iJRASET
International Journal For Research in
Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 11 **Issue:** II **Month of publication:** February 2023

DOI: https://doi.org/10.22214/ijraset.2023.49096

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.538
Volume 11 Issue II Feb 2023- Available at www.ijraset.com

Non-Extendability of Special DIO 3-Tuples Involving Nonaganal Pyramidal Number

S. Vidhya[1], T. Gokila[2]
[1]Assistant Professor, PG and Research Department of Mathematics, Cauvery College for Women (Autonomous), Affiliated to Bharathidasan University, Trichy – 18.
[2]PG Student, PG and Research Department of Mathematics, Cauvery College for Women (Autonomous), Affiliated to Bharathidasan University, Trichy – 18.

Abstract: This paper concerns with the construction of three distinct polynomials with integer coefficients (a_1, a_2, a_3) such that the product of any two contribution of the set subtracted to their sum and improved by a non-zero integer (or a polynomial with integer coefficients) is a perfect square and this shows the non-extendability of Special Dio Quadruple.
Keywords: Special Dio triples, Pyramidal number, Polynomials, Pell equation, Special Dio Quadruples.

I. INTRODUCTION

Diophantine Analysis is the mathematical study of Diophantine Problems, which was initiated by Diophantus in third century. A set of *m* distinct positive integers $\{a_1, a_2, \ldots, a_m\}$ is said to have the property D(*n*) if the product any two members of the set is decreased by their sum and increased by a non-zero integer *n*, is a perfect square for all *m* elements. Such a set is called Diophantine *m*-tuples of size *m*. Many mathematicians considered the extension problem of Diophantine quadruples with the property D(*n*) for any arbitrary integer *n* and also for any linear polynomial.

In this communication, we have presented three sections, in each of which we find the Diophantine triples for nonagonal Pyramidal number with distinct ranks and the non-extendability of Special Dio quadruple.

A. Notation

$\mathrm{PY}_n^9 =$ Pyramidal number on nonagonal of rank n$=\frac{1}{6}n(n+1)(7n-4)$

B. Basic Definition

A set of three distinct polynomials with integer coefficients (a_1, a_2, a_3) is said to be a Special Dio 3-tuple with property D (n) if $a_i * a_j - (a_i + a_j) + n$ is a perfect square for all $1 \le i \le j \le 3$, where n may be non-zero integer or polynomial with integer coefficients.

II. METHOD OF ANALYSIS

A. Section-I

Construction of Special Dio 3-tuples for pyramidal number on nonagonal of rank $n-1$ and n.

Let $a = 6\mathrm{PY}_{n-1}^9$, $b = 6\mathrm{PY}_n^9$ be Pyramidal numbers on nonagonal of rank $n-1$ and n respectively, such that

$ab - (a+b) - 7n^5 - n^4 + 3n^3 + 38n^2 - 3n + 1$ is a perfect square, say α^2.

Let c be any non-zero integer such that

$$ac - (a+c) - 7n^5 - n^4 + 3n^3 + 38n^2 - 3n + 1 = \beta^2 \quad (1)$$

$$bc - (b+c) - 7n^5 - n^4 + 3n^3 + 38n^2 - 3n + 1 = \gamma^2 \quad (2)$$

On solving equations (1) and (2), we get

$$(a-b)(7n^5 + n^4 + 3n) + (b-a)(3n^3 + 38n^2) = (b-1)\beta^2 - (a-1)\gamma^2 \quad (3)$$

Assume $\beta = x + (a-1)y$ and $\gamma = x + (b-1)y$ and it reduces to,

$$x^2 = (a-1)(b-1)y^2 - 7n^5 - n^4 + 3n^3 - 38n^2 + 3n \quad (4)$$

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.538
Volume 11 Issue II Feb 2023- Available at www.ijraset.com

The initial solution of the equation (4) is given by,

$x_0 = 7n^3 - 8n^2 - 5n + 1, \quad y_0 = 1$

Therefore, $\beta = 14n^3 - 26n^2 + 6n$

On substituting the values of 'a' and 'β' in equation (1), we get

$$c = 24PY_n^9 - 43n^2 + 13n + 1$$

Hence, the triple $(6PY_{n-1}^9, 6PY_n^9, 24PY_n^9 - 43n^2 + 13n + 1)$ is a Special Dio 3-tuple with the property $D(-7n^5 - n^4 + 3n^3 + 38n^2 - 3n + 1)$.

