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New Characterizations of Topological Spaces

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Abstract: In this paper, we introduce and investigate topological spaces called *sgw-compactness Spaces* and *sgw-connectedness space* and we get several characterizations and some of their properties. Also we investigate its relationship with other types of functions.

Mathematics Subject Classification: 54D05, 54D30.

Keywords: *sgw-open set, sgw-closed sets, sgw-compact spaces, sgw-connectedness*

I. INTRODUCTION

The notions of compactness and connectedness are useful and fundamental notions of not only general topology but also of other advanced branches of mathematics. Many researchers have investigated the basic properties of compactness and connectedness. The productivity and fruitfulness of these notions of compactness and connectedness motivated mathematicians to generalize these notions. In the course of these attempts many stronger and weaker forms of compactness and connectedness have been introduced and investigated. D. Andrijevic [1] introduced a new class of generalized open sets in a topological space called *b-open sets*. The class of *b-open sets* generates the same topology as the class of *b-open sets*. Since the advent of this notion, several research paper with interesting results in different respects came into existence. M. Ganster and M. Steiner [5] introduced and studied the properties of *gb-closed sets* in topological spaces. The aim of this paper is to introduce the concept of *sgw-compactness* and *sgw-connectedness* in topological spaces and is to give some characterizations of *sgw-compact spaces* in terms of nets and filter bases. S.S. Benchalli, T.D. Rayanagoudar and P.G. Patil introduced the concept of *g*-closed sets* and S.S. Benchalli, T.D. Rayanagoudar and P.G. Patil and Shik John studied the concept of *g*-preregular*, *g*-pre normal* and obtained their properties by utilizing *g*-closed sets*. The notation of closed set is fundamental in the study of topological spaces. In 1970, Levine introduced the concept of generalized closed sets in the topological space by comparing the closure of subset with its open supersets. The investigation on generalization of closed set has lead to significant contribution to the theory of separation axiom, covering properties and generalization of continuity. T. Kong, R. Kopperman and P. Meyer shown some of the properties of generalized closed set have been found to be useful in computer science and digital topology. Caw, Ganster and Reilly and has shown that generalization of closed set is also useful to characterize certain classes of topological spaces and there variations, for example the class of extremely disconnected spaces and the class of submaximal spaces. In 1990, S.P. Arya and T.M. Nour define generalized semi-open sets, generalized semi closed sets and use them to obtain some cauterization of *s-normal spaces*.

In 1993, N. PalaniInappan and K. Chandrasekhara Rao introduced regular generalized closed (briefly *rg-closed*) sets and study there properties relative to union, intersection and subspaces. In 2000, A. Pushpalatha introduce new class of closed set called weakly closed (briefly *w-closed*) sets and study there properties. In 2007, S.S. Benchalli and R.S. Wali introduced the new class of the set called regular *w-closed* (briefly *rw-closed*) sets in topological spaces. In this this paper is to introduce and study two new classes of spaces, namely *Semi weakly generalized-normal* and *Semi weakly generalized-regular spaces* and obtained their properties by utilizing *Semi weakly generalized-closed sets*.

II. PRELIMINARY NOTES

Throughout this paper (X, τ) , (Y, σ) are topological spaces with no separation axioms assumed unless otherwise stated. Let $A \subseteq X$. The closure of A and the interior of A will be denoted by $Cl(A)$ and $Int(A)$ respectively.

1) **Definition 2.1:** A subset A of X is said to be *b-open* [1] if $A \subseteq Int(Cl(A)) \cup Cl(Int(A))$. The complement of *b-open set* is said to be *b-closed*. The family of all *b-open sets* (respectively *b-closed sets*) of (X, τ) is denoted by $bO(X, \tau)$ [respectively $bCL(X, \tau)$].

2) **Definition 2.2:** Let A be a subset of X . Then

(i) *b-interior* [1] of A is the union of all *b-open sets* contained in A .

(ii) *b-closure* [1] of A is the intersection of all *b-closed sets* containing A .

The *b-interior* [respectively *b-closure*] of A is denoted by $b-Int(A)$ [respectively $b-Cl(A)$].

- 3) *Definition 2.3:* Let A be a subset of X . Then A is said to be *sgw-closed* [12] if $\text{cl}(\text{int}(A)) \subseteq U$ whenever $A \subseteq U$ and U regular-semi open in (X, τ) . The complement of *sgw-closed* [12] set is called *sgw-open*. The family of all *sgw-open* [respectively *sgw-closed*] sets of (X, τ) is denoted by $\text{sgwO}(X, \tau)$ [respectively, $\text{sgw-CL}(X, \tau)$].
- 4) *Definition 2.4:* The *sgw-closure* [14] of a set A , denoted by $\text{sgw-Cl}(A)$, is the Intersection of all *sgw-closed* sets containing A .
- 5) *Definition 2.5:* The *sgw-interior* [14] of a set A , denoted by $\text{sgw-Int}(A)$, is the Union of all *sgw-open* sets contained in A .
- 6) *Remark 2.6:* Every pre-closed set is *sgw-closed*.

III. SGW-COMPACTNESS.

- 1) *Definition 3.1:* A collection $\{A_i; i \in \Lambda\}$ of *sgw-open* sets in a topological space X is called a *sgw-open cover* of a subset B of X if $B \subset \{A_i; i \in \Lambda\}$ holds.
- 2) *Definition 3.2:* A topological space X is *sgw-compact* if every *sgw-open cover* of X has a finite sub-cover.
- 3) *Definition 3.3:* A subset B of a topological space X is said to be *sgw-compact relative to X* if, for every collection $\{A_i; i \in \Lambda\}$ of *sgw-open* subsets of X such that $B \subset \cup \{A_i; i \in \Lambda\}$ there exists a finite subset Λ_0 of Λ such that $B \subseteq \cup \{A_i; i \in \Lambda_0\}$.
- 4) *Definition 3.4:* A subset B of a topological space X is said to be *sgw-compact* if B is *sgw-compact* as a subspace of X .
- 5) *Theorem 3.5:* Every *sgw-closed* subset of a *sgw-compact* space is *sgw-compact Relative to X* .

Proof: Let A be *sgw-closed* subset of *sgw-compact* space X . Then A^c is *sgw-open* in X . Let $M = \{G_\alpha; \alpha \in \Lambda\}$ be a cover of A by *sgw-open* sets in X . Then $M^* = M \cup A^c$ is a *sgw-open cover* of X . Since X is *sgw-compact* M^* is reducible to a finite subcover of X , say $X = G_{\alpha_1} \cup G_{\alpha_2} \cup \dots \cup G_{\alpha_m} \cup A^c$, $G_{\alpha_k} \in M$. But A and A^c are disjoint hence $A \subset G_{\alpha_1} \cup \dots \cup G_{\alpha_m}$, $G_{\alpha_k} \in M$, which implies that any *sgw-open cover* M of A contains a finite sub-cover.

Therefore A is *sgw-compact relative to X* . Thus every *sgw-closed* subset of a *sgw-compact* space X is *sgw-compact*.

- 6) *Definition 3.6:* A function $f : X \rightarrow Y$ is said to be *sgw-continuous* [5] if $f^{-1}(V)$ is *sgw-closed* in X for every closed set V of Y .
- 7) *Definition 3.7:* A function $f : X \rightarrow Y$ is said to be *sgw-irresolute* [5] if $f^{-1}(V)$ is *sgw-closed* in X for every *sgw-closed* set V of Y .

