iJRASET
International Journal For Research in
Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH
IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 9 Issue: XII Month of publication: December 2021

DOI: https://doi.org/10.22214/ijraset.2021.39265

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.429
Volume 9 Issue XII Dec 2021- Available at www.ijraset.com

An Inventory Model for Deteriorating Items with Stock Dependent Demand and Partial Backlogging

Vikas Kumar[1], Vipin Kumar[2], Anupama Sharma[3]
[1]Research Scholar, Department of Mathematics, JJT University, Jhunjhunu, Rajasthan (India),
[2, 3]Department of Mathematics BKBIET, Pilani (India),

Abstract: In this paper, we formulate a deteriorating inventory model with stock-dependent demand Moreover, it is assumed that the shortages are allowed and partially backlogged, depending on the length of the waiting time for the next replenishment. The objective is to find the optimal replenishment to maximizing the total profit per unit time. We then provide a simple algorithm to find the optimal replenishment schedule for the proposed model. Finally, we use some numerical examples to illustrate the model.
Keywords- Inventory, Deteriorating items, Stock dependent demand, Partial backlogging

I. INTRODUCTION

Most researchers have considered demand in their models as fixed or varies as time whether it is linear or quadratic or other pattern. But one more parameter effect the demand and that is stock of a particular inventory. We see that demand may be increase or decrease due to stock level. The big stock level of a certain product some time increases the rate of demand and low stock level of such kind of products reduce the demand because the customers made a perception that the product is in last stage and will not be fresh. The vegetables, eggs, sweets etc. are the examples of that kind of products.

The fundamental result in the development of economic order quantity model with deterioration is that of Ghare and Schrader [1963] who considered an exponentially decaying inventory for a constant demand. However, as evident by chemical and basic sciences, the rate of deterioration especially with regard to perishable food items is seldom constant. Goyal and Giri [2001] presented several tendencies of the modeling of deteriorating inventory. Zauberman et al. [1989] presented a method for color retention of Litchi fruits with SO2 fumigation. In order to reduce the deterioration rate and to extend the expiration date of the product, preservation technologies like procedural changes and specialized equipment acquisition have been mathematically modeled by many researchers. In recent years, deteriorating inventory problems have been widely studied by many researchers. As presented by Wee [1995], deterioration was defined as decay, damage, spoilage, evaporation, obsolescence, pilferage, loss of utility or loss of marginal value of commodities that result in decreasing usefulness. Yang et al. [2015] developed the trade-off between preservation technology investment and the optimal dynamic trade credit for a deteriorating inventory model. Hsu et al. [2010] designed a deteriorating inventory model considering constants deterioration and demand rates, where in preservation technology also included. Optimization of the product portfolio has been recognized as a critical problem in industry, management, economy and so on. It aims at the selection of an optimal mix of the products to offer in the target market. Vipin Kumar [2020] developed an inventory model for deteriorating items with multivariate demand. Sanjay Sharma et al. developed a production inventory model for deteriorating Items with effect of price discount under the stock dependent demand, So while solving some problems a multi objective integer non-linear constraint model was developed by Ahmadi and Nikabadi [2019]. Having taken some realistic problem many researcher as Nadjafikhah [2017]; Ezzati et al. [2017]; Kazemi and Asl [2015] presented some special model. Wu et al. [2006] derived an optimal replenishment policy for items with non-instantaneous deterioration, stock-dependent demand and partial backlogging. Zhang et al. [1995] proposed a pricing policies for deteriorating items with preservation technology investment without shortage and stock. Pal et al. [2014] derived a deteriorating inventory model with stock and price-sensitive demand, where they assumed inflation and delay in payment. Shah and Shah [2014] attempt the same problem. However, they could not prove the existence of the optimal solution analytically. Moreover, they considered deterioration to start from the very beginning of replenishment time. Mishra [2014] developed an inventory model with controllable deterioration rate under time-dependent demand and time-varying holding cost. Liu et al. [2015] provided joint dynamic pricing and investment strategy for foods perishing at a constant rate with price and quality dependent demand. Zhang et al. [20] studied the integrated supply chain model for deteriorating items, in this model both manufacturer and retailer cooperatively invest in preservation technology in order to reduce their deterioration cost under different realistic scenarios.