A few dimensional examples of the Special Dio 3-tuples satisfying the above property are mentioned below.

Table-I

n	(a,b,c)	D(n)
2	(6,60,95)	-69
3	(60,204,469)	-1367
4	(204,480,1285)	-6635
5	(480,930,2711)	-21139
6	(930,1596,4915)	-53729

We present below, some of the Special Dio 3-tuple for pyramidal number on nonagonal of rank mentioned above with suitable properties,

Table-II

a	b	c	D(n)
$6PY_{n-1}^9$	$6PY_n^9$	$24PY_n^9 - 45n^2 + 9n - 7$	$D(-21n^5 - 12n^4 - 7n^3 + 132n^2 + 49n + 9)$
$6PY_{n-1}^9$	$6PY_n^9$	$24PY_n^9 - 47n^2 + 7n - 11$	$D(-35n^5 - 7n^4 - n^3 + 193n^2 + 87n + 25)$
$6PY_{n-1}^9$	$6PY_n^9$	$24PY_n^9 - 49n^2 + 5n - 17$	$D(-49n^5 - 5n^3 + 286n^2 + 151n + 64)$

1) Non-Extendability

Let us show that the Special Dio 3-tuples cannot be extended to Special Dio Quadruples.

Consider,

$$ad - (a + d) - 7n^5 - n^4 + 3n^3 + 38n^2 - 3n + 1 = l^2 \quad (5)$$

$$bd - (b + d) - 7n^5 - n^4 + 3n^3 + 38n^2 - 3n + 1 = m^2 \quad (6)$$

$$cd - (c + d) - 7n^5 - n^4 + 3n^3 + 38n^2 - 3n + 1 = n^2 \quad (7)$$

Eliminating 'd' from (5) and (7), we get

$$(a - c)(7n^5 + n^4 - 3n^3 - 38n^2 + 3n) = (c - 1)l^2 - (a - 1)n^2 \quad (8)$$

Assuming $l = x + (a - 1)y$ and $n = x + (c - 1)y$ and it reduces to,

$$x^2 = (a - 1)(c - 1)y^2 - 7n^5 - n^4 + 3n^3 + 38n^2 - 3n \quad (9)$$

The initial solution of the equation (9) is given by

$x_0 = 14n^3 - 26n^2 + 6n, \quad y_0 = 1$

Therefore, $l = 21n^3 - 44n^2 + 17n - 1$.

On substituting the values of 'l' and 'a' in equation (5), we get

$$d = 63n^3 - 101n^2 + 20n$$

Substituting the above value in equation (6), we get

$$bd - (b + d) - 7n^5 - n^4 + 3n^3 + 38n^2 - 3n + 1 = 441n^6 - 525n^5 - 416n^4 + 397n^3 + 56n^2 - 19n + 1$$

, but this is not a perfect square.

Hence, $\{6PY^9_{n-1}, 6PY^9_n, 24PY^9_n - 43n^2 + 13n + 1, 63n^3 - 101n^2 + 20n\}$ is not a Special Dio Quadruple.

Thus, we cannot extend $\{6PY^9_{n-1}, 6PY^9_n, 24PY^9_n - 43n^2 + 13n + 1\}$ into a Special Dio Quadruple.

B. *Section-II*

Construction of Special Dio 3-tuples for pyramidal number on nonagonal of rank $n - 2$ and $n - 1$.

Let $a = 6PY^9_{n-2}$, $b = 6PY^9_{n-1}$ be Pyramidal numbers on nonagonal of rank $n - 2$ and $n - 1$ respectively, such that $ab - (a + b) - 7n^5 + 6n^4 - 3n^3 + 201n^2 - 305n + 133$ is a perfect square, say α^2.

