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Linear System Order Reduction Model Using Stability Equation Method and Factor Division Algorithm for MIMO Systems

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Abstract: A New mixed method is proposed which combines the advantages of Stability Equation method and factor division algorithm in deriving the reduced order models for higher-order linear dynamic systems. The denominator of the reduced order model is obtained by the stability equation method and the numerator values are calculated using factor division algorithm. The reduced order models retain the steady-state value and guarantees stability of the original system. The proposed algorithm has also been extended for the design of PID controller for MIMO systems. The numerical examples are solved in literature to show the flexibility and effectiveness over other existing methods
Keywords: Stability equation, Factor division algorithm, Multivariable system, Order reduction, Stability, PID controller

I. INTRODUCTION

Since recent years design, Control and Analysis of large-scale systems is emerging as an essential area of research. Involvement of large number of variables in the high order system makes the analysis process computationally tedious. Majority of available analysis fail to give reasonable results when applied to large-scale systems. At this juncture the advantageous features of order reduction make the application of reduction procedures inevitable. The order reduction procedures are mainly classified into either time domain or frequency domain. Basing on the simplicity and amicability the frequency domain dependent methods have become more prominent. Further, the extension of single-input single-output (SISO) methods to reduce multi-input multi-output (MIMO) systems has also been carried out in [1]-[9].Each of these methods has both advantages and disadvantages when tried on a particular system. In the field of engineering practical systems available are of higher order. The analysis, design and simulation of these higher order systems become computationally tedious. In order to overcome this problem an approximate reduced model of the original higher order model is used in spite of the original higher order model. There are several methods available in literature for the reduction of higher order SISO systems, but very few methods are available for the reduction of higher order MIMO systems.

The existing methods for the reduction of SISO systems like Pade Approximation, Continued Fraction expansion involve simple computations but have serious drawback of generating unstable reduced models sometimes.

In this paper, a new mixed method for the reduction of higher order MIMO systems has been introduced.Many of the methods available in the international literature can be easily extended for the reduced of linear MIMO (Multi input–Multi output systems).S. Mukherjee and R. N. Mishra [3] are proposed a method “Reduced order modeling of linear multivariable systems using an error minimization technique”. This method becomes complex when the input polynomial is of high order.GirishParmar and Manisha Bhandari [9] proposed “Reduced order modeling of linear dynamic systems using Eigen spectrum analysis and modified cauer continued fraction method”. A combined method using the advantages of the stability equation method and factor division algorithm is proposed for single as well as multivariable linear dynamic systems. In this method the reduced denominator is obtained by The Stability Equation and numerator of the reduced model is determined by the factor division algorithm. The proposed algorithm has also been extended for the design of PID controller for MIMO systems

II. REDUCTION PROCEDURE FOR PROPOSED METHOD

Let the transfer function of the original high-order system (HOS) of order ‘n’ be:

$$G_n(s) = \frac{N(s)}{D(s)} = \frac{a_0 + a_1 s + a_2 s^2 \ldots\ldots\ldots + a_{n-1} s^{n-1}}{b_0 + b_1 s + b_2 s^2 \ldots\ldots\ldots + b_n s^n} \qquad (1)$$

The low-order system (LOS) of order ‘r’ to be synthesized is:

$$G_r(s) = \frac{\alpha_0 + \alpha_1 s + \alpha_2 s^2 + \ldots\ldots\ldots + \alpha_{k-1} s^{r-1}}{d_0 + d_1 s + \ldots\ldots\ldots + d_r s^r} \qquad (2)$$

Further, the method consists of following steps:

1) *Step-1*Determination of the denominator coefficients of LOS:

For stable original system $G_n(s)$, the denominator $D(s)$ of the HOS is separating in the even and odd parts in the form of stability equations as [7]:

$$D_e(s) = \sum_{i=0,2,4}^{n} b_i s^i = b_0 \prod_{i=1}^{m1}\left(1+\frac{s^2}{z_i^2}\right)$$
$$D_0(s) = \sum_{i=1,3,5}^{n} b_i s^i = b_1 s \prod_{i=1}^{m2}\left(1+\frac{s^2}{p_i^2}\right) \qquad (3)$$

Where m_1 and $m_2$are the integer parts of n/2 and (n-1)/2, respectively and $z_i^2 < p_i^2 < z_2^2 < p_2^2$……..