Thereafter, Lu et al. [2016] presented an inventory model, in which they suggested the joint dynamic pricing and replenishment policy for a deteriorating item under limited capacity. Khedlekar et al. [2016] established an inventory model with declining demand under preservation technology investment. The chapter is organised after the introduction part as, we describe the fundamental notations and assumptions for the proposed model in Section 2. In Section 3, a mathematical model is established and solution procedure is discussed for maximizing the total profit, based on which an algorithm for finding the optimal policy is suggested. To illustrate the proposed model, numerical examples are provided in Section 4. In Section 5, sensitivity analysis of the optimal solution with respect to major parameters is carried out. Lastly, in Section 6, we draw the conclusions and give suggestions for future research.

II. ASSUMPTIONS AND NOTATIONS

Assumption and Notations: In this section, the following are the assumption and notations used to study and develop the model.

A. *Assumptions*

The following are the assumptions

1) The planning horizon is finite
2) The rate of deterioration is a two parameter Weibull distribution, $\theta(t) = \alpha\beta t^{\beta-1}$, where $0 \leq \alpha \ll 1$ and $\beta > 0$ are the scale and shape parameter respectively and $0 \leq \theta(t) << 1$
3) The demand rate $D(I(t), s) = \begin{cases} a + bI(t) - s, & 0 \leq t \leq v \\ a - s, & v \leq t \leq T \end{cases}$ where a, b demand parameters are and s is selling price. also $a > 0, 0 < b << 10 < b << 1$ and $a > s$
4) During the negative inventory period, the unfulfilled demand of products are backlogged; The rate of backlogging is a function of time that depends on the length of waiting for the next replenishment. Backlogging rate is $B(T - t) = \frac{1}{1+\delta(T-t)}$
5) Replenishment rate is instantaneous.
6) Holding cost is constant as $h(t) = h$ where $h > 0$
7) Lead time is zero

B. *Notations*

The following are the notations

1) A: Ordering cost per cycle.
2) p: Unit purchasing cost
3) s : Unit selling price
4) l: Unit lost sale cost.
5) C: Unit shortage cost.
6) Q_1: Inventory level during (0,v).
7) Q_2: Inventory level during [v,T].
8) Q the total retailer's order quantity
9) v: the time from the shortages starts
10) T: inventory cycle length
11) $I_1(t)$: Positive Inventory function at $t \in [0, v]$
12) $I_2(t)$: Negative Inventory function at $t \in [v, T]$
13) $P(s, v)$ the retailer total cost optimal value

III. MATHEMATICAL FORMULATION AND SOLUTION

In this section, the Behaviour of the inventory level with respect to time is shown in fig.1. The replenishment cycle length is T and Q_1 denote the maximum positive inventory and Q_2 denote the Maximum backorder quantity. The total cycle time interval $[0, T]$is divided into two subintervals, the first sub interval$[0, v]$is the part which show the positive inventory and $[v, T]$is the second subinterval which represent the backorders. The cycle is start at $t = 0$ with maximum inventory level Q_1 and the inventory stocks goes to zero level at $t = v$ due to cumulative effect of demand and instantaneous deterioration. The shortages occur in second subinterval$[v, T]$. the shortages start at time v under the partial backlogging effect. The occurring unsatisfied demand is partially backlogged due to some specific reason of the customers. The following are instantaneous state represented by the differential equations of the inventory system