Let c be any non-zero integer such that

$$ac - (a + c) - 7n^5 + 6n^4 - 3n^3 + 201n^2 - 305n + 133 = \beta^2 \quad (10)$$

$$bc - (b + c) - 7n^5 + 6n^4 - 3n^3 + 201n^2 - 305n + 133 = \gamma^2 \quad (11)$$

On solving equations (10) and (11), we get

$$(a - b)(7n^5 + 3n^3 + 305n) + (b - a)(6n^4 + 201n^2 + 132) = (b - 1)\beta^2 - (a - 1)\gamma^2 \quad (12)$$

Assume $\beta = x + (a - 1)y$ and $\gamma = x + (b - 1)y$ and it reduces to,

$$x^2 = (a - 1)(b - 1)y^2 - 7n^5 + 6n^4 - 3n^3 + 201n^2 - 305n + 132 \quad (13)$$

The initial solution of the equation (13) is given by,

$x_0 = 7n^3 - 29n^2 + 30n - 13, \quad y_0 = 1$

Therefore, $\beta = 14n^3 - 68n^2 + 98n - 50$

On substituting the values of 'a' and 'β' in equation (10), we get

$$c = 24PY^9_n - 127n^2 + 145n - 63$$

Hence, the triple $(6PY^9_{n-2}, 6PY^9_{n-1}, 24PY^9_n - 127n^2 + 145n - 63)$ is a Special Dio 3-tuple with property $D(-7n^5 + 6n^4 - 3n^3 + 201n^2 - 305n + 133)$.

A few dimensional examples of the Special Dio 3-tuples satisfying the above property are mentioned below.

Table-III

n	(a,b,c)	D(n)
3	(6,60,75)	-269
4	(60,204,445)	-3695
5	(204,480,1257)	-14867
6	(480,930,2679)	-41765
7	(930,1596,4879)	-96425

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.538
Volume 11 Issue II Feb 2023- Available at www.ijraset.com

We present below, some of the Special Dio 3-tuple for pyramidal number on nonagonal of rank mentioned above with suitable properties,

Table-IV			
a	b	c	D(n)
$6PY^9_{n-2}$	$6PY^9_{n-1}$	$24PY^9_n - 129n^2 + 147n - 89$	$D(-21n^5 + 9n^4 - 5n^3 + 783n^2 - 877n + 640)$
$6PY^9_{n-2}$	$6PY^9_{n-1}$	$24PY^9_n - 131n^2 + 137n - 127$	$D(-35n^5 - 13n^3 + 1778n^2 - 1415n + 1989)$
$6PY^9_{n-2}$	$6PY^9_{n-1}$	$24PY^9_n - 133n^2 + 129n - 157$	$D(-49n^5 + 7n^4 - 9n^3 + 2676n^2 - 1565n + 3564)$

1) Non-Extendability

Let us show that the Special Dio 3-tuples cannot be extended to Special Dio Quadruples.

Consider,

$$ad - (a + d) - 7n^5 + 6n^4 - 3n^3 + 201n^2 - 305n + 133 = l^2 \quad (14)$$

$$bd - (b + d) - 7n^5 + 6n^4 - 3n^3 + 201n^2 - 305n + 133 = m^2 \quad (15)$$

$$cd - (c + d) - 7n^5 + 6n^4 - 3n^3 + 201n^2 - 305n + 133 = n^2 \quad (16)$$

Eliminating 'd' from (14) and (16), we get

$$(a - c)(7n^5 - 6n^4 + 3n^3 - 201n^2 + 305n - 132) = (c - 1)l^2 - (a - 1)n^2 \quad (17)$$

Assuming $l = x + (a - 1)y$ and $n = x + (c - 1)y$ and it reduces to,

$$x^2 = (a - 1)(c - 1)y^2 - 7n^5 + 6n^4 - 3n^3 + 201n^2 - 305n + 132 \quad (18)$$

The initial solution of the equation (18) is given by

$x_0 = 14n^3 - 68n^2 + 98n - 50, \qquad y_0 = 1$

Therefore, $l = 21n^3 - 107n^2 + 166n - 87$.