- 8) *Theorem 3.8:* A *sgw-continuous* image of a *sgw-compact* space is compact

Proof. Let $f : X \rightarrow Y$ be a *sgw-continuous* map from a *sgw-compact* space X onto a topological space Y . Let $\{A_i; i \in \Lambda\}$ be an open cover of Y . Then $\{f^{-1}(A_i); i \in \Lambda\}$ is a *sgw-open cover* of X . Since X is *sgw-compact* it has a finite sub-cover say $\{f^{-1}(A_1), f^{-1}(A_2), \dots, f^{-1}(A_n)\}$. Since f is onto $\{A_1, \dots, A_n\}$ is a cover of Y , which is finite. Therefore Y is compact.

- 9) *Theorem 3.9:* If a map $f : X \rightarrow Y$ is *sgw-irresolute* and a subset B of X is *sgw-compact relative to X* , then the image $f(B)$ is *sgw-compact relative to Y* .

Proof: Let $\{A_\alpha; \alpha \in \Lambda\}$ be any collection of *sgw-open* subsets of Y such that $f(B) \subset \cup \{A_\alpha; \alpha \in \Lambda\}$. Then $B \subset \cup \{f^{-1}(A_\alpha); \alpha \in \Lambda\}$ holds. Since by hypothesis B is *sgw-compact relative to X* there exists a finite subset Λ_0 of Λ such that $B \subset \cup \{f^{-1}(A_\alpha); \alpha \in \Lambda_0\}$ Therefore we have $f(B) \subset \cup \{A_\alpha; \alpha \in \Lambda_0\}$, which shows that $f(B)$ is *sgw compact relative to Y* .

IV. SGW-CONNECTEDNESS

- 1) *Definition 4.1:* A topological space X is said to be *sgw-connected* if X cannot be expressed as a disjoint union of two non-empty *sgw-open* sets. A subset of X is *sgw-connected* if it is *sgw-connected* as a subspace.
- 2) *Example 4.2:* Let $X = \{a, b\}$ and let $\tau = \{X, \phi, \{a\}\}$. Then it is *sgw-connected*.
- 3) *Remark 4.3:* Every *sgw-connected* space is connected but the converse need not be true in general, which follows from the following example.
- 4) *Example 4.4:* Let $X = \{a, b\}$ and let $\tau = \{X, \phi\}$. Clearly (X, τ) is connected. The *sgw-open* sets of X are $\{X, \phi, \{a\}, \{b\}\}$. Therefore (X, τ) is not a *sgw-connected* space, because $X = \{a\} \cup \{b\}$ where $\{a\}$ and $\{b\}$ are non-empty *sgw-open* sets.
- 5) *Theorem 4.5:* For a topological space X the following are equivalent.
 - (i) X is *sgw-connected*.
 - (ii) X and ϕ are the only subsets of X which are both *sgw-open* and *sgw-closed*.
 - (iii) Each *sgw-continuous* map of X into a discrete space Y with at least two

Points are a constant map.

Proof:

(i) \Rightarrow (ii) : Let O be any sgw-open and sgw-closed subset of X . Then O^c is both sgw-open and sgw-closed. Since X is disjoint union of the sgw-open sets O and O^c implies from the hypothesis of (i) that either $O = \emptyset$ or $O = X$.

(ii) \Rightarrow (i) : Suppose that $X = A \cup B$ where A and B are disjoint non-empty sgw-open subsets of X . Then A is both sgw-open and sgw-closed. By assumption $A = \emptyset$ or X . Therefore X is sgw-connected.

(ii) \Rightarrow (iii) : Let $f : X \rightarrow Y$ be a sgw-continuous map. Then X is covered by sgw-open and sgw-closed covering $\{f^{-1}(Y) : y \in (Y)\}$. By assumption $f^{-1}(y) = \emptyset$ or X for each $y \in Y$. If $f^{-1}(y) = \emptyset$ for all $y \in Y$, then f fails to be a map. Then there exists only one point $y \in Y$ such that $f^{-1}(y) \neq \emptyset$ and hence $f^{-1}(y) = X$. This shows that f is a constant map.

(iii) \Rightarrow (ii) : Let O be both sgw-open and sgw-closed in X . Suppose $O \neq \emptyset$. Let $f : X \rightarrow Y$ be a sgw-continuous map defined by $f(O) = y$ and $f(O^c) = \{w\}$ for some distinct points y and w in Y . By assumption f is constant. Therefore we have $O = X$.

6) *Theorem 4.6:* If $f : X \rightarrow Y$ is a sgw-continuous and X is sgw-connected, then Y is connected.

Proof: Suppose that Y is not connected. Let $Y = A \cup B$ where A and B are disjoint non-empty open set in Y . Since f is sgw-continuous and onto, $X = f^{-1}(A) \cup f^{-1}(B)$ where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint non-empty sgw-open sets in X . This contradicts the fact that X is sgw-connected. Hence Y is connected.

7) *Theorem 4.7:* If $f : X \rightarrow Y$ is a sgw-irresolute surjection and X is sgw-connected, then Y is sgw-connected.

Proof: Suppose that Y is not sgw-connected. Let $Y = A \cup B$ where A and B are disjoint non-empty sgw-open set in Y . Since f is sgw-irresolute and onto, $X = f^{-1}(A) \cup f^{-1}(B)$ where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint non-empty sgw-open sets in X . This contradicts the fact that X is sgw-connected. Hence Y is connected.

8) *Theorem 4.8:* In a topological space (X, τ) with at least two points, if $\text{sgw-O}(X, \tau) = \text{sgw-CL}(X, \tau)$ then X is not sgw-connected.

Proof: By hypothesis we have $\text{sgw-O}(X, \tau) = \text{sgw-CL}(X, \tau)$ and by Remark 2.6 we have every pre-closed set is sgw-closed, there exists some non-empty proper subset of X which is both sgw-open and sgw-closed in X . So by last Theorem 4.5 we have X is not sgw-connected.

9) *Definition 4.9:* A topological space X is said to be T_{sgw} -space if every sgw-closed subset of X is closed subset of X .

10) *Theorem 4.10:* Suppose that X is a T_{sgw} -space then X is connected if and only if it is sgw-connected.

Proof: Suppose that X is connected. Then X cannot be expressed as disjoint union of two non-empty proper subsets of X . Suppose X is not a sgw-connected space. Let A and B be any two sgw-open subsets of X such that $X = A \cup B$, where $A \cap B = \emptyset$ and $A \subset X, B \subset X$. Since X is T_{sgw} -space and A, B are sgw-open, A, B are open subsets of X , which contradicts that X is connected. Therefore X is sgw-connected. Conversely, every open set is sgw-open. Therefore every sgw-connected space is connected.

11) *Theorem 4.11:* If the sgw-open sets C and D form a separation of X and if Y is sgw-connected subspace of X , then Y lies entirely within C or D .

Proof: Since C and D are both sgw-open in X the sets $C \cap Y$ and $D \cap Y$ are sgw-open in Y these two sets are disjoint and their union is Y . If they were both non-empty, they would constitute a separation of Y . Therefore, one of them is empty. Hence Y must lie entirely in C or in D .

12) *Theorem 4.12:* Let A be a sgw-connected subspace of X . If $A \subset B \subset \text{sgw-Cl}(A)$ then B is also sgw-connected.

Proof: Let A be sgw-connected and let $A \subset B \subset \text{sgw-Cl}(A)$. Suppose that $B = C \cup D$ is a separation of B by sgw-open sets. Then by Theorem 4.11 above A must lie entirely in C or in D . Suppose that $A \subset C$, then $\text{sgw-Cl}(A) \subseteq \text{sgw-Cl}(C)$. Since $\text{sgw-Cl}(C)$ and D are disjoint, B cannot intersect D . This contradicts the fact that D is non-empty subset of B . So $D = \emptyset$ which implies B is sgw-connected.

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