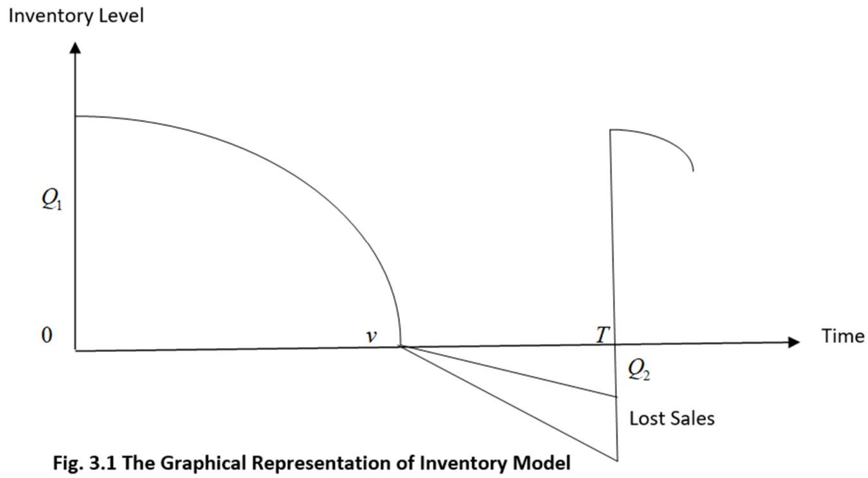


Fig. 3.1 The Graphical Representation of Inventory Model

$$\frac{dI_1(t)}{dt} - \theta(t)I_1(t) = -D(s, I(t)) \qquad 0 \le t \le v \qquad 3.1$$

$$\frac{dI_2(t)}{dt} = -D(s, I(t))B(T-t) \qquad v \le t \le T \qquad 3.2$$

The boundary conditions

$I_1(v) = I_2(v) = 0$

The solution of above differential equation are

$$I_1(t) = (a-s)\left[\begin{array}{c}(v-t) + \frac{b}{2}(v^2 - t^2)\\ + \frac{\alpha}{\beta+1}\left(v^{\beta+1} - t^{\beta+1}\right) - bt(v-t) - \alpha t^{\beta}(v-t)\end{array}\right] \qquad 0 \le t \le v \qquad 3.3$$

$$I_2(t) = \frac{(a-s)}{\delta} log\left[\frac{1+\delta(T-t)}{1+\delta(T-v)}\right] \qquad v \le t \le T \qquad 3.4$$

The maximum positive inventory, in time interval$[0, v]$ is

$$Q_1 = I_1(0) = (a-s)\left[v + \frac{b}{2}v^2 + \frac{\alpha}{\beta+1}v^{\beta+1}\right] \qquad 3.5$$

The maximum backordered units are in time interval$[v, T]$

$$Q_2 = -I_2(T) = \frac{(a-s)}{\delta} log[1 + \delta(T-v)] \qquad 3.6$$

Hence, during the time interval$[0, T]$, the total order size units are

$$Q = Q_1 + Q_2 = (a-s)\left\{v + \frac{b}{2}v^2 + \frac{\alpha}{\beta+1}v^{\beta+1} + \frac{1}{\delta}log[1 + \delta(T-v)]\right\} \qquad 3.7$$

The total profit consists of the following revenue and costs

A. Ordering Cost

$$OC = A \qquad 3.8$$

B. Holding Cost

$$HC = h\int_0^v I_1(t)e^{-Rt}dt\text{v}$$

$$HC = h\int_0^v \left[(a-s)\left[\begin{array}{c}(v-t) + \frac{b}{2}(v^2 - t^2) + \frac{\alpha}{\beta+1}\left(v^{\beta+1} - t^{\beta+1}\right)\\ -b(tv - t^2) - \alpha\left(vt^{\beta} - t^{\beta+1}\right)\end{array}\right]\right]e^{-Rt}dt$$

$$= h(a-s)\left[\frac{v^2}{2} + \frac{bv^3}{6} - \frac{Rv^3}{6} + \frac{v^{2+\beta}\alpha\beta}{(1+\beta)(2+\beta)}\right] \qquad 3.9$$