On substituting the values of 'a' and 'l' in equation (14), we get

$$d = 63n^3 - 290n^2 + 403n - 200$$

Substituting the above value in equation (15), we get

$$bd - (b + d) - 7n^5 + 6n^4 - 3n^3 + 201n^2 - 305n + 133 = 441n^6 - 3171n^5 + 8740n^4 - 11917n^3 + 8542n^2 - 2919n + 333$$

, but this is not a perfect square.

Hence, $\{6PY^9_{n-2}, 6PY^9_{n-1}, 24PY^9_n - 127n^2 + 145n - 63, 63n^3 - 290n^2 + 403n - 200\}$ is not a Special Dio Quadruple.

Thus, we cannot extend $\{6PY^9_{n-2}, 6PY^9_{n-1}, 24PY^9_n - 127n^2 + 145n - 63\}$ into a Special Dio Quadruple.

C. Section-III

Construction of Special Dio 3-tuples for pyramidal number on nonagonal of rank n and $n + 1$.

Let $a = 6PY^9_n$, $b = 6PY^9_{n+1}$ be Pyramidal numbers on nonagonal of rank n and $n + 1$ respectively, such that $ab - (a + b) + 7n^5 + 5n^4 - n^3 + 46n^2 + 39n + 10$ is a perfect square, say α^2.

Let c be any non-zero integer such that

$$ac - (a + c) + 7n^5 + 5n^4 - n^3 + 46n^2 + 39n + 10 = \beta^2 \quad (19)$$

$$bc - (b + c) + 7n^5 + 5n^4 - n^3 + 46n^2 + 39n + 10 = \gamma^2 \quad (20)$$

On solving equations (19) and (20), we get

$$(a-b)n^3+(b-a)(7n^5+5n^4+46n^2+39n+10)=(b-1)\beta^2-(a-1)\gamma^2 \quad (21)$$

Assume $\beta = x+(a-1)y$ and $\gamma = x+(b-1)y$ and it reduces to,

$$x^2=(a-1)(b-1)y^2+7n^5+5n^4-n^3+46n^2+39n+9 \quad (22)$$

The initial solution of the equation (22) is given by,

$x_0 = 7n^3+14n^2+n-2, \qquad y_0 = 1$

Therefore, $\beta = 14n^3+17n^2-3n-3$

On substituting the values of 'a' and 'β' in equation (19), we get

$$c = 24PY_n^9+43n^2+37n+1$$

Hence, the triple $(6PY_n^9, 6PY_{n+1}^9, 24PY_n^9+43n^2+37n+1)$ is a Special Dio 3-tuple with the property $D(7n^5+5n^4-n^3+46n^2+39n+10)$.

A few dimensional examples of the Special Dio 3-tuples satisfying the above property are mentioned below.

Table-V

n	(a,b,c)	D(n)
1	(6,60,105)	106
2	(60,204,487)	568
3	(204,480,1315)	2620
4	(480,930,2757)	9286
5	(930,1596,4981)	26230

We present below, some of the Special Dio 3-tuple for Pyramidal number on nonagonal of rank mentioned above with suitable properties,

Table-VI

a	b	c	D(n)
$6PY_n^9$	$6PY_{n+1}^9$	$24PY_n^9+45n^2+31n+15$	$D(21n^5-8n^4+9n^3+255n^2+23n+31)$
$6PY_n^9$	$6PY_{n+1}^9$	$24PY_n^9+47n^2+27n+25$	$D(35n^5-5n^4+11n^3+437n^2-37n+106)$
$6PY_n^9$	$6PY_{n+1}^9$	$24PY_n^9+49n^2+23n+35$	$D(49n^5+5n^3+647n^2-137n+231)$

1) Non-Extendability

Let us show that the Special Dio 3-tuples cannot be extended to Special Dio Quadruples.

