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Lehmer-4 Mean Labeling of Graphs

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***Abstract:** A graph G=(V,E) with p vertices and q edges is called Lehmer-4 mean graph, if it is possible to label verticesx€V with distinct label g(x) from 2,4,6,8,……….. 2p in such a way thatwhen eachedge e=uv islabeledwithg(e=uv)=*$\left\lfloor \frac{g(u)^4+g(v)^4}{g(u)^3+g(v)^3} \right\rfloor$ *(or)*$\left\lceil \frac{g(u)^4+g(v)^4}{g(u)^3+g(v)^3} \right\rceil$*,then the edge labels are distinct. In this case,g is called Lehmer-4 meanlabelingof G. Inthis paper, Lehmer-4 mean labeling have been introduced.*
***Keywords:** Labeling, Graceful Graph, Multiplicative Labeling*

I. INTRODUCTION

Graph labeling is an assignment of integer to its vertices or edges subject to some certain condition.All Graphs in this paper are considered as finite and undirected. The symbols V(G) and E(G) denote the vertex set and edge set of a graph G. The cardinality of the vertex set is called the order of G denoted by p. The cardinality of the edge set is called the size of G denoted by q edges is called a (p,q) graph. A graph labeling is an assignment of integers to the vertices or edges. Bloom and Hsu[2] extended the notion of graceful labeling to directed graphs. Graceful signed graphs f(uv) is the difference between f(v) and f(v), that is f(uv) = f(v) – f(u). Shalini, Paul Dhayabaran [14] introduced the concept A Study on Root Mean Square Labelings in Graphs. Shalini, Paul Dhayabaran [13] defined An Absolute Differences of Cubic and Square Difference Labeling. Shalini, Gowri, Paul Dhayabaran [15] discussed An Absolute Differences of Cubic and Square Difference LabelingFor Some Families of Graphs. Shalini, Sri Harini, Paul Dhayabaran [19] introduced Sum of an Absolute Differences of Cubic AndSquare Difference Labeling For Cycle Related Graphs. Shalini, Gowri, Paul Dhayabaran [16] studied An Absolute Differences of Cubic and Square Difference Labeling for Some Shadow and Planar Graphs. Shalini, Subha, Paul Dhayabaran [20] investigated A Study on Disconnected Graphs for an Absolute Difference Labeling. Shalini, Subha, Paul Dhayabaran [22] discussed A Study on Disconnected Graphs for Sum of an Absolute Difference of Cubic and Square Difference Labeling. Shalini, Sri Harini, Paul Dhayabaran [21] extended Sum of an Absolute Differences of Cubic And Square Difference Labeling For Path Related Graphs.For detailed survey J.A Gallian survey [1] is referred and for standard terminologies and notations HararyF [2] is referred.

II. BASIC DEFINITIONS

1) *Definition2.1:* A graph G is said to be Lehmer-4 mean graph if it admits lehmer-4 mean labeling.
2) *Definition2.2:* A pathis represented by a walk in which vertices are distinct. A path with n vertices is denoted byp_n
3) *Definition2.3:* The Comb $P_n \Theta K_1$ is a graph obtained by joining a single pendant edge to each vertex of a path
4) *Definition2.4:* The graph $P_n \Theta K_{1,2}$is obtained by attaching complete bipartite graph $K_{1,2}$ to each vertex of path P_n
5) *Definition2.5:* The graph $P_n \Theta K_{1,3}$is obtained by attaching complete bipartite graph $K_{1,3}$ to each vertex of path P_n

III. MAIN RESULTS

A. *Theorem3.1*

The Path P_n is a Lehmer-4 mean graph for $n \geq 2$
Proof:
Let G be a graph of path P_n
The path P_n consists of n vertices and n-1 edges
Define f:V(G) → {2,4,6,8,……,2n} by $f(v_i) = 2i$; $1 \leq i \leq n$
Then the edge labels as $f(e_i) = 2i + 1$; $1 \leq i \leq n$
The edges of the path graph receive distinct numbers
Hence, the pathP_n is a Lehmer-4 mean graph.

Example3.1

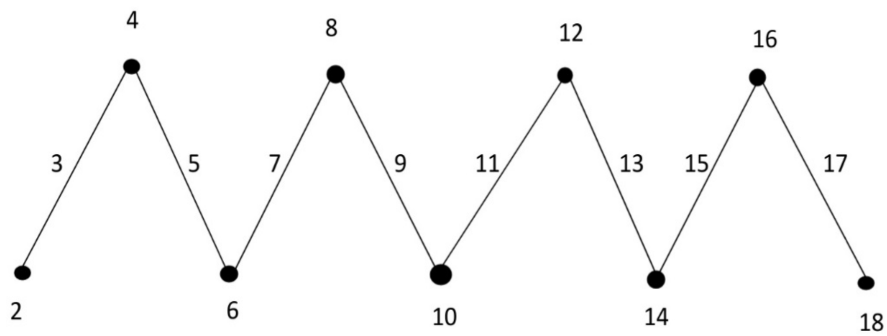

Figure3.1 : Path P_9

B. Theorem3.2

The Comb $P_n \Theta K_1$ is a Lehmer-4 mean graph forn ≥ 2

Proof:

Let G be a graph of comb ($P_n \Theta K_1$)

Let ($P_n \Theta K_1$) be a comb with vertices as $v_1, v_2, \ldots, v_n; w_1, w_2, \ldots, w_n$

Definef:V(G) $\rightarrow$ {2,4,6,8,……,4n} by ,

$f(v_i)= 4i-2$; $1 \leq i \leq n$

$f(w_i)=4i$; $1 \leq i \leq n$

The edges of the comb graph receive distinct numbers

Hence, the comb ($P_n \Theta K_1$) is said aLehmer-4 mean graph.

Example 3.2

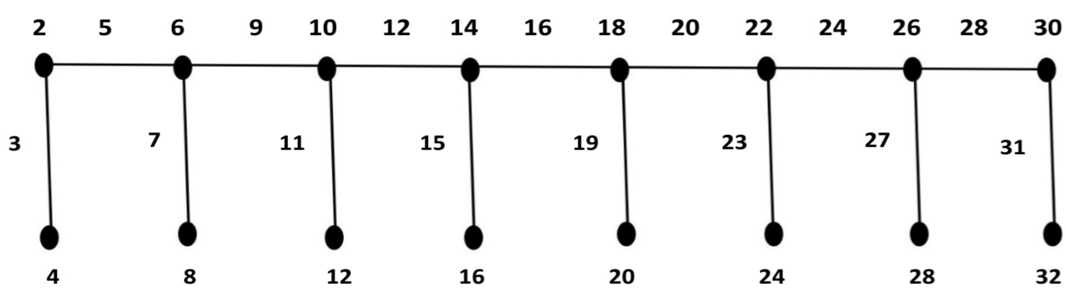

Figure3.2 : Comb $P_8 \Theta K_1$

C. *Theorem3.3*

$P_n \Theta K_{1,2}$ is a Lehmer-4 mean graph for $n \geq 2$

Proof:

Let G be a graph of $P_n \Theta K_{1,2}$

Let $P_n \Theta K_{1,2}$ with vertices as $v_1, v_2, \ldots, v_n; w_1, w_2, \ldots, w_n; u_1, u_2, \ldots, u_n$

Definef:V(G) $\rightarrow$ {2,4,6,8,……,6n} by ,

$f(v_i)=6i-4$,$1 \leq i \leq n$

$f(w_i)=6i-2$,$1 \leq i \leq n$

$f(u_i)=6i$,$1 \leq i \leq n$

The edges of the graph $P_n \Theta K_{1,2}$ receive distinct numbers

Hence, $P_n \Theta K_{1,2}$ is a Lehmer-4 mean graph.

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Example 3.3

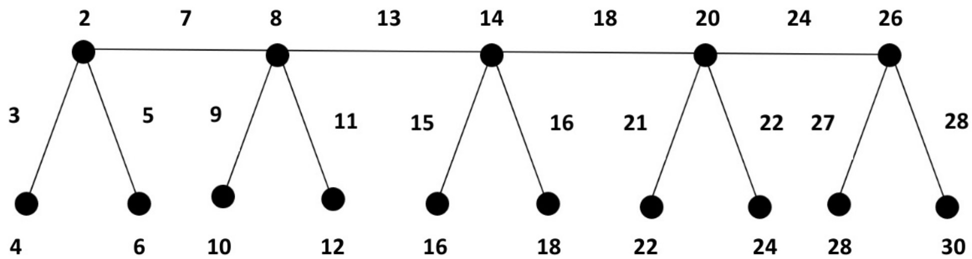

Figure3.3 :$P_5 \Theta K_{1,2}$

D. *Theorem 3.4*

$P_n \Theta K_{1,3}$ is a Lehmer-4 meangraph for $n \geq 2$

Proof:

Let G be a graph of $P_n \Theta K_{1,3}$

Let $P_n \Theta K_{1,3}$with vertices as $v_1, v_2, \ldots, v_n$; $w_1, w_2, \ldots, w_n$; $u_1, u_2, \ldots, u_n$; $x_1, x_2, \ldots, x_n$

Define f:V(G) $\rightarrow$ {2,4,6,8,……,8n} by ,

$f(v_i)=8i-6$,$1 \leq i \leq n$

$f(w_i)=8i-4$,$1 \leq i \leq n$

$f(u_i)=8i-2$,$1 \leq i \leq n$

$f(x_i)=8i$,$1 \leq i \leq n$

The edges of the graph$P_n \Theta K_{1,3}$ receive distinct numbers

Hence, $P_n \Theta K_{1,3}$ is a Lehmer-4 mean graph

Example 3.4

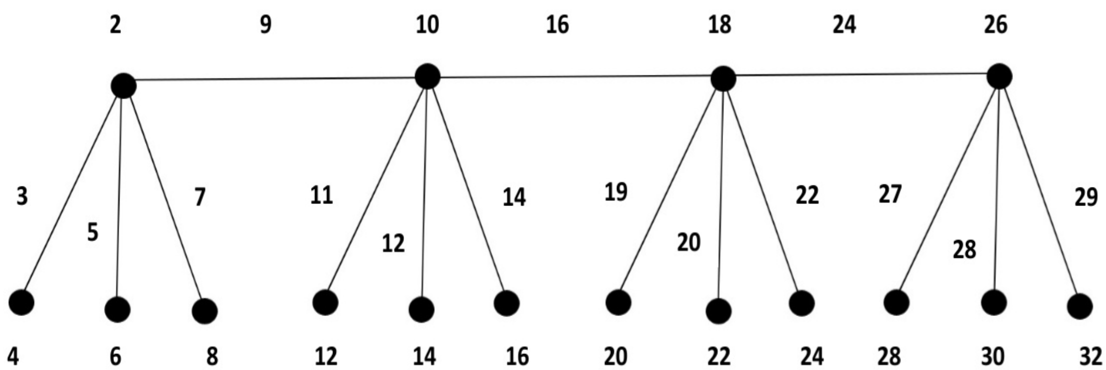

Figure3.4 :$P_4 \Theta K_{1,3}$

IV. CONCLUSION

Finally, we conclude that path, comb, $P_n \Theta K_{1,2}$, $P_n \Theta K_{1,3}$is a Lehmer-4 mean graph.

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