C. Shortage Cost

$$SC = c_2 \int_v^T -I_2(t) e^{-Rt} dt$$

$$SC = c_2 \int_v^T -\left\{\frac{(a-s)}{\delta} log\left[\frac{1+\delta(T-t)}{1+\delta(T-v)}\right]\right\} e^{-Rt} dt$$

$$= -c_2(a-s)\begin{bmatrix} T - v - \frac{3RT^2}{4} + \frac{RTv}{2} + \frac{Rv^2}{4} \\ + \frac{R(v-T)}{2\delta} + \left(\frac{R}{2\delta^2} + \frac{RT-1}{\delta}\right) log[1 + (T-v)\delta] \end{bmatrix} \quad 3.10$$

D. Lost Sales Cost

$$LSC = l \int_v^T (1 - B(T-t)) D(t) e^{-Rt} dt$$

$$LSC = l \int_v^T \left(1 - \frac{1}{1+\delta(T-t)}\right)(a-s) e^{-Rt} dt$$

$$= l(a-s)\begin{bmatrix} T - v + \frac{R(v^2-T^2)}{2} + \frac{R(v-T)}{\delta} \\ + \left[\frac{R}{\delta^2} + \frac{RT-1}{\delta}\right] log[1 + (T-v)\delta] \end{bmatrix} \quad 3.11$$

E. Purchase Cost

$$PC = pQe^{-RT}$$

$$PC = p[Q_1 + Q_2]e^{-RT}$$

$$= p(a-s)\left[v + \frac{bv^2}{2} + \frac{\alpha v^{\beta+1}}{\beta+1} + \frac{log(1+\delta(T-v))}{\delta}\right] e^{-RT} \quad 3.12$$

F. Sales Revenue Cost

The sales revenue cost is calculated as

$$SR = s\left[\int_0^v D(t) e^{-Rt} dt + \int_v^T D(t) e^{-Rt} dt\right]$$

$$SR = s\begin{bmatrix} \int_0^v \left\{ a + b\left\{ a - s) \begin{bmatrix} (v-t) + \frac{b}{2}(v^2 - t^2) \\ + \frac{\alpha}{\beta+1}\left(v^{\beta+1} - t^{\beta+1}\right) - \left(bt - t^\beta \alpha\right)(v-t) \end{bmatrix} \right\} - s \right\} e^{-Rt} dt \\ + \int_v^T (a-s) e^{-Rt} dt \end{bmatrix}$$

$$= s(a-s)\left(\begin{matrix} T - v - \frac{1}{2}(T^2 - v^2)R \\ + \left(v + \frac{1}{24} bv^2 \left(\begin{matrix} 12 - 4Rv + bv(4 - Rv) \\ - \frac{12 v^\beta \alpha\beta (Rv(1+\beta) - 2(3+\beta))}{(1+\beta)(2+\beta)(3+\beta)} \end{matrix}\right)\right) \end{matrix}\right) \quad 3.13$$

