iJRASET
International Journal For Research in
Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 11 **Issue:** VII **Month of publication:** July 2023

DOI: https://doi.org/10.22214/ijraset.2023.54917

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.538
Volume 11 Issue VII Jul 2023- Available at www.ijraset.com

New Maps in Fuzzy Topological Spaces

Dr. Vivekananda Dembre
Assistant Professor, Department of Mathematics, BLDEA'S BHS Arts and TGP Science College, Jamakhandi, Bagalkote-587103, Karnataka State , INDIA.

Abstract: In this paper, we introduce fuzzy sgw-open and fuzzy sgw-closed maps in fts and obtain certain characterization of the sgw- closed and sgw-open maps.
2010 Mathematics Subject Classification: 54A40
Keywords and phrases:Fuzzy sgw-open maps,fuzzy sgw-closed maps,fuzzy sgw-closed sets, fuzzy sgw-open sets.

I. INTRODUCTION

The concept of a fuzzy subset was introduced and studied by L.A.Zadeh in the year 1965. The subsequent research activities in this area and related areas have found applications in many branches of science and engineering. In the year 1965, L.A.Zadeh [1] introduced the concept of fuzzy subset as a generalization of that of an ordinary subset. The introduction of fuzzy subsets paved the way for rapid research work in many areas of mathematical science. In the year 1968, C.L.Chang [2] introduced the concept of fuzzy topological spaces as an application of fuzzy sets to topological spaces . Subsequently several researchers contributed to the development of the theory and applications of fuzzy topology. The theory of fuzzy topological spaces can be regarded as a generalization theory of topological spaces. An ordinary subset A or a set X can be characterized by a function called characteristic function

$\mu_A : X \longrightarrow [0,1]$ of A, defined by
$\mu_A (x) = 1$, if $x \in A$.
$= 0$, if $x \in A$.
Thus an element xX is in A if $\mu_A (x) = 1$ and is not in A if $\mu_A (x) = 0$. In general if X is a set and A is a subset of X then A has the following representation. $A = \{ (x ,\mu_A (x)): x \in X\}$, here $\mu_A (x)$ may be regarded as the degree of belongingness of x to A, which is either 0 or 1. Hence A is the class of objects with degree of belongingness either 0 or 1 only. Prof. L.A.Zadeh [1] introduced a class of objects with continuous grades of belongingness ranging between 0 and 1 ; he called such a class as fuzzy subset.
A fuzzy subset A in X is characterized as a membership function $\mu_A : X \longrightarrow [0,1]$, which associates with each point in x a real number $\mu_A(x)$ between 0 and 1 which represents the degree or grade membership of belongingness of x to A.The purpose of this paper is to introduce a new class of fuzzy sets called fuzzy sgw-closed sets in fuzzy topological spaces and investigate certain basic properties of these fuzzy sets. Among many other results it is observed that every fuzzy closed set is fuzzy sgw-closed but not conversely.Also we introduce fuzzy sgw-open sets in fuzzy topological spaces and study some of their properties.

A. Preliminaries

1.1Definition:[1]A fuzzy subset A in a set X is a function $A : X \to [0, 1]$. A fuzzy subset in X is empty iff its membership function is identically 0 on X andis denoted by 0 or μ_ϕ.The set X can be considered as a fuzzy subset of X whose membership function is identically 1 on X and is denoted by $\mu_x$or I_x. In fact everysubset of X is a fuzzy subset of X but not conversely. Hence the concept of a fuzzy subset is a generalization of the concept of a subset.

1.2 Defnition:[1] If A and B are any two fuzzy subsets of a set X , thenA is said to be included in B or A is contained in B iff $A(x) \le B(x)$ for all x in X. Equivalently, $A \le B$ iff $A(x) \le B(x)$ for all x in X.

1.3 Defnition:[1]Two fuzzy subsets A and B are said to be equal if $A(x) = B(x)$ for every x in X.
Equivalently $A = B$ if $A(x) = B(x)$ for every x in X.

1.4Defnition:[1] The complement of a fuzzy subset A in a set X, denoted by A' or $1 - A$, is the fuzzy subset of X defined by $A'(x) = 1 - A(x)$ for all x in X. Note that$(A')' = A$.

1.5 Defnition:[1] The union of two fuzzy subsets A and B in X, denoted by $A \vee B$, is a fuzzy subset in X defined by $(A \vee B)(x) = \text{Max}\{\mu_A(x), \mu_B(X)\}$ for all x in X.
1.6 Defnition:[1]The intersection of two fuzzy subsets A and B in X, denoted by $A \wedge B$, is a fuzzy subset in X defined by $(A \wedge B)(x) = \text{Min}\{A(x), B(x)\}$ for all x in X.
1.7 Defnition:[1] A fuzzy set on X is 'Crisp' if it takes only the values 0 and 1 on X.

1.8 Defnition:[2]Let X be a set and be a family of fuzzy subsets of X. τ is called a fuzzy topology on X iff τ satisfies the following conditions.
(i)μ_ϕ ; $\mu_X \in \tau$: That is 0 and 1 $\in \tau$
(ii) If $G_i \in \tau$ for $i \in I$ then $\bigvee_{i \in I} G_i \in \tau$
(iii) If $G, H \in \tau$ then $G \wedge H \in \tau$
The pair (X, τ) is called a fuzzy topological space (abbreviated as fts). The membersof $\tau$are called fuzzy open sets and a fuzzy set A in X is said to be closed iff $1 - A$ is an fuzzy open set in X.

1.9 Remark:[2] Every topological space is a fuzzy topological space but not conversely.
1.10 Defnition:[2]Let X be a fts and A be a fuzzy subset in X. Then $\wedge$ {B : B is a closed fuzzy set in Xand $B \geq A$} is called the closure of A and is denoted by A or cl(A).
1.11 Defnition:[2]Let A and B be two fuzzy sets in a fuzzy topological space $(X,\tau)$and let $A \geq B$. Then B is called an interior fuzzy set of A if there exists $G \in \tau$ suchthat $A \geq G \geq B$, the least upper bound of all interior fuzzy sets of A is called theinterior of A and is denoted by A^0.
1.12 Definition[3] A fuzzy set A in a fts X is said to be fuzzy semiopen if and only if there exists a fuzzy open set V in X such that $V \leq A \leq cl(V)$.
1.13 Definition[3] A fuzzy set A in a fts X is said to be fuzzy semi-closed if and only if there exists a fuzzy closed set V in X such that $int(V) \leq A \leq V$. It is seen that a fuzzy set A is fuzzy semiopen if and only if 1-A is a fuzzy semi-closed.

1.14 Theorem:[3] The following are equivalent:
(a) $\mu$is a fuzzy semiclosed set,
(b) μ^C is a fuzzy semiopen set,
(c) $int(cl(\mu)) \leq \mu$.
(b) $int(cl(\mu)) \geq \mu^c$

1.15 Theorem [3] Any union of fuzzy semiopen sets is a fuzzy semiopen set and (b) any intersection of fuzzy semi closed sets is a fuzzy semi closed.

1.16 Remark[3]
(i) Every fuzzy open set is a fuzzy semiopen but not conversely.
(ii) Every fuzzy closed set is a fuzzy semi-closed set but not conversely.
(iii) The closure of a fuzzy open set is fuzzy semiopen set
(iv) The interior of a fuzzy closed set is fuzzy semi-closed set

1.17Definition:[3] A fuzzy set $\mu$of a fts X is called a fuzzy regular open set of X if $int(cl(\mu).= \mu$.
1.18 Definition:[3] A fuzzy set $\mu$of fts X is called a fuzzy regular closed set of X if $cl(int(\mu)) = \mu$.
1.19 Theorem:[3] A fuzzy set $\mu$of a fts X is a fuzzy regular open if and only if μ^C fuzzy regular closed set.

1.20 Remark:[3]
(i) Every fuzzy regular open set is a fuzzy open set but not conversely.
(ii) Every fuzzy regular closed set is a fuzzy closed set but not conversely.

1.21 Theorem:[3]
(i) The closure of a fuzzy open set is a fuzzy regularclosed.
(ii) The interior of a fuzzy closed set is a fuzzy regular open set.

1.22 Definition:[4]A fuzzy set α in fts X is called fuzzy rw-closed if cl(α)$\leq$ μ whenever $\alpha \leq \mu$ and μ is regular semi-open in X.

1.23 Definition [5]:A fuzzy set αin fts X is called fuzzy sgw closed if p-cl(α)$\leq$ μ whenever $\alpha \leq \mu$ and μ is rgα $-$open set in X.
1.24 Defintion [5]:A fuzzy set α of a fts X is fuzzy sgw-open set, if it's complement α^c is a fuzzy sgw-closed in fts X.
1.25 definition [2]:Let X and Y be fts. A map f: X $\rightarrow$Y is said to be a fuzzy continuous mapping if $f^{-1}(\mu)$ is fuzzy open in X for each fuzzy open set μ in Y.
1.26 definition[6]:A mapping f: X$\rightarrow$Y is said to be fuzzy irresolute iff f^{-1} (B) is fuzzy semi-open in X is fuzzy open in Y.
1.27 definition[7]:A mapping f: X $\rightarrow$Y is said to be fuzzy sgw irresolute if the inverse image of every fuzzy sgw-open in Y is a fuzzy sgw-open set in X.

B. Fuzzy Sgw-Open Maps And Fuzzy Sgw-Closed Maps İn Fuzzy Topological Spaces

Definition 2.1: Let X and Y be two fts. A map f: $(X,T_1) \rightarrow (Y,T_2)$ is called fuzzy sgw-open map if the image of every fuzzy open set in X is fuzzy sgw-open in Y.
Theorem 2.2: Every fuzzy open map is a fuzzy sgw-open map.
Proof:Let f :$(X,T_1) \rightarrow (Y,T_2)$ be a fuzzy open map and and μ be a fuzzy open set in fts X. Then f(μ) is a fuzzy open set in fts Y. Since every fuzzy open set is fuzzy sgw-open,f(μ) is a fuzzy sgw-open set in fts Y. Hence f is a fuzzy sgw-open map.
The converse of the above theorem need not be true in general as seen from the following example.

Example 2.3:Let X= Y={a, b, c} and the functions α, β: X $\rightarrow$ [0, 1] be defined as
α (x) = 1 if x = a,c
0 otherwise
β(x) = 1 if x = a
0 otherwise

Consider T_1= {1,0,α}, T_2 = {1,0,β}then $(X,T_1) \rightarrow (Y,T_2)$are fts.Let map f: X$\rightarrow$ Y be the identity map.Then this function is sgw-open map but it is not fuzzy open .Since the image of the fuzzy open set α (x) in X is fuzzy set β in Y which is not fuzzy open.

Remark 2.4 :If f: $(X,T_1) \rightarrow (Y,T_2)$ and g:$(Y,T_2) \rightarrow (Z,T_3)$ be two fuzzy sgw—open map then composition gof: $(X,T_1) \rightarrow (Z,T_3)$ need not be fuzzy sgw-open as seen from the following example

Example 2.5: Let X=Y=Z={a,b,c} and the function $\alpha, \beta, \gamma, \delta$: X $\rightarrow$[0,1] be defined as

α (x) = 1 if x = a
0 otherwise

β(x) = 1 if x = c
0 otherwise

γ(x) = 1 if x = a,c
0 otherwise
δ(x) = 1 if x = a,b
0 otherwise

Consider T_1 = {1,0,α, δ} , T_2 ={1,0,α}and T_3 = { 0,1, α, β, γ}.Let map f: (X T_1) $\rightarrow$ (Y,T_2)

and g: $(Y,T_2) \rightarrow (Z,T_3)$ be the identity maps then f and g are fuzzy sgw-open map but their composition gof:$(X,T_1) \rightarrow (Z,T_3)$ is not fuzzy sgw-open as β: X$\rightarrow$ [0,1] is fuzzy open in X
But (gof)(β)=β is not fuzzy sgw-open in Z.

Theorem 2.6 : If f: $(X,T_1) \rightarrow (Y,T_2)$ is fuzzy open map and g:$(Y,T_2) \rightarrow (Z,T_3)$ is fuzzy sgw—open map then their composition gof: $(X,T_1) \rightarrow (Z,T_3)$ is fuzzy sgw-open map.

Proof:Let α be fuzzy open set in (X,T_1). since f is fuzzy open map,f(α) is a fuzzy open set in (Y,T_2) .Since g is a fuzzy sgw-open map g(f(α))= (gof)(α). Thus gof is a fuzzy sgw-open map.

Definition 2.7: Let X and Y be two fuzzy topological spaces. A map f:$(X,T_1) \rightarrow (Y,T_2)$ is called fuzzy sgw-closed map if the image of every fuzzy closed set in X is a fuzzy closed set in Y.
Theorem 2.8 :Let f: $(X,T_1) \rightarrow (Y,T_2)$be fuzzy closed map then f is a fuzzy sgw closed map.
Proof:
Let α be a fuzzy closed set in (X,T_1). since f is fuzzy closed map,f(α) is a fuzzy closed map,
f(α) is a fuzzy closed set in (Y,T_2). since every fuzzy closed set is fuzzy sgw-closed ,f(α) is a fuzzy sgw-closed set in (Y,T_2).Hence f is fuzzy sgw-closed map.
The converse of the theorem need not be true in general as seen from the following example

Example 2.9:Let X= Y={a, b, c} and the functions α, β: X $\rightarrow$ [0, 1] be defined as
α (x) = 1 if x = a,c
0 otherwise
β(x) = 1 if x = a
0 therwise
Consider T_1= {1,0,α} and T_2= {1,0,β}; then (X,T_1) and (Y,T_2) are fts .Let f: $(X,T_1) \rightarrow (Y,T_2)$
be the identity map. Then f is a fuzzy sgw-closed map but it is not a fuzzy closed map, since the image of the closed set α in X is not a fuzzy closed set in Y.

Remark 2.10: The composition of two fuzzy sgw-closed maps need not be a fuzzy sgw-closed map
Example 2.11: Consider the fts (X,T_1),$(Y,T_2)$$(Z,T_3)$ and mappings defined in 2.5. then maps f and g are fuzzy sgw-closed but their composition is not fuzzy sgw-closed as β:X$\rightarrow$ [0,1]
is a fuzzy closed set in X but (gof)(β)=β is not fuzzy sgw-closed in Z.
Theorem 2.12:If a map f:($X,T_1) \rightarrow (Y,T_2)$ and g : $(Y,T_2) \rightarrow (Z,T_3)$ be two maps then composition of two sgw-closed maps isfuzzy sgw-closed map.
Proof: Let α be a fuzzy closed set in (X,T_1). Since f is a fuzzy closed map, f(α) is a fuzzy closed set in (Y,T_2). Since g is a fuzzy sgw-closed map but g(f(α)) is a fuzzy sgw-closed set in (Z,T_3) but g(f(α))=(gof)(α) .Thus gof is fuzzy sgw-closed map.

Theorem 2.13: A map f: X $\rightarrow$ Y is fuzzy sgw-closed map if for each fuzzy set δ of Y and for each fuzzy open set μ of X s.t $\mu \geq f^{-1}(\delta)$, there is a fuzzy sgw-open map set α of Y s.t $\delta \leq \alpha$ and
$f^{-1}(\alpha) \leq \mu$.
Proof: Suppose that f is fuzzy sgw-closed map.Let δ be a fuzzy subset of Y and μ be a fuzzy open set of X s.t $f^{-1}(\delta) \leq \mu$. Let α = 1 – f(1- μ) is fuzzy sgw-open set in fts Y. note that
$f^{-1}(\delta) \leq \mu$.Which implies $\delta \leq \alpha$ and $f^{-1}(\alpha) \leq \mu$.

For the converse, Suppose that μ is a fuzzy closed set in X, Then f^{-1}(1-f (μ)≤1- μ and 1- μ is fuzzy open by hypothesis, there is a fuzzy sgw-open set α of Y s.t (1-f (μ)≤ α and $f^{-1}(\alpha) \leq 1-\mu$. Therefore $\mu \leq 1- f^{-1}(\alpha)$. Hence $1-\alpha \leq \mu$, $f(1-f^{-1}(\alpha)) \leq 1-\alpha$ Which implies f (μ) = 1- α . since 1- α is fuzzy sgw-closed f (μ) is fuzzy sgw-closed and thus f is fuzzy sgw-closed.

Lemma 2.14: Let f :($X,T_1) \rightarrow (Y,T_2)$ be fuzzy irresolute map and α be fuzzy rgα open in Y then $f^{-1}(\alpha)$ is fuzzy rgα open in X.

Proof: Let α be fuzzy rgα-open in Y. To prove $f^{-1}(\alpha)$ is fuzzy rgα open in X that is to prove $f^{-1}(\alpha)$ is both fuzzy semi-open and fuzzy semi-closed in X.Now α is fuzzy semi-open in Y. Since f is fuzzy irresolute map,$f^{-1}(\alpha)$ is fuzzy semi-open in X.

Now α is fuzzy semi-closed in Y as fuzzy rgα open set is fuzzy semi-closed then 1- α is Fuzzy semi-open in Y. Since f is fuzzy irresolute map, f^{-1}(1- α) is a fuzzy semi open in X. But $f^{-1}(1-\alpha) = 1 - f^{-1}(\alpha)$ is fuzzy semi-open in X and so $f^{-1}(\alpha)$ is semi closed in X. Thus $f^{-1}(\alpha)$ is both fuzzy semi-open and fuzzy semi-closed in X and hence $f^{-1}(\alpha)$ is fuzzy rgα open in X.

Thorem 2.15:If a map f: $(X,T_1) \rightarrow (Y,T_2)$ is fuzzy irresolute map and fuzzy sgw-closed and α is fuzzy sgw-closed set of X,then f(α) is a fuzzy sgw-closed set in Y.

Proof:Let α be a fuzzy closed set of X.Let $f(\alpha) \leq \mu$. Where μ is fuzzy rgα open in Y.Since Since f is fuzzy irresolute map, $f^{-1}(\mu)$ is a fuzzy rgα open in X, by lemma 2.14 and $\alpha \leq f^{-1}(\mu)$. Since α is a fuzzy sgw-closed set in X,P-cl(α) $\leq f^{-1}(\mu)$. Since f is a fuzzy sgw-closed set contained in the fuzzy rgα open set μ, which implies cl(f(p-cl(α)) $\leq \mu$ and hence cl(f(α)) $\leq \mu$. Therefore f(α) is a fuzzy sgw-closed set in Y.

Corollary 2.16: If a map f: $(X,T_1) \rightarrow (Y,T_2)$ is fuzzy irresolute map and fuzzy closed and α is a fuzzy sgw-closed set in fts.

Proof :The proof follows from the theorem 2.15 and the fact that every fuzzy closed map is a fuzzy sgw-closed map.

Theorem 2.17: Let f: X $\rightarrow$ Y and g: Y $\rightarrow$ Z be two mappings s.t

(i) if f is fuzzy continuous map and surjective, then g is fuzzy sgw-closed map.

(ii) if g is fuzzy sgw-irresolute map and injective then f is fuzzy sgw-closed map.

Proof :(i) Let μ be a fuzzy closed set in Y ,since f is fuzzy continuous map,$f^{-1}(\mu)$ is a fuzzy closed set in X. Since gof is a fuzzy sgw closed map,(gof)($f^{-1}(\mu)$) is a fuzzy sgw-closed set in Z but (gof) ($f^{-1}(\mu)$) = g(μ) as f is surjective thus g is fuzzy sgw closed map.

(ii) Let β be a fuzzy closed set of X then (gof)(β) is a fuzzy sgw closed set in Z, Since (gof) is a fuzzy sgw-closed map. Since g is fuzzy sgw-irresolute map g^{-1} (gof)(β) is fuzzy sgw-closed in Y. But g^{-1} (gof)(β) = f(β) as g is injective. Thus f is fuzzy sgw-closed map.

REFERENCES

[1] L.A.Zadeh, Fuzzy sets, Information and control, 8 (1965) 338-353.
[2] C.L.Chang,Fuzzy topological spaces, JI. Math. Anal. Appl., 24(1968), 182-190.
[3] K.K.Azad,On fuzzy semi continuity, fuzzy almost continuity and fuzzy weakly continuity. JI. Math. Anal. Appl. 82 No. 1 (1981), 14-32.
[4] S. S. Benchalli, R. S. Wali and Basavaraj M. Ittanagi on fuzzy rw-closed sets and fuzzy rw-open sets infuzzy topological spacesInt. J. of Mathematical Sciences and Applications, Vol. 1, No. 2, May 2011.
[5] N. Nagaveni, Studies on Generalizations of Homeomorphisms in Topological Spaces, Ph.D.Thesis, Bharathiar University, Coimbatore, 1999.
[6] Munkerjee, M.N.Sinha,,S.P.Irresolute and almost open functions between fts,fuzzy sets and systems , 29(1989)381-388.
[7] R. S. Wali and Vivekananda Dembre;fuzzy sgw-continuous maps and fuzzy sgw-irresolute in fts.
[8] A.Vadivel & K.vairamamanickam,rgα-Closed sets& rgα-open sets in Topological Spaces,Int J of math ,Analysis Vol 3, (2009)37,1803-1819.
[9] Thakur S.S. and Bajpai Pandey Jyoti“Intuitionistic Fuzzy rgα-closed sets”, International Journal of Fuzzy system and Rough System 4(1), 67-73.