46529n^2S^4 - 279174n^2S^2T^2 + 277440n^2S^3T - 277440n^2ST^3$$
$$+46529n^2T^4 - 104159nS^4 + 671874nS^2T^2 - 98912nS^3T$$
$$+98912nST^3 - 104159nT^4 + 30600S^4 - 199920S^2T^2$$
$$+77758S^3T - 77758ST^3 + 30600T^4)$$
$$R = 289S^2 + 289T^2$$

Observation

1) $-\dfrac{[E(S,S,1)+H(S,S,1)]}{S^2} - 61 = 959$ is a Kynea number.
2) 2R(1,1) is a perfect square.
3) $E(S,S,1) + H(S,S,1) + 1020C_{2,S} + 510Gno_S \equiv 0(\text{mod } 510)$
4) $E(S,S,1) + 238\Pr_S - 119Gno_S \equiv 0(\text{mod}119)$
5) $E(s,1,1) + R(s,1) - 325T_{6,s} - 2127s + 361 = 0$

Some numerical examples are listed below.

TABLE 3								
s	t	n	E	H	K	L	R	L.H.S=R.H.S
1	1	1	-238	-782	-554876	554884	578	-370753059840.00
2	2	1	-952	-3128	-8878076	8878084	2312	-94912783319040.00
2	2	2	-5032	-5304	-2811388	2811396	2312	- 150278573588480.00
3	3	1	-2142	-7038	-44945276	44945284	5202	-2432510825610240

D. Pattern: IV

Write 1 as,

$$1 = \frac{6^2 + 8^2}{10^2} \qquad (8)$$

Utilizing (8) in (5) and proceeding as in pattern-II, we get

$$c = -2ns^2 + 8s^2 + 2nt^2 - 8t^2 - 28nst + 12st$$
$$d = 14ns^2 - 6s^2 - 14nt^2 + 6t^2 - 4nst + 16st$$

Realizing that the goal is to determine the integer result, we get from taking s=10S, t=10T

$$E = 120nS^2 + 20S^2 - 120nT^2 - 20T^2 - 320nST + 280ST$$
$$H = -160nS^2 + 140S^2 + 160nT^2 - 140T^2 - 240nST - 40ST$$
$$k = 4(1-2800n^2S^4 + 16800n^2S^2T^2 - 38400n^2S^3T + 38400n^2ST^3$$
$$-2800n^2T^4 + 12400nS^4 - 74400nS^2T^2 + 27200nS^3T$$

$$-27200nST^3+12400nT^4-4800S^4+28800S^2T^2+5600S^3T$$
$$-5600ST^3-4800T^4)$$

$$l=4(1+2800n^2S^4-16800n^2S^2T^2+38400n^2S^3T-38400n^2ST^3$$
$$+2800n^2T^4-12400nS^4+74400nS^2T^2-27200nS^3T+27200nST^3$$
$$-12400nT^4+4800S^4-28800S^2T^2-5600S^3T+5600ST^3+4800T^4)$$

$$R=100S^2+100T^2$$

Observation

1) $\frac{-[H(S,S,1)]+7S^2}{S^2}=287$ is a kynea number.
2) $2[R(2,2)+H(1,1,1)+E(1,1,1)]=1000$ is a cube root.
3) $l(S,S,1)-480S^2E(S,1,1)-576R(1,1)-11200So_S$ $-4400Gno_S\equiv 0(\bmod 23596)$
4) $-2[k(1,1,1)+l(1,1,1)]=16$ is a perfect square.
5) $E(s,1,1)-140\Pr_s+90Gno_s+230=0$
6) $R(s,1)-25T_{10,s}-75s\equiv 0(\bmod 100)$

Some numerical examples are listed below.

TABLE 4

s	t	n	E	H	K	l	R	L.H.S=R.H.S
1	1	1	-40	-280	-76796	76804	200	-6144000000
2	1	2	60	-1580	-2492796	2492804	500	-6232000000000.00
6	2	1	4000	-4000	4	4	4000	0
-3	4	2	2500	7500	-49999996.00	50000004	2500	-3125000000000000.00

IV. CONCLUSION

We have constructed an infinite number of non intriguing numerical solutions to the bi-quadratic Diophantine equation with five unknowns. For the equation beneath cognizance, various pattern of solutions can always be obtained.

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