Therefore, the total profit per unit items per unit time is calculated as

$$P(s, v) = \frac{1}{T}\{S.R. - O.C. - H.C. - B.C. - L.S.C. - P.C.\}$$

$$P(s, v) = \frac{1}{T} s(a-s)\left(\begin{matrix} T - v - \frac{1}{2}(T^2 - v^2)R \\ + \left(v + \frac{1}{24} bv^2 \left(\begin{matrix} 12 - 4Rv + bv(4 - Rv) \\ - \frac{12 v^\beta \alpha\beta (Rv(1+\beta) - 2(3+\beta))}{(1+\beta)(2+\beta)(3+\beta)} \end{matrix}\right)\right) \end{matrix}\right) - \frac{1}{T} A - \frac{1}{T} h(a-s)\left[\frac{v^2}{2} + \frac{bv^3}{6} - \frac{Rv^3}{6} + \frac{v^{2+\beta}\alpha\beta}{(1+\beta)(2+\beta)}\right] +$$
$$\frac{1}{T} c_2(a-s)\left[T - v - \frac{3RT^2}{4} + \frac{RTv}{2} + \frac{Rv^2}{4} + \frac{R(v-T)}{2\delta} + \left(\frac{R}{2\delta^2} + \frac{RT-1}{\delta}\right) log[1 + (T-v)\delta]\right] - \frac{1}{T} l(a-s)\left[T - v + \frac{R(v^2-T^2)}{2} + \frac{R(v-T)}{\delta} + \right.$$
$$\left.\left[\frac{R}{\delta^2} + \frac{RT-1}{\delta}\right] log[1 + (T-v)\delta]\right] - \frac{1}{T} p(a-s)\left[v + \frac{bv^2}{2} + \frac{\alpha v^{\beta+1}}{\beta+1} + \frac{log(1+\delta(T-v))}{\delta}\right] e^{-RT} \quad 3.14$$

IV. SOLUTION PROCEDURE

We wish to maximize the total profit function $P(s, v)$

1) Step 1. The necessary conditions to maximize the total profit function $P(s, v)$ are

$$\frac{\partial P(s,v)}{\partial s} = 0 \text{ and } \frac{\partial P(s,v)}{\partial v} = 0 \quad 3.15$$

2) Step 2.Using the software Mathematica 11, we calculated the optimal value of s* and v* by equation 3.15.
3) Step 3. The optimal value of s*and v*, satisfy the sufficient conditions for maximizing the total inventory profit function $\frac{\partial^2 P(s,v)}{\partial s^2} < 0, \frac{\partial^2 P(s,v)}{\partial v^2} < 0 \text{and} \left(\frac{\partial^2 P(s,v)}{\partial s^2}\right)\left(\frac{\partial^2 P(s,v)}{\partial v^2}\right) - \left(\frac{\partial^2 P(s,v)}{\partial s \partial v}\right) > 0$
4) Step 4. In addition, at $s = s*$ and $v = v*$ the optimal value of the total Inventory cost is determined by equation 3.14 and the optimal value is $P(s, v) = P*(s*, v*)$

V. NUMERICAL EXAMPLE

Consider the following numerical values of parameters to illustrate the profit function $\alpha = 0.3$, $\beta = 6$, $h = 25$, $A = 500$, $a = 22$, $b = 30$, $\delta = 0.04$, $p = 10$, $c = 40$, $l = 50$, $R = 0.06$, $T = 3$Use Mathematica-11 to obtain the optimal solution for v and s Based on the above numerical values of used parameters the optimal solution is $s^* = 19.519 v^* = 1.521$ $P(s^*, v^*) = 35493.68$

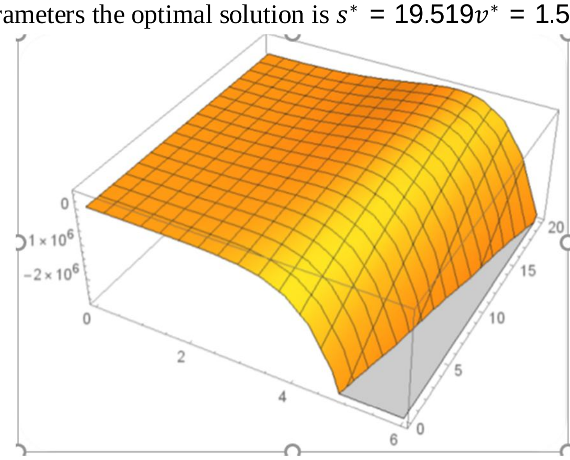


Fig. 3.2 $P(s, v)$ v/s S and V

From fig. 3.2, the concavity of the total cost function provide that the total profit function of the model is maximize with respect to s and v.

VI. SENSITIVITY ANALYSIS

In this part, the sensitivity analysis has been discussed by taking the different values of the used parameter as -50%, -25%,0, +25%,50% with respect to the optimal value of the total profit and keeping remaining parameter unchanged.

Parameter	%	Value	Total Profit	Parameter	%	Value	Total Profit
∝	50%	0.45	7658.08	δ	50%	0.6	7670.3
	25%	0.375	7664.07		25%	0.5	7670.18
	0	0.3	7670.05		0	0.4	7670.05
	-25%	0.225	7676.03		-25%	0.3	7669.92
	-50%	0.15	7682.02		-50%	0.2	7669.79
β	50%	9	7631.05	p	50%	15	7537.19
	25%	7.5	7655.56		25%	12.5	7603.62
	0	6	7670.05		0	10	7670.05
	-25%	4.5	7683.87		-25%	7.5	7736.5

	-50%	3	7683.87		-50%	5	7802.91
h	50%	37.5	7471.55	C	50%	60	7655.26
	25%	31.25	7570.8		25%	50	7662.66
	0	25	7670.05		0	40	7670.05
	-25%	18.75	7769.3		-25%	30	7677.44
	-50%	12.5	7868.55		-50%	20	7684.84
A	50%	750	7752.32	l	50%	45	7669.59
	25%	625	7714.91		25%	37.5	7669.82
	0	500	7670.05		0	30	7670.05
	-25%	375	7625.76		-25%	22.5	7670.28
	-50%	250	7575.6		-50%	15	7670.51
a	50%	33	41676.7	R	50%	0.09	7597.84
	25%	27.5	24673.5		25%	0.075	7634.02
	0	22	7670.05		0	0.06	7670.05
	-25%	16.5	9333.29		-25%	0.045	7705.36
	-50%	11	26336.6		-50%	0.03	7740.1
b	50%	45	17779.5	T	50%	4.5	5074.5
	25%	37.5	12204.4		25%	3.75	6117.43
	0	30	7670.05		0	3	7670.05
	-25%	22.5	4176.36		-25%	2.25	10239.1
	-50%	15	1725.38		-50%	1.5	15344.2

VII. OBSERVATIONS

From tables (3), the following facts are apparent

1) With increment of a, $b$and δ, the total profit $P(s,v)$ shows increasing Behaviour

2) If α, β, h, A, and T are increases then the total profit function $P(s,v)$ decreases.

VIII. CONCLUSION

In this chapter, the model is profit maximize policy for deteriorating products by taking the multivariate demand which is price, stock and time dependent. The rate of deterioration is two parameter Weibull function. In this model shortages is also considered and partial backlogged with variable rate. The model is solved analytically to check the optimality. Numerical examples and graphs are also demonstrated to validate the policy along with sensitivity with respect to different parameters used in the model. Some other extension can be made by assuming more realistic assumptions like as non-zero lead time, stochastic demand rate etc.

REFERENCES

[1] C. T. Yang, C. Y. Dye and J. F. Ding, Optimal dynamic trade credit and preservation technology allocation for a deteriorating inventory model,Comp. Ind. Eng. 87 (2015) 356–369.

[2] G. Liu, J. Zhang and W. Tang, Joint dynamic pricing and investment strategy for perishable foods with price quality dependent demand, Ann. of Oper. Res. 226(1) (2015) 397–416.

[3] G. Zauberman, R. Ronen, M. Akerman and Y. Fuchs, Low PH treatment protects litchi fruit color, Int. Soc. Hort. Sci. 269, (1989) 309–314.

[4] H. M. Wee, A deterministic lot-size inventory model for deteriorating items with shortages and a declining market, Comp Oper. Res. 22(3) (1995) 345–356.

[5] J. X. Zhang, Z. Y. Bai and W. S. Tang, Optimal pricing policy for deteriorating items with preservation technology investment, J. Ind. Manag. Optim. 10(4) (2014) 1261–1277.