Consider,

$$ad-(a+d)+7n^5+5n^4-n^3+46n^2+39n+10=l^2 \quad (23)$$

$$bd-(b+d)+7n^5+5n^4-n^3+46n^2+39n+10=m^2 \quad (24)$$

$$cd-(c+d)+7n^5+5n^4-n^3+46n^2+39n+10=n^2 \quad (25)$$

Eliminating 'd' from (23) and (25), we get

$$(a-c)(-7n^5-5n^4+n^3-46n^2-39n-9)=(c-1)l^2-(a-1)n^2 \quad (26)$$

Assuming $l = x + (a-1)y$ and $n = x + (c-1)y$ and it reduces to,

$$x^2 = (a-1)(c-1)y^2 + 7n^5 + 5n^4 - n^3 + 46n^2 + 39n + 9 \tag{27}$$

The initial solution of the equation (27) is given by

$x_0 = 14n^3 + 17n^2 - 3n - 3, \quad y_0 = 1$

Therefore,l$= 21n^3 + 20n^2 - 7n - 4$.

On substituting the values of 'a' and 'l' in equation (23), we get

$$d = 63n^3 + 92n^2 + 11n - 6$$

Substituting the above value in equation (24), we get

$bd - (b+d) + 7n^5 + 5n^4 - n^3 + 46n^2 + 39n + 10 = 441n^6 + 2163n^5 + 3739n^4 + 2645n^3 + 591n^2 - 67n - 26$, but this is not a perfect square.

Hence, $\{6PY_n^9,\ 6PY_{n+1}^9, 24PY_n^9 + 43n^2 + 37n + 1, 63n^3 + 92n^2 + 11n - 6\}$ is not a Special Dio Quadruple.

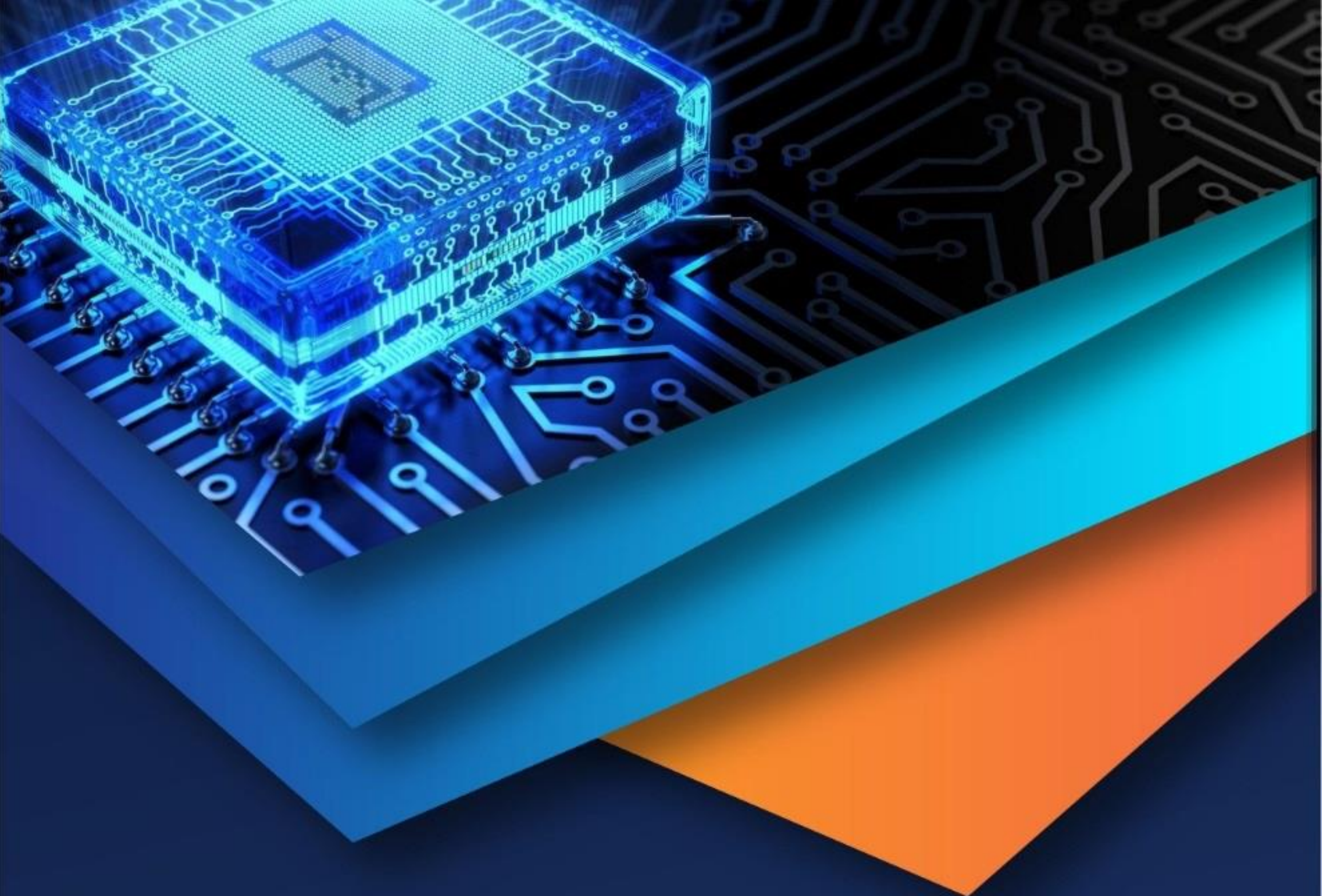
Thus, we cannot extend $\{6PY_n^9,\ 6PY_{n+1}^9, 24PY_n^9 + 43n^2 + 37n + 1\}$into a Special Dio Quadruple.

III. CONCLUSION

In this paper, we have presented the construction of a special dio 3-tuples for Pyramidal number with suitable properties and the non-extendability of Special Dio Quadruple. To conclude that one may search for Special Dio 3- tuples for higher order Pyramidal number with their corresponding suitable properties.

REFERENCES

[1] Carmichael R.D, History of Theory of numbers and Diophantine Analysis, *Dover Publication*, New York, 1959.
[2] Mordell L.J, Diophantine equations, *Academic press*, London, 1969.
[3] Nagell T, Introduction to Number Theory, *Chelsea publishing company*, New York, 1982.
[4] Brown E, "Sets in which xy + k is always a perfect square", Math. Comp, 1985, 45, 613-620
[5] Beardon A.F, Deshpande M.N, "Diophantine Triples", the Mathematical Gazette, 2002, 86, 258-260.
[6] Vidhya S, Janaki G (2017), Special Dio 3-tuples for Pronic number-I, *International Journal for Research in Applied Science and Engineering Technology*, *5*(XI), 159-162.
[7] Janaki G, Vidhya S (2017), Special Dio 3-tuples for Pronic number-II, *International Journal of Advanced Science and Research*, *2*(6), 8-12.
[8] Vidhya S, Janaki G (2019), Elevation of Stella Octangula number as a Special Dio 3-tuples and the non-extendability of special Dio quadruple, *Adalya Journal*, *8*(8), 621-624.
[9] Janaki G, Saranya C (2017), Special Dio 3-tuples for Pentatope number, *Journal of Mathematics and Informatics*, *11*, 119-123.
[10] Gopalan M.A, Geetha V and Vidhyalakshmi S (2014), Special Dio 3-tuples for Special Numbers-I, *The Bulletin of Society for Mathematical Services and Standards*, *10*, 1-6.
[11] Gopalan M.A, Geetha K and Somnath M (2014), Special Dio 3-tuples, *The Bulletin of Society for Mathematical Services and Standards*, *10*, 22-25.



10.22214/IJRASET

INDEX COPERNICUS
45.98

ISRA JIF
IMPACT FACTOR:
7.129

TOGETHER WE REACH THE GOAL
IMPACT FACTOR:
7.429

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY