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# Lehmer-4 Mean Labeling of Graphs

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**Abstract:** A graph  $G=(V,E)$  with  $p$  vertices and  $q$  edges is called Lehmer-4 mean graph, if it is possible to label vertices  $x \in V$  with distinct label  $g(x)$  from  $2,4,6,8,\dots,2p$  in such a way that when each edge  $e=uv$  is labeled with  $g(e=uv) = \left\lfloor \frac{g(u)^4 + g(v)^4}{g(u)^2 + g(v)^2} \right\rfloor$

(or)  $\left\lfloor \frac{g(u)^4 + g(v)^4}{g(u)^2 + g(v)^2} \right\rfloor$ , then the edge labels are distinct. In this case,  $g$  is called Lehmer-4 mean labeling of  $G$ . In this paper, Lehmer-4 mean labeling have been introduced.

**Keywords:** Labeling, Graceful Graph, Multiplicative Labeling

## I. INTRODUCTION

Graph labeling is an assignment of integer to its vertices or edges subject to some certain condition. All Graphs in this paper are considered as finite and undirected. The symbols  $V(G)$  and  $E(G)$  denote the vertex set and edge set of a graph  $G$ . The cardinality of the vertex set is called the order of  $G$  denoted by  $p$ . The cardinality of the edge set is called the size of  $G$  denoted by  $q$  edges is called a  $(p,q)$  graph. A graph labeling is an assignment of integers to the vertices or edges. Bloom and Hsu[2] extended the notion of graceful labeling to directed graphs. Graceful signed graphs  $f(uv)$  is the difference between  $f(v)$  and  $f(u)$ , that is  $f(uv) = f(v) - f(u)$ . Shalini, Paul Dhayabaran [14] introduced the concept A Study on Root Mean Square Labelings in Graphs. Shalini, Paul Dhayabaran [13] defined An Absolute Differences of Cubic and Square Difference Labeling. Shalini, Gowri, Paul Dhayabaran [15] discussed An Absolute Differences of Cubic and Square Difference Labeling For Some Families of Graphs. Shalini, Sri Harini, Paul Dhayabaran [19] introduced Sum of an Absolute Differences of Cubic And Square Difference Labeling For Cycle Related Graphs. Shalini, Gowri, Paul Dhayabaran [16] studied An Absolute Differences of Cubic and Square Difference Labeling for Some Shadow and Planar Graphs. Shalini, Subha, Paul Dhayabaran [20] investigated A Study on Disconnected Graphs for an Absolute Difference Labeling. Shalini, Subha, Paul Dhayabaran [22] discussed A Study on Disconnected Graphs for Sum of an Absolute Difference of Cubic and Square Difference Labeling. Shalini, Sri Harini, Paul Dhayabaran [21] extended Sum of an Absolute Differences of Cubic And Square Difference Labeling For Path Related Graphs. For detailed survey J.A Gallian survey [1] is referred and for standard terminologies and notations HararyF [2] is referred.

## II. BASIC DEFINITIONS

- 1) *Definition 2.1:* A graph  $G$  is said to be Lehmer-4 mean graph if it admits lehmer-4 mean labeling.
- 2) *Definition 2.2:* A path is represented by a walk in which vertices are distinct. A path with  $n$  vertices is denoted by  $P_n$
- 3) *Definition 2.3:* The Comb  $P_n \circ K_1$  is a graph obtained by joining a single pendant edge to each vertex of a path
- 4) *Definition 2.4:* The graph  $P_n \circ K_{1,2}$  is obtained by attaching complete bipartite graph  $K_{1,2}$  to each vertex of path  $P_n$
- 5) *Definition 2.5:* The graph  $P_n \circ K_{1,3}$  is obtained by attaching complete bipartite graph  $K_{1,3}$  to each vertex of path  $P_n$

## III. MAIN RESULTS

### A. Theorem 3.1

The Path  $P_n$  is a Lehmer-4 mean graph for  $n \geq 2$

Proof:

Let  $G$  be a graph of path  $P_n$

The path  $P_n$  consists of  $n$  vertices and  $n-1$  edges

Define  $f: V(G) \rightarrow \{2,4,6,8,\dots,2n\}$  by  $f(v_i) = 2i$ ;  $1 \leq i \leq n$

Then the edge labels as  $f(e_i) = 2i + 1$ ;  $1 \leq i \leq n$

The edges of the path graph receive distinct numbers

Hence, the path  $P_n$  is a Lehmer-4 mean graph.

Example3.1

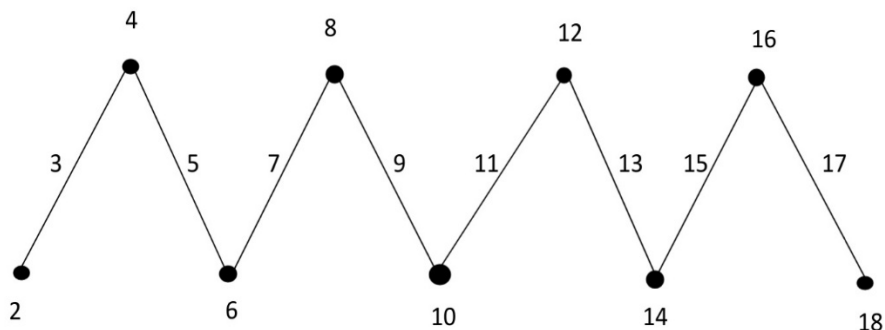


Figure3.1 : Path  $P_9$

B. Theorem3.2

The Comb  $P_n \odot K_1$  is a Lehmer-4 mean graph for  $n \geq 2$

Proof:

Let  $G$  be a graph of comb  $(P_n \odot K_1)$

Let  $(P_n \odot K_1)$  be a comb with vertices as  $v_1, v_2, \dots, v_n; w_1, w_2, \dots, w_n$

Define:  $V(G) \rightarrow \{2, 4, 6, 8, \dots, 4n\}$  by ,

$$f(v_i) = 4i - 2 ; 1 \leq i \leq n$$

$$f(w_i) = 4i ; 1 \leq i \leq n$$

The edges of the comb graph receive distinct numbers

Hence, the comb  $(P_n \odot K_1)$  is said a Lehmer-4 mean graph.

Example 3.2

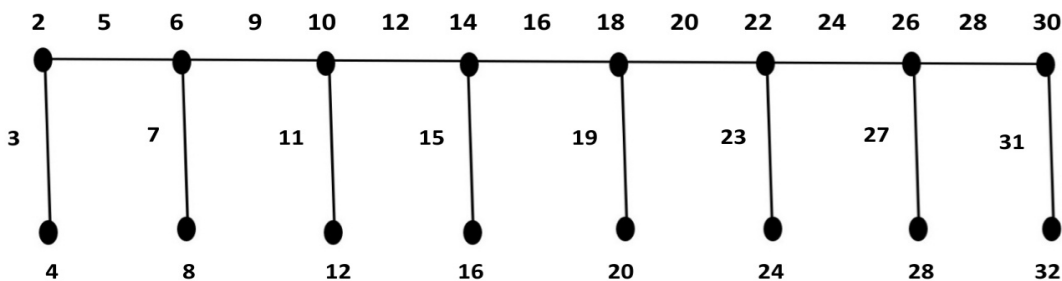


Figure3.2 : Comb  $P_8 \odot K_1$

C. Theorem3.3

$P_n \odot K_{1,2}$  is a Lehmer-4 mean graph for  $n \geq 2$

Proof:

Let  $G$  be a graph of  $P_n \odot K_{1,2}$

Let  $P_n \odot K_{1,2}$  with vertices as  $v_1, v_2, \dots, v_n; w_1, w_2, \dots, w_n; u_1, u_2, \dots, u_n$

Define:  $V(G) \rightarrow \{2, 4, 6, 8, \dots, 6n\}$  by ,

$$f(v_i) = 6i - 4 , 1 \leq i \leq n$$

$$f(w_i) = 6i - 2 , 1 \leq i \leq n$$

$$f(u_i) = 6i , 1 \leq i \leq n$$

The edges of the graph  $P_n \odot K_{1,2}$  receive distinct numbers

Hence,  $P_n \odot K_{1,2}$  is a Lehmer-4 mean graph.

Example 3.3

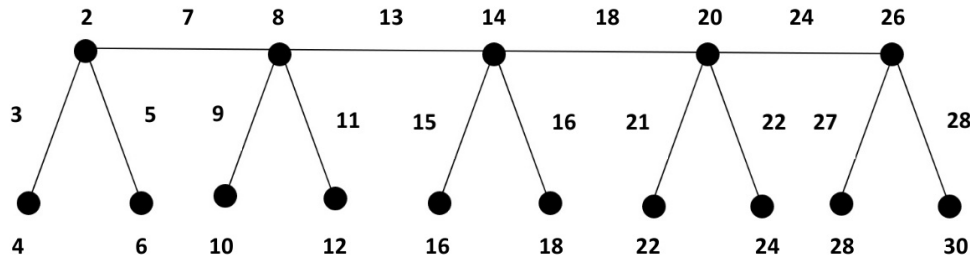


Figure3.3 : $P_5OK_{1,2}$

D. Theorem 3.4

$P_nOK_{1,3}$  is a Lehmer-4 meangraph for  $n \geq 2$

Proof:

Let  $G$  be a graph of  $P_nOK_{1,3}$

Let  $P_nOK_{1,3}$  with vertices as  $v_1, v_2, \dots, v_n; w_1, w_2, \dots, w_n; u_1, u_2, \dots, u_n; x_1, x_2, \dots, x_n$

Define  $f: V(G) \rightarrow \{2, 4, 6, 8, \dots, 8n\}$  by ,

$$f(v_i) = 8i - 6, 1 \leq i \leq n$$

$$f(w_i) = 8i - 4, 1 \leq i \leq n$$

$$f(u_i) = 8i - 2, 1 \leq i \leq n$$

$$f(x_i) = 8i, 1 \leq i \leq n$$

The edges of the graph  $P_nOK_{1,3}$  receive distinct numbers

Hence,  $P_nOK_{1,3}$  is a Lehmer-4 mean graph

Example 3.4

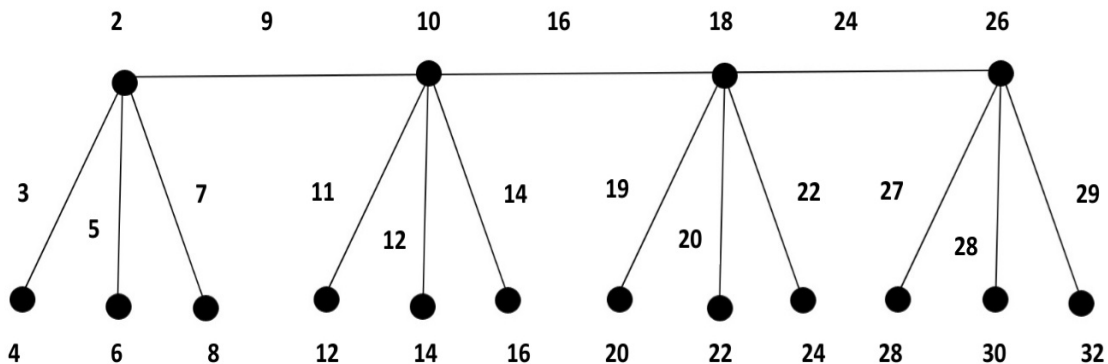


Figure3.4 : $P_4OK_{1,3}$

IV. CONCLUSION

Finally, we conclude that path, comb,  $P_nOK_{1,2}$ ,  $P_nOK_{1,3}$  is a Lehmer-4 mean graph.

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