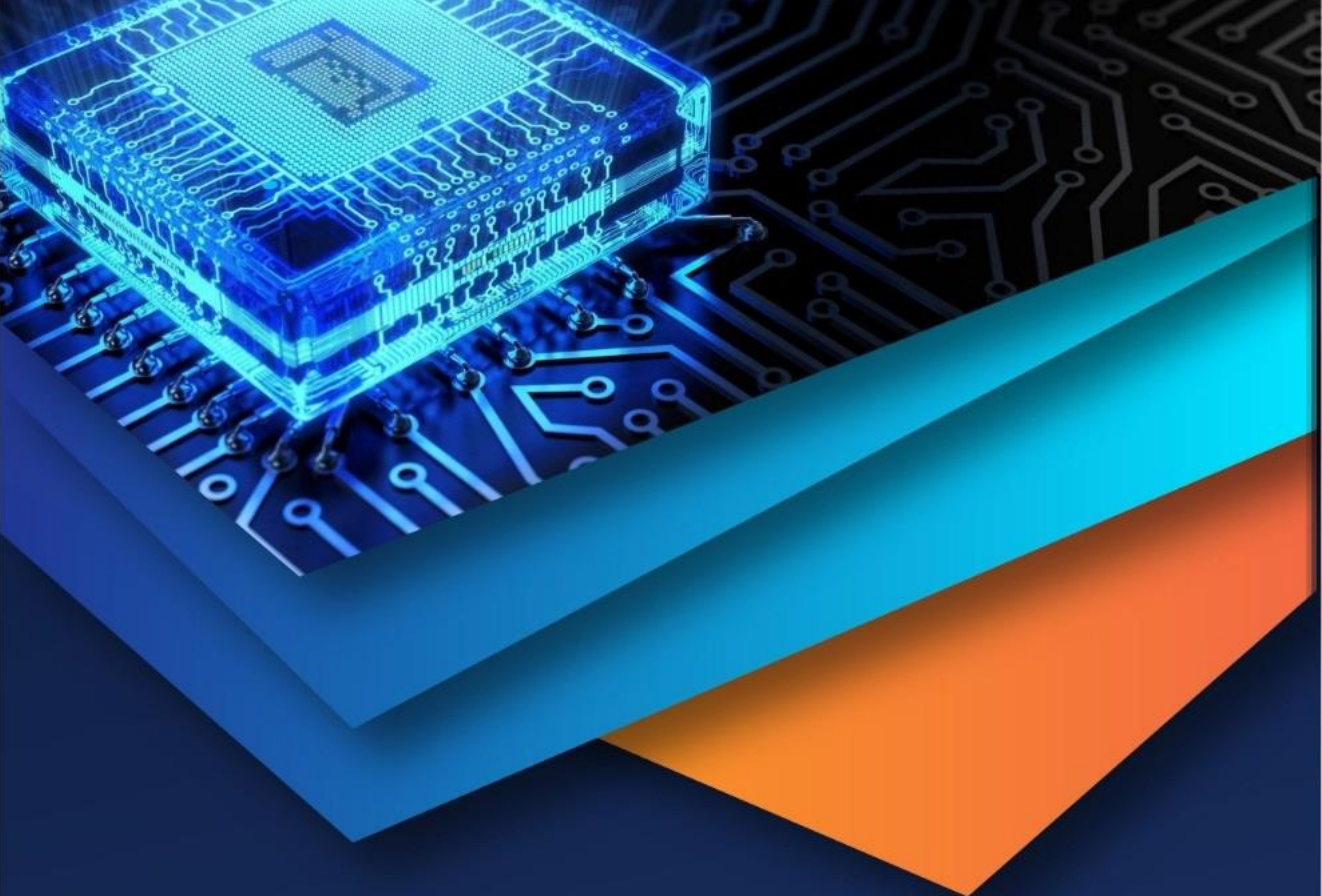
[6] J. Zhang, Q. Wei, Q. Zhang and W. Tang, Pricing, service and preservation technology investments policy for deteriorating items under common resource constraints, Comp. Ind. Eng. 95 (2016) 1–9.

[7] K. S. Wu, L. Y. Ouyang and C. T. Yang, An optimal replenishment policy for non- instantaneous deteriorating items with stock-dependent demand and partial backlogging, Int. J. Prod. Econ. 101(2) (2006) 369–384.

[8] L. Lu, J. Zhang and W. Tang, Optimal dynamic pricing and replenishment policy for perishable items with inventory-level-dependent demand, Int. J. Sys. Sci. 47(6) (2016) 1480–1494.

[9] M. Nadjafikhah and S. Shagholi, Mathematical modeling of optimized SIRS epidemic model and some dynamical behaviors of the solution, Int. J. Nonlinear Anal. Appl. 8(2) (2017) 125–134.

[10] N. D. Ahmadi, M. S. Nikabadi, A non-linear multi objective model for the product portfolio optimization: An integer programming, Int. J. Nonlinear Anal. Appl. 8(2) (2019) 231–239.
[11] N. H. Shah and A. D. Shah, Optimal cycle time and preservation technology investment for deteriorating items with price-sensitive stock-dependent demand under inflation, J. Phy.: Conf. Ser. 495(1) (2014) 1–10.
[12] P. H. Hsu, H. M. Wee and H. M. Teng, Preservation technology investment for deteriorating inventory, Int. J. Prod. Econ. 124(2) (2010) 388–394.
[13] P. M. Ghare and G. H. Schrader, A model for exponentially decaying inventory system,J. Ind. Engrg. 14 (1963) 238–243.
[14] P. S. You, Inventory policy for products with price and time-dependent demands,J. Oper. Res. Soc. 56(7) (2005). 870–873.
[15] R. Ezzati, S. Abbasbandy and H. Behforooz, Interpolation of fuzzy data by using at end fuzzy splines, Int. J. Nonlinear Anal. Appl. 8(2) (2017) 89–97.
[16] R. Kazemi and M. Q. V. Asl, Probabilistic analysis of the asymmetric digital search trees, Int. J. Non-linear Anal. Appl. 6(2) (2015) 161–173.
[17] S. K. Goyal and B. C. Giri, Recent trends in modeling of deteriorating inventory, Eur. J. Oper. Res. 134(10) (2001) 1–16.
[18] S. Pal, G. S. Mahapatra and G.P. Samanta, An inventory model of price and stock dependent demand rate with deterioration under in nation and delay in payment, Int. J. Sys. Ass. Eng. Manag. 5(4) 2014 591–601.
[19] Sanjay Sharma, Vipin Kumar, Anand Tyagi , A Production Inventory Model for Deteriorating Items with Effect of Price Discount under the Stock Dependent Demand, RT&A, Special Issue No 2(64), Volume 16, November 2021, 213-224
[20] U. K. Khedlekar, D. Shukla and A. Namdeo Pricing policy for declining demand using item preservation technology, SpringerPlus 5(1) (2016) 19–57.
[21] U. Mishra, L. E. Barron, S. Tiwari, A. A. Shaikh and G. T. Garza,An inventory model under price and stock dependent demand for controllable deterioration rate with shortage and preservation technology investment, Ann. Oper. Res. 254(2) (2017) 165–190.
[22] V. K. Mishra, Deteriorating inventory model with controllable deterioration rate for time dependent demand and time varying holding cost, Yugosl. J. Oper. Res. 24(1) (2014) 87–-98.
[23] Vipin Kumar, Anupama Sharma, Amit Kumar, And C. B. Gupta, An Inventory Model For Deteriorating Items With multivariate Demand And Trade Credit, Advances In Mathematics: Scientific Journal9(2020), No.9, 7501–7514



10.22214/IJRASET

45.98

IMPACT FACTOR:
7.129

IMPACT FACTOR:
7.429

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY