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# New Forms of $\alpha g$ - continuous Maps in Topology

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**Abstract:** In this paper, we describe a novel class of continuous maps called as strongly alpha generalised continuous maps in topological spaces. Furthermore, we investigate their properties with existing continuous maps, and we also study their decomposition of continuous functions.

**Keywords:** Semi closed sets,  $M$ - $\alpha g$ -closed sets, continuous functions, strongly  $\alpha g$ -continuous map, strongly  $gp$ -continuous map.  
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## I. INTRODUCTION

In mathematics and applied sciences, general topology is very significant. Many topologists use the generalised form of open and closed sets to generalize concept like the continuity and separation axioms, among others, in this approach.

N. Levine [16] established the concept of generalized closed sets in 1970, and it now plays an important role in general topology. Following the introduction of generalized closed sets, several scholars produced research publications that dealt with various types of sets and studied various features. H. Maki et al. [18] defined and established the idea of  $gp$ -closed sets in topology in 1993. Furthermore, H. Maki et al. [20] established the concept of  $\alpha$  - generalized closed sets in topological spaces in 1994. This class of  $\alpha g$ -closed sets is suitably lies between  $g$ -closedness and  $gs$ -closedness. Noiri [28] introduced and studied the strong forms of continuous functions in 1984. Following that, several mathematicians, including Levine.N [14], S. P. Arya and R. Gupta [5,] B.M.Munshi and D.S.Bassan [24], proposed and studied the idea of strongly continuous, perfectly continuous, and completely continuous functions, which are stronger than continuous functions. P. Sundaram [31] later defined and investigated the strongly  $g$ -continuous and perfectly  $g$ -continuous continuous functions in topological spaces.

## II. PRELIMINARIES

Throughout this article,  $(X, \tau)$ ,  $(Y, \sigma)$  and  $(Z, \eta)$  (or simply,  $X$ ,  $Y$ , and  $Z$ ) represent topological spaces for which no separation axioms are assumed unless explicitly stated.

The closure of  $A$  and the interior of  $A$  with respect to topology is denoted by  $Cl(A)$  and  $Int(A)$  for a subset  $A$  of  $X$ , respectively.

We recall the following known definitions and results which are very useful in the sequel.

1) *Definition 2.1:* A subset  $A$  of a topological space  $X$  is called a

- (i) semi-open [15] if  $A \subseteq Cl(Int(A))$  and semi-closed [10] if  $Int(Cl(A)) \subseteq A$ .
- (ii) pre-open [21] if  $A \subseteq Int(Cl(A))$  and pre-closed [23] if  $Cl(Int(A)) \subseteq A$ .
- (iii)  $\alpha$ -open [27] if  $A \subseteq Int(Cl(Int(A)))$  and  $\alpha$ -closed [22] if  $Cl(Int(Cl(A))) \subseteq A$ .
- (iv) semipre-open [3] (=  $\beta$ -open [1]) if  $A \subseteq Cl(Int(Cl(A)))$  and semipre-closed [3] (=  $\beta$ -closed [1]) if  $Int(Cl(Int(A))) \subseteq A$ .

The Pre-closure of a subset  $A$  of  $X$ , denoted by  $pcl(A)$  is the intersection of all pre-closed sets containing a subset  $A$ . The pre-interior of a subset  $A$  of  $X$  and is denoted by  $pInt(A)$ , is the union of all pre-open sets of  $X$  contained in  $A$ .

2) *Definition 2.2:* A subset  $A$  of a topological space  $(X, \tau)$  is called a

- (i) generalized closed (briefly,  $g$ -closed) [16] if  $Cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $X$ .
- (ii)  $\alpha$ -generalised closed (briefly,  $\alpha g$ -closed) [20] if  $\alpha Cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $X$ .
- (iii) generalised preclosed (briefly,  $gp$ -closed) [19] if  $pCl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $X$ .
- (iii) generalised semipre-closed (briefly,  $gsp$ -closed) [13] if  $spCl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $X$ .
- (iv) strongly generalised closed (briefly,  $g^*$ -closed [32]) [29] if  $Cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $g$ -open in  $X$ .