Now by discarding the factors with large magnitudes of z_i^2 and p_i^2 in (3), the stability equations for r^th order LOS are obtained as;

$$D^r{}_e(s) = b_0 \prod_{i=1}^{m3}\left(1+\frac{s^2}{z_i^2}\right)$$
$$D^r{}_o(s) = b_1 s \prod_{i=1}^{m4}\left(1+\frac{s^2}{p_i^2}\right)\ldots.. \qquad (4)$$

Where m_3 and $m_4$are the integer parts of r/2 and (r-1)/2, respectively.

Combining these reduced stability equations and therefore proper normalizing it, the r^th order denominator D(s) of LOS is obtained as:

$$D_r(s) = D^r{}_e(s) + D^r{}_o(s) = \sum_{i=0}^{r-1} b_i s^i + s^r$$

Therefore, the denominator polynomial in (2) is now known, which is given by

$$D_r(s) = d_0 + d_1 s + d_2 s^2 \ldots\ldots\ldots.. + d_r s^r \qquad (5)$$

2) *Step-2:* Determination of the numerator coefficients of reduced order models.

After obtaining the reduced denominator the numerator of reduced model is determined by factor division algorithm.

$$N(s) D_r(s) = (a_0 + a_1 s + a_2 s^2 + \ldots\ldots\ldots.. + a_{n-1} s^{n-1})$$
$$(d_0 + d_1 s^1 + d_2 s^2 \ldots\ldots.. + d_r s^r)$$

$$= p_0 + p_1 s + p_2 s^2 + \ldots\ldots\ldots.. + p_{n-1} s^{n-1} + \ldots\ldots.. + p_{n+r-1} s^{n+r-1} \qquad (6)$$

And

$$D(s) = b_0 + b_1 s + b_2 s^2 + \ldots.. + b_n s^n$$

$$\alpha_0 = \frac{p_0}{b_0}\left(\begin{matrix} p_0\ p_1 \cdots \cdots p_{r-1} \\ b_0\ b_1 \cdots \ldots b_{r-1}\end{matrix}\right.$$

$$\alpha_1 = \frac{q_0}{b_0}\left(\begin{matrix} q_0\ q_1 \cdots \cdots q_{r-2} \\ b_0\ b_1 \cdots \ldots b_{r-2}\end{matrix}\right.$$

$$\alpha_2 = \frac{r_0}{b_0}\left(\begin{matrix} r_0\ r_1 \cdots \ldots r_{r-3} \\ b_0\ b_1 \ldots \ldots b_{r-3}\end{matrix}\right.$$

$$\alpha_3 = \frac{u_0}{b_0}\left(\begin{matrix} u_0\ u_1 \\ b_0\ b_1\end{matrix}\right.$$

$$\alpha_{r-1} = \frac{v_0}{b_0}\left(\begin{matrix} v_0 \\ b_0\end{matrix}\right.$$

Where

$$q_i = p_{i+1} - \alpha_0 b_{i+1} \quad i = 0,1,\ldots\ldots.r-2$$
$$r_i = q_{i+1} - \alpha_1 b_{i+1} \quad i = 0,1,\ldots\ldots.r-3$$
$$v_0 = u_1 - \alpha_{r-2} b_1$$

Now finally the rth order reduced model can written as

$$G_r(s) = \frac{\alpha_0 + \alpha_1 s + \alpha_2 s^2 + \ldots\ldots\ldots\ldots + \alpha_{r-1} s^{r-1}}{D_r(s)} \qquad (8)$$

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III. DESIGN PROCEDURE FOR PID CONTROLLER

The following procedure can be followed to design a PID controller [12] when the specifications are damping ratio, ζ and natural frequency of oscillation ω_n.

1) *Step 1:* Determine the dominant pole, S_d and calculate its magnitude and phase:

$S_d = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$ D = $|S_d|$ and $\beta = \angle S_d$ (9)

By considering the dominant pole at $-\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$ we get,

D = $\sqrt{\zeta^2 w_n^2 + w_n^2(1-\zeta^2)}$ and $\beta = \tan^{-1}\frac{\sqrt{1-\zeta^2}}{-\zeta}$ (10)

2) *Step 2:* Determine the magnitude and phase of G(s) at s = S_d

Let $A_d = |G(s)|$ at s = S_d (i.e. $A_d = |G(s_d)|$)

And, $\Phi_d = \angle G(s)$ at s = S_d (i.e. $A_d = \angle G(s_d)$) (11)

3) *Step- 3:* Determine the transfer function of PID controller:

$$G_c = K_p + \frac{K_i}{s} + K_d s = \frac{K_p s + K_i + K_d s^2}{s}$$

$$= \frac{K_d\left(s^2 + \frac{K_p}{K_d}s + \frac{K_i}{K_d}\right)}{s} \quad (12)$$

Determine K_i from the specified error constant, such that the compensated system meets the error requirement.

For example, if the system is type-0 system and velocity error constant, K_v is specified then K_i is obtained by evaluating the following expression. $K_i = \lim_{s\to 0} s\,G_c(s)G(s)$, (13)

$$k_v \geq \frac{1}{e_{ss}} = \frac{1}{0.08}\ 12.5$$

Calculate the parameter K_d and K_p using the following the equations;

Proportional constant, $K_p = \frac{-\sin(\beta+\phi_d)}{A_d \sin\beta} - \frac{2K_i\cos\beta}{D}$ (14)

Derivative constant, $K_d = \frac{\sin\phi_d}{DA_d\sin\beta} + \frac{K_i}{D^2}$ (15)

4) *Step-4:* Verify the design

Open loop transfer functions of compensated system, $G_0(s) = G_c(s)$G(s). (16)

The design is accepted if the root locus of compensated system passes through the dominant pole, s_d. This can be verified from the magnitude condition, which states that the point s=s_d will be a point on root locus if 1+$G_0(s_d)$ =0, where $G_0(s_d)$is the value of $G_0(s)$ at s=s_d. It can be shown that $G_0(s_d)$ = -1

IV. MULTIVARIABLE SYSTEMS

Let, the transfer function matrix of the HOS of order 'n' having 'p' inputs and 'm' outputs be:

$$[G(s)] = \frac{1}{D(s)} = \begin{bmatrix} a_{11}(s) & a_{12}(s) & a_{13}(s)\cdots & a_{1p}(s) \\ a_{21}(s) & a_{22}(s) & a_{23}(s)\cdots & a_{2p}(s) \\ \vdots & & \vdots & \vdots \\ a_{m1}(s) & a_{m2}(s) & a_{m3}(s) & a_{mp}(s) \end{bmatrix} \quad (17)$$

Or,[G(s)]=[$g_{ij}(s)$],i=1,2,…..,m;j=1,2,..P (18)

Is am×p transfer matrix.

The general form of $g_{ij}(s)$ of [G(s)] in (9) is taken as:

$$g_{ij}(s) = \frac{a_{ij}(s)}{D(s)} = \frac{a_0 + a_1 s + a_2 s^2 \dots\dots\dots\dots + a_{n-1} s^{n-1}}{b_0 + b_1 s + b_2 s^2 \dots\dots\dots\dots + b_n s^n} \qquad (19)$$

Let, the transfer function matrix of the LOS of order 'r' having 'p' inputs and 'm' outputs be synthesized is:

$$[R(s)] = \frac{1}{D(s)} = \begin{bmatrix} b_{11}(s) & b_{12}(s) & b_{13}(s) \cdots & b_{1p}(s) \\ b_{21}(s) & b_{22}(s) & b_{23}(s) \cdots & b_{2p}(s) \\ \vdots & & \vdots & \vdots \\ b_{m1}(s) & b_{m2}(s) & b_{m3}(s) & b_{mp}(s) \end{bmatrix}$$

Or,[R(s)]=[$r_{ij}(s)$],i=1,2,..,m;j=1,2…..p (20)

Is am×p transfer matrix.

The general form of $r_{ij}(s)$ of [R(s)] in (12) is taken as:

$$r_{ij}(s) = \frac{b_{ij}(s)}{D(s)} = \frac{\alpha_0 + \alpha_1 s + \alpha_2 s^2 + \dots\dots\dots\dots + \alpha_{k-1} s^{r-1}}{d_0 + d_1 s + d_2 s^2 \dots\dots\dots\dots + d_r s^r} \qquad (21)$$

V. NUMERICAL EXAMPLE

Example: Consider a sixth-order two output system [12] Described by the transfer function matrix:

$$[G(s)] = \begin{bmatrix} \frac{2(s+2)}{(s+1)(s+10)} & \frac{(s+4)}{(s+2)(s+5)} \\ \frac{(s+10)}{(s+1)(s+20)} & \frac{(s+6)}{(s+2)(s+3)} \end{bmatrix}$$

$$= \frac{1}{D(s)} \begin{bmatrix} a_{11}(s) & a_{12}(s) \\ a_{21}(s) & a_{22}(s) \end{bmatrix}$$

Where, the common denominator D(s) is given by:

D(s) = (s+1) (s+2) (s+3) (s+5) (s+10) (s+20)

Where $g_{11}(s) = 2s^5 + 70s^4 + 762s^3 + 3616s^2 + 7820s + 6000$

$g_{12}(s) = s^5 + 38s^4 + 459s^3 + 2182s^2 + 4160s + 2400$

$g_{21}(s) = s^5 + 30s^4 + 331s^3 + 1650s^2 + 3700s + 3000$

$g_{22}(s) = s^5 + 42s^4 + 601s^3 + 3660s^2 + 9100s + 6000$

And $D6(s) = s^6 + 41s^5 + 571s^4 + 3491s^3 + 10060s^2 + 13100s + 6000$

And $G_1(s) = g_{11}(s) + g_{12}(s)$ $G_2(s) = g_{21}(s) + g_{22}(s)$

Where $G_1(s) = 3s^5 + 108s^4 + 1221s^3 + 5792s^2 + 11860s + 8400$

$G_2(s) = 2s^5 + 72s^4 + 932s^3 + 5310s^2 + 12300s + 9000$

For getting the second order model, first denominator is obtained using stability equation as described before in Step1

1) *Step-1:* Divide the denominator of the above HOS in even and odd parts; we get the stability equations as:

$$D_e(s) = 6000 + 10060s^2 + 571s^4 + s^6$$

$$= 6000\left(1 + \frac{s^2}{0.6181}\right)$$

$$D_o(s) = 13100s + 3491s^3 + 41s^5$$

$$= 13100s$$

$$= 9707.167s^2 + 13100s + 6000$$

Now by discarding the factors with large magnitudes of z_i^2 and p_i^2 in $D_e(s)$ and $D_o(s)$ respectively, the stability equations for the second-order reduced model are given by: $D_r(s) = s^2 + 1.3493s + 0.6181$

2) *Step-2:* By using the factor division algorithm as

$$N(s)D_r(s) = (8400+11860s+5792s^2+1221s^3+108s^4+3s^5)\times(0.6181+1.3493s+s^2)$$

$$= 5192.04 + 18704.266s + 16058.44s^2 + \cdots \dots\dots\dots\dots$$

$$\alpha_0 = 0.86534 \left(\frac{5192.04}{6000}\;\frac{18704.266}{13100}\right.$$

$$\alpha_1 = 1.22174 \left(\frac{11335.954}{6000}\right.$$

Numerator of second order model is show as $N_r(s) = 0.86534+1.22174s$

The reduced second order models obtained by using proposed method for first output is

$$R_1(s) = \frac{1.22174s + 0.86534}{s^2 + 1.3493s + 0.6181} \qquad \text{(Proposed)}$$

The reduced second order models obtained by using proposed method for second output is

$$R_2(s) = \frac{1.267067s + 0.927122}{s^2 + 1.7856s + 1.1904} \qquad \text{(Proposed)}$$

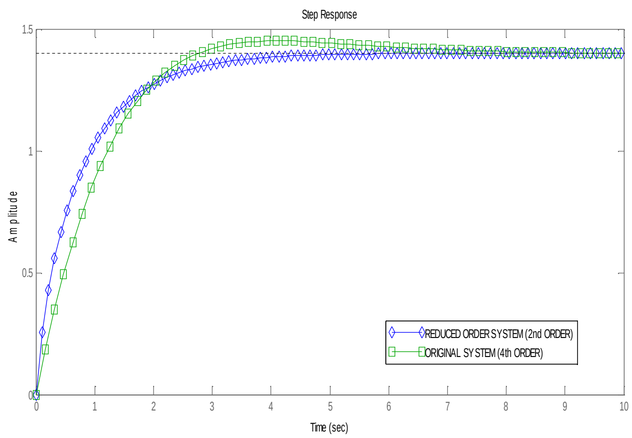

Fig. a

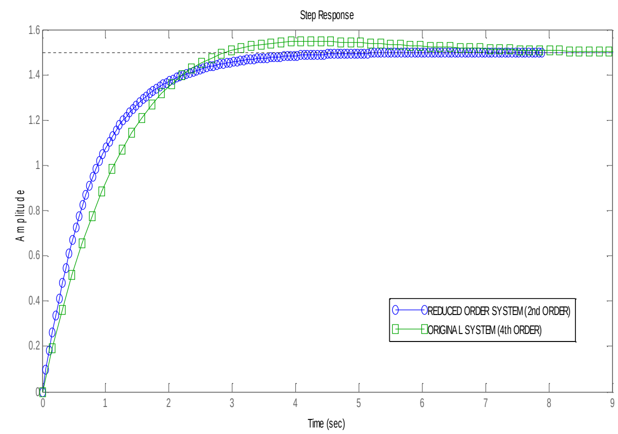

Fig. b

VI. COMPARISON OF PROPOSED METHOD WITH OTHER EXISTING METHODS

The reduced second order models obtained modal methods and Pade type approximation by R.Prasad [11].

$$N_1 = \frac{1.183s + 2}{s^2 + 3s + 2} \qquad \text{(For 1st output)}$$

$$N_2 = \frac{1.247s + 2}{s^2 + 3s + 2} \qquad \text{(For 2nd output)}$$

The 2[nd] order reduced models obtained by Eigen Spectrum Analysis and Modified Cauer Continued FractionCethod Proposed by GirishParmar and ManishaBhandari [8] are:

$$P_1 = \frac{3s + 11.859}{s^2 + 13.66667s + 8.4707} \qquad \text{(for 1st output)}$$

$$P_2 = \frac{2s + 12.7061}{s^2 + 6.510934s + 8.4707} \qquad \text{(for 2nd output)}$$

The step responses of the models obtained by the proposed method, modal methods and Pade type approximations by R.Prasad, Eigen Spectrum Analysis and Modified Cauer Continued Fraction Method Proposed by GirishParmar and ManishaBhandari are compared with that of the original system are shown in Figs. (C, d)

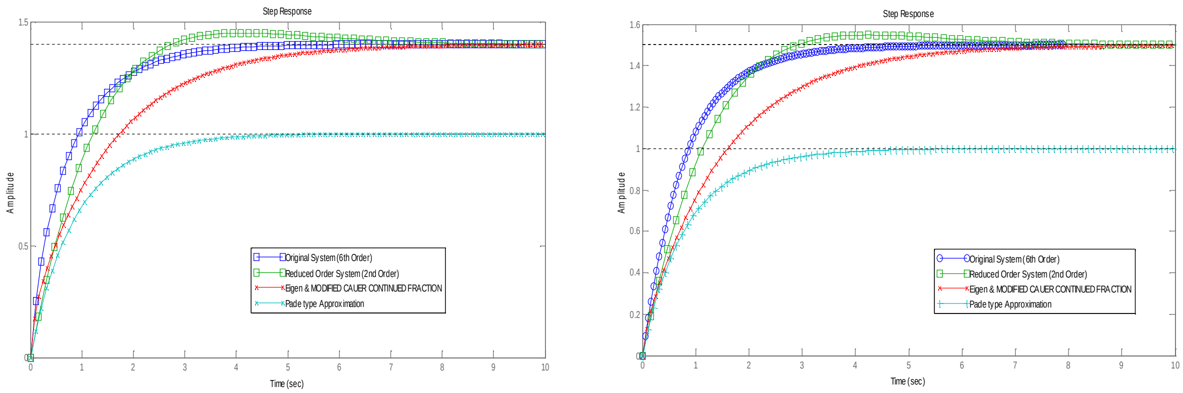

Fig. b

A. Design of PID Controller

1) Step 1: Determine the dominant pole, S_d and calculate its magnitude and phase:

$S_d = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$ D =2.9495 and β = 132.69

2) Step 2: Determine the magnitude and phase of G(s) at s = S_d Let A_d = 0.3133 And, Φ_d = -124.56

3) Step- 3: Determine the transfer function of PID controller:

Determine K_i from the specified error constant, such that the compensated system meets the error requirement.

For example, if the system is type-0 system and velocity error constant, K_v is specified then K_i is obtained by evaluating the following expression. $K_v = 12.5$

Calculate the parameter K_d and K_p using the following the equations; $K_p = 5.133$, $K_d = 0.2244$

The proposed second order model using PID controller is obtained as $G_c(s) = \frac{0.1729 + 4.204s^2 + 13.08s + 7.726}{1.173s^3 + 5.553s^2 + 13.7s + 7.726}$

4) Step-4: Verify the design

Open loop transfer functions of compensated system,

$$G_0(s) = \frac{0.4362s^7 + 25.63 + 550.5s^5 + 5574s^4 + 29730s^3 + 85960s^2 + 127048s + 75000}{1.436s^7 + 66.33s^6 + 1122s^5 + 9065s^4 + 39490s^3 + 99060s^2 + 133048s + 75000}$$

The proposed algorithm is successively applied to each element of remaining two transfer function matrix of above multivariable system and the reduced order models $r_{ij}(s)$ of the LOS [R(s)] are obtained. The general form of second-order reduced transfer function matrix is taken as:

FOR NUMERATOR 2 $a_{12} = 2400 + 4160s + 2182s^2 + 459s^3 + 38s^4 + s^5$

$D_r(s) = s^2 + 1.3493s + 0.6181$

The proposed second order model is obtained as

$$G_r(s) = \frac{0.247s + 0.429}{s^2 + 1.3493s + 0.6181}$$

PID controller $G_{c12}(s) = \frac{0.3759s^3 + 6.651s^2 + 17.21s + 7.726}{1.376s^3 + 8s^2 + 17.21s + 7.726}$

Open loop transfer functions of compensated system,

$$G_{012}(s) = \frac{0.8762s^7 + 48.29s^6 + 1003s^5 + 4985s^4 + 50730s^3 + 132800s^2 + 166100s + 75072}{1.876s^7 + 89.29s^6 + 1574s^5 + 13480s^4 + 60790s^3 + 145900s^2 + 172100s + 75072}$$

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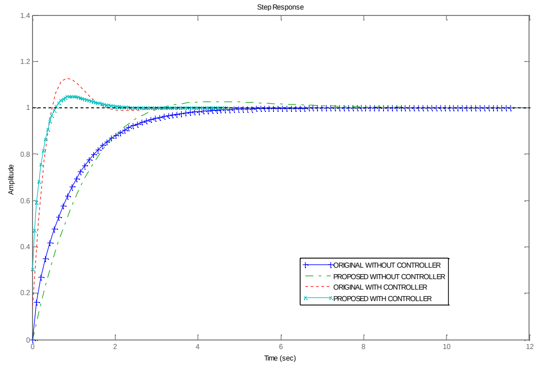
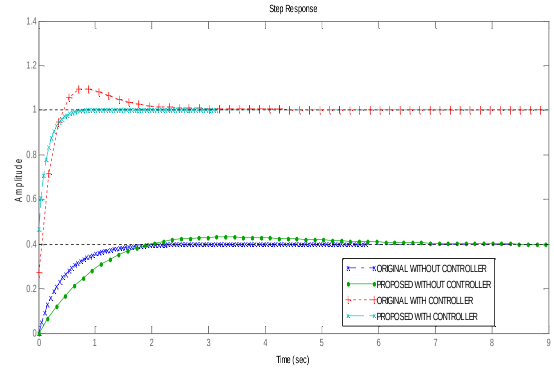




FOR NUMERATOR 3

The proposed second order model is obtained as

Where, $D_r(s) = s^2 + 1.3495s + 0.6181$

$b_{13}(s) = 0.38126s + 0.309$

PID controller $G_{c13}(s) = \dfrac{0.1296s^3 + 4.04s^2 + 12.72s + 7.726}{1.13s^3 + 5.389s^2 + 13.24s + 7.726}$

$$G_{013}(s) = \frac{0.3398s^7 + 20.51s^6 + 447.1s^5 + 4727s^4 + 26560s^3 + 80450s^2 + 123460s + 75000}{1.34s^7 + 61.51s^6 + 1013s^5 + 8218s^4 + 36620s^3 + 93550s^2 + 129460s + 75000}$$

FOR NUMERATOR 4

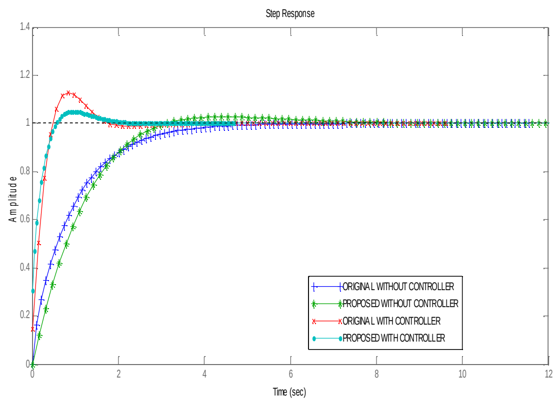
The proposed second order model is obtained as

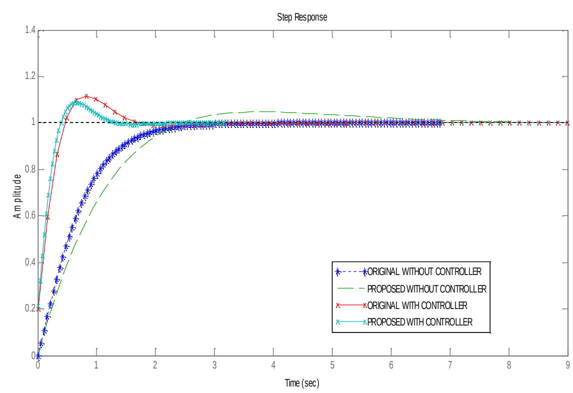
Where, $D_r(s) = s^2 + 1.3495s + 0.6181$

$b_{14}(s) = 0.937s + 0.6181$

PID controller $G_{c14}(s) = \dfrac{0.2522s^3 + 4.701s^2 + 14.7s + 7.726}{1.252s^3 + 6.05s^2 + 15.32s + 7.726}$

$$G_{014}(s) = \frac{0.2692s^7 + 16.155s^6 + 377.6s^5 + 4416s^4 + 27680s^3 + 91410s^2 + 142790s + 75000}{1.269s^7 + 57.15s^6 + 948.6s^5 + 7910s^4 + 37740s^3 + 104500s^2 + 148790s + 75000}$$





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