The compliment of the above mentioned closed sets are their respective open sets.

3) *Definition 2.3:* A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is called a

- (i) semi-continuous[15] if  $f^{-1}(V)$  semi-closed in  $X$  for every closed subset  $V$  of  $Y$ .
- (ii)  $\alpha$ -continuous[22] if  $f^{-1}(V)$   $\alpha$ -closed in  $X$  for every closed subset  $V$  of  $Y$ .
- (iii) pre-continuous[21] if  $f^{-1}(V)$  pre-closed in  $X$  for every closed subset  $V$  of  $Y$ .
- (iv)  $g$ -continuous [6] if  $f^{-1}(V)$   $g$ -closed in  $X$  for every closed subset  $V$  of  $Y$ .
- (v)  $\alpha g$ -continuous [20] if  $f^{-1}(V)$   $\alpha g$ -closed in  $X$  for every closed subset  $V$  of  $Y$ .
- (vi)  $gp$ -continuous [4] if  $f^{-1}(V)$   $gp$ -closed in  $X$  for every closed subset  $V$  of  $Y$ .
- (vii) strongly continuous [14] if  $f^{-1}(V)$  is both open and closed in  $X$  for every subset  $V$  of  $Y$ .
- (viii) strongly  $\alpha$ -continuous [7] if  $f^{-1}(V)$  is  $\alpha$ -open in  $X$  for every semi-open set  $V$  of  $Y$ .
- (ix) strongly pre-continuous [9] if  $f^{-1}(V)$  is preopen in  $X$  for every semi-open set  $V$  of  $Y$ .
- (x) strongly semi-continuous [2] if  $f^{-1}(V)$  is open in  $X$  for every semi-open set  $V$  of  $Y$ .
- (xi) strongly  $g$ -continuous [31] if  $f^{-1}(V)$  is open in  $X$  for every  $g$ -open set  $V$  of  $Y$ .
- (xii)  $\beta w g$ -continuous.[27],if  $g^{-1}(V)$  is  $\beta w g$ -open set in  $X$ , for every open set  $V$  of  $Y$

4) *Definition 2.4:* A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is called a

- (i) irresolute[11] if  $f^{-1}(V)$  semi-closed in  $X$  for every semi-closed subset  $V$  of  $Y$ .
- (ii) pre-irresolute[30] if  $f^{-1}(V)$  pre-closed in  $X$  for every pre-closed set  $V$  of  $Y$ .
- (iii)  $\alpha$ -irresolute[17] if  $f^{-1}(V)$   $\alpha$ -closed in  $X$  for every  $\alpha$ -closed subset  $V$  of  $Y$ .
- (iv) semi- $\alpha$ -irresolute[8] if  $f^{-1}(V)$  is semi-open in  $X$  for every  $\alpha$ -open set  $V$  of  $Y$ .
- (v)  $g$ c-irresolute [31] if  $f^{-1}(V)$   $g$ -closed in  $X$  for every  $g$ -closed set  $V$  of  $Y$ .
- (vi)  $\alpha g$ -irresolute [12] if  $f^{-1}(V)$   $\alpha g$ -closed in  $X$  for every  $\alpha g$ -closed subset  $V$  of  $Y$ .
- (vii)  $gp$ -irresolute [25] if  $f^{-1}(V)$   $gp$ -closed in  $X$  for every  $gp$ -closed subset  $V$  of  $Y$ .

5) *Definition 2.5:* A space  $(X, \tau)$  is called

- (i) an  $\alpha T_b$  -space[20],[31] if every  $\alpha g$ -closed set in it is closed.
- (ii) a  $T_g$  -space[4], if every  $gp$ -closed set is  $g$ -closed.

### III. PROPERTIES OF STRONGLY $\alpha g$ -CONTINUOUS FUNCTIONS

In this section, we define and study a new type of continuous function using  $\alpha g$ -open sets in the following.

1) *Definition 3.1:* A function  $f: X \rightarrow Y$  is called strongly  $\alpha g$ -continuous if  $f^{-1}(V)$  is  $\alpha g$ -open in  $X$  for every semiopen set  $V$  of  $Y$ .

2) *Theorem 3.2:* Every strongly  $\alpha$ -continuous function is strongly  $\alpha g$ -continuous function.

Proof: Let  $V$  be a semiopen subset of  $Y$  and  $f$  be strongly  $\alpha$ -continuous function, then  $f^{-1}(V)$  is  $\alpha$ -open in  $X$ . Since  $f$  be strongly  $\alpha$ -continuous function. But as every  $\alpha$ -open set is  $\alpha g$ -open set so,  $f^{-1}(V)$  is  $\alpha g$ -open set in  $X$ . Therefore,  $f$  is strongly  $\alpha g$ -continuous function.

The converse of the Theorem need not be true as seen from the following example.

3) *Example 3.3:* Let  $X = Y = \{a, b, c\}$  with topologies,  $\tau = \{\emptyset, X, \{a, b\}\}$  and  $\sigma = \{\emptyset, Y, \{a\}, \{a, c\}\}$ ; Here  $\alpha O(Y) = \{\emptyset, Y, \{a\}, \{a, c\}, \{a, b\}\}$  and  $\alpha GO(X) = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ . Define a function  $f: X \rightarrow Y$  by  $f(a)=a, f(b)=c, f(c)=b$ . Then  $f$  is  $\alpha$ -continuous function. But not  $\alpha g$ -continuous, since  $f^{-1}(\{a,b\}) = \{a,c\}$  is not  $\alpha g$ -open set in  $X$  for the  $\alpha$ -open set  $\{a,b\}$  of  $Y$ .

4) *Theorem 3.4:* A function  $f: X \rightarrow Y$  is a strongly  $\alpha g$ -continuous if and only if for each  $x$  in  $X$  and each semi-open set  $V$  of  $Y$  with  $f(x) \in V$ , there exists an  $\alpha g$ -open set  $U$  of  $X$  such that  $x \in U, f(U) \subseteq V$ .

Proof: Let  $f(x) \in V$ . Since  $f$  is a strongly  $\alpha g$ -continuous, we have  $x \in f^{-1}(V) \in \alpha g$ -open of  $X$ . Let  $U = f^{-1}(V)$ . Then  $x \in V$  and  $f(U) \subseteq V$ .

Conversely, let  $V$  be an semi-open set in  $Y$  and let  $x \in f^{-1}(V)$ . Then  $f(x) \in V$  and thus there exists an  $\alpha g$ -open set  $U_x$  such that  $x \in U_x$  and  $f(U_x) \subseteq V$ . Now  $x \in U_x \subseteq f^{-1}(V)$  and  $f^{-1}(V) = \bigcup_{x \in f^{-1}(V)} U_x$ . Therefore,  $f^{-1}(V)$  is  $\alpha g$ -open in  $X$  and consequently,  $f$  is a strongly  $\alpha g$ -continuous function.

5) *Theorem 3.5:* Every strongly  $\alpha g$ -continuous function is continuous and thus strongly pre-continuous,  $\alpha g$ -continuous and strongly semi-continuous.

Proof: Follows from the definitions.

The converse of the above theorem need not be true as seen from the following example.

6) *Example 3.6:* Let  $X = Y = \{a, b, c\}$  with topologies,  $\tau = \{\emptyset, X, \{a\}, \{a, c\}\}$  and  $\sigma = \{\emptyset, Y, \{a\}, \{b\}, \{a, b\}\}$ ; Define a function  $f: X \rightarrow Y$  by  $f(a) = a, f(b) = c, f(c) = b$ . Then  $f$  is strongly pre-continuous (resp.  $\alpha g$ -continuous function, strongly semi-continuous) function but not strongly  $\alpha g$ -continuous, since for the semiopen set  $\{a, b\}$  of  $Y, f^{-1}(\{a, b\}) = \{a, c\}$  is not  $\alpha g$ -open set in  $X$ .

7) *Theorem 3.7:* Every strongly continuous function is strongly  $\alpha g$ -continuous but not conversely.

Proof: Follows from the definitions.

8) *Example 3.8:* Let  $X = Y = \{a, b, c\}$  with topologies,  $\tau = \{\emptyset, X, \{a\}, \{a, c\}\}$  and  $\sigma = \{\emptyset, Y, \{a\}\}$ . Define a function  $f: X \rightarrow Y$  be an Identity function. Then  $f$  is strongly  $\alpha g$ -continuous function but not strongly continuous, since for the subset  $\{b, c\}$  in  $Y, f^{-1}(\{b, c\}) = \{b, c\}$  is closed but not open in  $X$ .

9) *Theorem 3.9:* Every strongly  $\alpha$ -continuous function is an irresolute but not conversely.

Proof: Let  $f: X \rightarrow Y$  be a strongly  $\alpha$ -continuous function and let  $V$  be a semiopen set in  $Y$ . Then  $f^{-1}(V)$  is  $\alpha$ -open and hence semiopen in  $X$ . Hence  $f$  is an irresolute.

10) *Example 3.10:* Let  $X = Y = \{a, b, c\}$  with topologies,  $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$  and  $\sigma = \{\emptyset, Y, \{a\}\}$ . Define a function  $f: X \rightarrow Y$  by  $f(a) = a, f(b) = c$  and  $f(c) = b$ . Then  $f$  is an irresolute function but not strongly  $\alpha$ -continuous, since for the subset  $\{a, b\}$  of  $Y, f^{-1}(\{a, b\}) = \{a, c\}$  is open in  $X$ .

11) *Theorem 3.11:* Let  $(Y, \sigma)$  be any topological spaces,  $(X, \tau)$  is a  ${}_a T_b$ -space and  $f: X \rightarrow Y$  be any function. Then the following are equivalent:

- (i)  $f$  is strongly  $\alpha g$ -continuous
- (ii)  $f$  is strongly semi-continuous

Proof: (i)  $\Rightarrow$  (ii): Follows from the Theorem 3.5.4.

(ii)  $\Rightarrow$  (i): Let  $U$  be any semiopen set in  $Y$ . Since  $f$  is strongly  $\alpha g$ -continuous. Then  $f^{-1}(U)$  is  $\alpha g$ -open in  $X$ . But  $X$  is a  ${}_a T_b$ -space, then  $f^{-1}(U)$  is open in  $X$ , which implies that  $f$  is strongly semi-continuous function.

12) *Remark 3.12:* Strongly  $\alpha g$ -continuity and strongly  $g$ -continuity are independent of each other as shown in the following examples.

13) *Example 3.13:* Let  $X = Y = \{a, b, c\}$  with topologies,  $\tau = \{\emptyset, X, \{a, b\}\}$  and  $\sigma = \{\emptyset, Y, \{a\}\}$ . Define a function  $f: X \rightarrow Y$  by  $f(a) = a, f(b) = b$  and  $f(c) = c$ . Then  $f$  is strongly  $g$ -continuous but not strongly  $\alpha g$ -continuous, since for the semiopen subset  $\{a, c\}$  of  $Y, f^{-1}(\{a, c\}) = \{a, c\}$  is not  $\alpha g$ -open in  $X$ .

14) *Example 3.14:* Let  $X = Y = \{a, b, c\}$  with topologies,  $\tau = \{\emptyset, X, \{a\}, \{a, c\}\}$  and  $\sigma = \{\emptyset, Y, \{a\}, \{a, b\}\}$ . Define a function  $f: X \rightarrow Y$  by  $f(a) = a, f(b) = c$  and  $f(c) = b$ . Then  $f$  is strongly  $\alpha g$ -continuous but not strongly  $g$ -continuous, since for the subset  $\{a, c\}$  of  $Y, f^{-1}(\{a, c\}) = \{a, b\}$  is not  $g$ -open in  $X$ .

Now, we derive decomposition of strongly  $\alpha g$ -continuous functions in the following

15) *Theorem 3.15:* If  $f: X \rightarrow Y$  is strongly  $\alpha g$ -continuous and  $g: Y \rightarrow Z$  is strongly semi-continuous. Then their composition  $g \circ f: X \rightarrow Z$  is strongly  $\alpha g$ -continuous function.

Proof: Let  $U$  be any open set in  $Z$ . Since  $g$  is strongly semi-continuous,  $g^{-1}(U)$  is open set in  $Y$ . Since every open set is semiopen set and hence  $g^{-1}(U)$  is semiopen in  $Y$ . Again,  $f$  is strongly  $\alpha g$ -continuous, and  $f^{-1}(g^{-1}(U)) = (g \circ f)^{-1}(U)$  is  $\alpha g$ -open in  $X$ . Thus,  $g \circ f$  is strongly  $\alpha g$ -continuous function.

16) *Theorem 3.16:* If  $f: X \rightarrow Y$  is strongly  $\alpha g$ -continuous and  $g: Y \rightarrow Z$  is strongly  $\alpha$ -continuous. Then the composition  $g \circ f: X \rightarrow Z$  is strongly  $\alpha g$ -continuous function.

Proof: Let  $V$  be any semiopen set in  $Z$ . Since  $g$  is strongly  $\alpha$ -continuous,  $g^{-1}(V)$  is  $\alpha$ -open set of  $Y$ . Since every  $\alpha$ -open set is semiopen set and hence  $g^{-1}(V)$  is  $\alpha$ -open in  $Y$ . Again,  $f$  is strongly  $\alpha g$ -continuous,  $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$  is  $\alpha g$ -open in  $X$ . Thus, the composition  $g \circ f$  is strongly  $\alpha g$ -continuous function.

17) *Theorem 3.17:* If the function  $f: X \rightarrow Y$  is  $\alpha g$ -continuous and the function  $g: Y \rightarrow Z$  is strongly semi-continuous. Then the composition  $g \circ f: X \rightarrow Z$  is strongly  $\alpha g$ -continuous function.

Proof: Let  $U$  be any semiopen set in  $Z$ . Since  $g$  is strongly semi-continuous,  $g^{-1}(U)$  is an open set of  $Y$ . Again,  $f$  is strongly  $\alpha g$ -continuous,  $f^{-1}(g^{-1}(U)) = (g \circ f)^{-1}(U)$  is  $\alpha g$ -open in  $X$ . Thus, the composition  $g \circ f$  is strongly  $\alpha g$ -continuous function.

We define the following

18) *Definition 3.18:* A function  $f: X \rightarrow Y$  is said to be  $M$ - $\alpha g$ -continuous if the inverse image of each  $\alpha g$ -open set of  $Y$ , is  $\alpha g$ -open in  $X$ .

19) *Theorem 3.19:* If the function  $f: X \rightarrow Y$  is  $M$ - $\alpha g$ -continuous and the function,  $g: Y \rightarrow Z$  is strongly  $\alpha g$ -continuous. Then the composition  $g \circ f: X \rightarrow Z$  is strongly  $\alpha g$ -continuous.

Proof: Let  $V$  be an arbitrary semi-open set in  $Z$ . Since  $g$  is strongly  $\alpha g$ -continuous,  $g^{-1}(V)$  is  $\alpha g$ -open set in  $Y$ . Again,  $f$  is  $M$ - $\alpha g$ -continuous,  $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$  is  $\alpha g$ -open in  $X$ . Thus, the composition  $g \circ f: X \rightarrow Z$  is strongly  $\alpha g$ -continuous.

20) *Theorem 3.20:* Let  $X$  and  $Z$  be any two topological spaces, and let  $Y$  be an  ${}_a T_b$ -space. Then the composition  $g \circ f: X \rightarrow Z$  is strongly  $\alpha g$ -continuous if the function  $f: X \rightarrow Y$  is  $M$ - $\alpha g$ -continuous and the function  $g: Y \rightarrow Z$  is strongly  $\alpha g$ -continuous.

Proof: Let  $F$  be any semiopen set in  $Z$ . Since  $g$  is strongly  $\alpha g$ -continuous,  $g^{-1}(F)$  is  $\alpha g$ -open in  $Y$ . But  $Y$  is  ${}_a T_b$ -space and so  $g^{-1}(F)$  is open set in  $Y$ . Since  $f$  is  $M$ - $\alpha g$ -continuous,  $f^{-1}(g^{-1}(F)) = (g \circ f)^{-1}(F)$  is  $\alpha g$ -open in  $X$ . Hence,  $g \circ f$  is strongly  $\alpha g$ -continuous.

Also, further we define and studied the followings

21) *Definition 3.21:* A function  $f: X \rightarrow Y$  is called strongly  $gp$ -continuous if  $f^{-1}(V)$  is  $gp$ -open in  $X$  for every semiopen set  $V$  of  $Y$ .

22) *Lemma 3.22:* Every strongly pre-continuous function is strongly  $gp$ -continuous function.

Proof: Let  $V$  be a semiopen subset of  $Y$  and  $f$  be strongly pre-continuous function, then  $f^{-1}(V)$  is preopen in  $X$ . Since  $f$  be strongly pre-continuous function. But as every preopen set is  $gp$ -open set and so,  $f^{-1}(V)$  is  $gp$ -open set in  $X$ . Therefore,  $f$  is strongly  $gp$ -continuous function.

23) *Theorem 3.23:* A function  $f: X \rightarrow Y$  is a strongly  $gp$ -continuous if and only if for each  $x$  in  $X$  and each semi-open set  $V$  of  $Y$  with  $f(x) \in V$ , there exists an  $gp$ -open set  $U$  of  $X$  such that  $x \in U$ ,  $f(U) \subseteq V$ .

Proof: Let  $f(x) \in V$ . Since  $f$  is a strongly  $gp$ -continuous, we have  $x \in f^{-1}(V) \in gp$ -open of  $X$ . Let  $U = f^{-1}(V)$ . Then  $x \in U$  and  $f(U) \subseteq V$ . Conversely, let  $V$  be a semi-open set in  $Y$  and let  $x \in f^{-1}(V)$ . Then  $f(x) \in V$  and thus there exists a  $gp$ -open set  $U_x$  such that  $x \in U_x$  and  $f(U_x) \subseteq V$ . Now  $x \in U_x \subseteq f^{-1}(V)$  and  $f^{-1}(V) = \bigcup_{x \in f^{-1}(V)} U_x$ . Therefore,  $f^{-1}(V)$  is  $gp$ -open in  $X$  and consequently,  $f$  is a strongly  $gp$ -continuous.

24) *Theorem 3.24:* Every strongly  $\alpha g$ -continuous function is strongly  $gp$ -continuous but not conversely.

Proof: Let  $V$  be a semiopen subset of  $Y$  and  $f$  be strongly  $\alpha g$ -continuous function, then  $f^{-1}(V)$  is  $\alpha g$ -open in  $X$  since  $f$  is strongly  $\alpha g$ -continuous function. But as every  $\alpha g$ -open set is  $gp$ -open set and so,  $f^{-1}(V)$  is  $gp$ -open set in  $X$ . Therefore,  $f$  is strongly  $gp$ -continuous function.

25) *Example 3.25:* Let  $X = Y = \{a, b, c\}$ ,  $\tau = \{\emptyset, X, \{a, b\}\}$  and  $\sigma = \{\emptyset, Y, \{a\}, \{a, b\}\}$ . Define a function  $f: X \rightarrow Y$  be an Identity function,  $f(a) = a$ ,  $f(b) = b$  and  $f(c) = c$ . Then  $f$  is strongly  $gp$ -continuous but not strongly  $\alpha g$ -continuous, since for the semiopen subset  $\{a, c\}$  of  $Y$ ,  $f^{-1}(\{a, c\}) = \{a, c\}$  is not  $\alpha g$ -open in  $X$ .

26) *Remark 3.26:* Strongly gp-continuity and strongly g-continuity are independent of each other as shown in the following examples.

27) *Example 3.27:* Let  $(X, \tau)$  be as in the Example 3.5.14. Define a function  $f: X \rightarrow Y$  be an Identity function,  $f(a) = a$ ,  $f(b) = b$  and  $f(c) = c$ . Then  $f$  is strongly gp - continuous but not strongly g - continuous, since for the subset  $\{a, b\}$  of  $Y$ ,  $f^{-1}(\{a, b\}) = \{a, b\}$  is not g - open in  $X$ .

28) *Example 3.28:* Let  $X = Y = \{a, b, c\}$ ,  $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$  and  $\sigma = \{\emptyset, Y, \{a\}, \{a, b\}\}$ . Define a function  $f: X \rightarrow Y$  be an Identity function. Then  $f$  is strongly g - continuous but not strongly gp-continuous, since for the semiopen subset  $\{a, c\}$  of  $Y$ ,  $f^{-1}(\{a, c\}) = \{a, c\}$  is not gp - open in  $X$ .

Now, we give decomposition of strongly gp-continuous functions in the following

29) *Theorem 3.29:* If  $f: X \rightarrow Y$  is strongly gp - continuous and  $g: Y \rightarrow Z$  is strongly pre-continuous. Then the composition  $g \circ f: X \rightarrow Z$  is strongly gp-continuous.

*Proof:* Let  $U$  be any open set in  $Z$ . Since  $g$  is strongly pre-continuous,  $g^{-1}(U)$  is preopen set in  $Y$ . Since every preopen set is gp-open set, and hence  $g^{-1}(U)$  is gp-open in  $Y$ . Again,  $f$  is strongly gp-continuous, and  $f^{-1}(g^{-1}(U)) = (g \circ f)^{-1}(U)$  is gp-open in  $X$ . Thus,  $g \circ f$  is strongly gp - continuous function.

30) *Theorem 3.30:* If  $f: X \rightarrow Y$  is strongly gp - continuous and  $g: Y \rightarrow Z$  is  $\beta$ wg-continuous. Then the composition  $g \circ f: X \rightarrow Z$  is gp-continuous.

*Proof:* Let  $U$  be any open set in  $Z$ . Since  $g$  is  $\beta$ wg -continuous,  $g^{-1}(U)$  is  $\beta$ wg-open set in  $Y$ . Since every  $\beta$ wg- open set is gp-open set, and hence  $g^{-1}(U)$  is gp-open in  $Y$ . Again,  $f$  is strongly gp-continuous, and  $f^{-1}(g^{-1}(U)) = (g \circ f)^{-1}(U)$  is gp-open in  $X$ . Thus,  $g \circ f$  is gp - continuous function.

31) *Theorem 3.31:* If  $f: X \rightarrow Y$  is gp-irresolute and  $g: Y \rightarrow Z$  is strongly  $\alpha$ g-continuous. Then the composition  $g \circ f: X \rightarrow Z$  is strongly gp-continuous.

*Proof:* Let  $V$  be any semiopen set in  $Z$ . Since  $g$  is strongly  $\alpha$ g-continuous,  $g^{-1}(V)$  is  $\alpha$ g-open set of  $Y$ . Since every  $\alpha$ g-open set is gp-open set and hence  $g^{-1}(V)$  is gp-open in  $Y$ . Again,  $f$  is gp-irresolute,  $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$  is gp-open in  $X$ . Thus, the composition  $g \circ f$  is strongly gp-continuous.

32) *Theorem 3.32:* Let  $f: X \rightarrow Y$  and  $g: Y \rightarrow Z$  be any two functions such that  $g \circ f: X \rightarrow Z$ . Then

- (i)  $g \circ f$  is strongly gp-continuous if  $f$  is strongly gp-irresolute and  $g$  is strongly gp-continuous
- (ii)  $g \circ f$  is strongly gp-continuous if  $f$  is strongly gp-continuous and  $g$  is strongly  $\alpha$ -continuous
- (iii)  $g \circ f$  is strongly gp-continuous if  $f$  is gp-continuous and  $g$  is strongly semi-continuous.

*Proof:* The proof follows from the definitions.

33) *Theorem 3.33:* Let  $X$  and  $Z$  be any two topological spaces, and let  $Y$  be an  $T_g$ - space. Then the composition  $g \circ f: X \rightarrow Z$  is strongly gp - continuous if the function  $f: X \rightarrow Y$  is  $(\alpha g, g)$  continuous and the function  $g: Y \rightarrow Z$  is strongly gp-continuous.

*Proof:* Let  $F$  be any semiopen set in  $Z$ . Since  $g$  is strongly gp-continuous,  $g^{-1}(F)$  is gp-open in  $Y$ . But  $Y$  is  $T_g$ - space and so  $g^{-1}(F)$  is g-open set in  $Y$ . Since  $f$  is  $(\alpha g, g)$ -continuous,  $f^{-1}(g^{-1}(F)) = (g \circ f)^{-1}(F)$  is  $\alpha g$ -open in  $X$ . Hence,  $g \circ f$  is strongly  $\alpha g$ -continuous.

#### IV. NEW FORMS OF CLOSED AND OPEN FUNCTIONS

In this present section we also define and studied the followings

- 1) *Definition 4.1:* A function  $f: X \rightarrow Y$  said to be Strongly  $\alpha$  - generalized-closed function (resp. Strongly  $\alpha g$ -open), if the image of every semi closed (resp. semi open) subset in  $X$  is  $\alpha g$ -closed (resp. is  $\alpha g$ -closed) set in  $Y$ .
- 2) *Definition 4.2:* A function  $f: X \rightarrow Y$  said to be Strongly generalized pre-closed function (resp. Strongly gp-open), if the image of every semi closed (resp. semi open) subset in  $X$  is gp-closed (resp. is gp-closed) set in  $Y$ .
- 3) *Theorem 4.3:* Every strongly  $\alpha g$ -open function is strongly pre open function, not conversely.

*Proof:* Let  $f: X \rightarrow Y$  be strongly  $\alpha g$ -open function. Let  $G$  be semi open set in  $X$ . Then  $f(G)$  is an  $\alpha g$ -open set in  $Y$ . Therefore  $f(G)$  is gp-open In  $Y$ . Hence  $f$  is strongly gp-open in  $Y$ .

- 4) *Example 4.4:* Let  $X = \{p, q, r\} = Y$ ,  $\tau = \{X, \phi, \{p\}, \{q\}, \{p, q\}\}$ , and  $\sigma = \{Y, \phi, \{p\}\}$ . Let  $f: X \rightarrow Y$  be the identity function. Then  $f$  is  $g_p$ -open, since for the semopen sets  $\{p, r\}, \{q, r\}$  in  $X$ , then  $f(\{p, r\}) = \{p, r\}$  and  $f(\{q, r\}) = \{q, r\}$  are not strongly  $\alpha g$ -open in  $Y$  but it is strongly  $g_p$ -open in  $Y$ .

## V. CONCLUSIONS

Every year many topologists introduced different types of sets and maps in topological spaces. In this paper, we studied a new forms of continuous functions in topological spaces, namely is strongly  $\alpha g$ -continuous,  $g_p$ -continuous functions introduced. Then some new examples and theorems in separation axioms on topological spaces are developed.

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