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Novel Linguistic Intuitionistic Fuzzy Score and Accuracy functions for Linguistic Intuitionistic Fuzzy TOPSIS Method

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Abstract: This work proposes various Linguistic Intuitionistic Fuzzy Score and Accuracy functions (LIF-Score and LIF-Accuracy), including linguistic degree, membership, non-membership, and hesitancy degree, for Linguistic Intuitionistic Fuzzy Sets (LIFSs) in Multiple Attribute Group Decision Making (MAGDM) problems. The proposed functions are integrated into the Linguistic Intuitionistic Fuzzy-Technique of Order Preference by Similarity to Ideal Solution (LIF-TOPSIS) MAGDM approach to rank the alternatives, and a numerical example demonstrating their use in ranking alternatives is shown in the paper.
Keywords: MAGDM, LIF-Score, LIF-Accuracy, LIF-TOPSIS, LIFS, Decision making.

I. INTRODUCTİON

Whenever attributes involving differences of opinion are involved in a decision system, one of the DSS's working principles is to rank the best option among those that are available, and if the distance is sought with the ideal solution, then the method is called TOPSIS technique. Based on ranking techniques that gauge proximity to either the positive or negative ideal answer, the TOPSIS method is where the decision-making problem will focus its methodology [3,5,6,7,8,9,11] in choosing the appropriate alternative which will better suit the problem solution. In recent days, intuitionistic fuzzy data proposed in [1,2] has gained the attention of researchers to a large extent. Score and accuracy functions are extremely important in Fuzzy Decision Making situations [4,10,12], especially in ranking the final alternatives. In this work, we have suggested several score and accuracy measures for Linguistic Intuitionistic Fuzzy Sets (LIFSs), and we employ them in the Linguistic Intuitionistic Fuzzy TOPSIS method for both attribute weight determination and ranking of the best alternatives from the available ones. Utilizing the suggested score and accuracy functions, various calculations are carried out, and new decision algorithms are established based on the proposed measures. According to the study, our novel score and accuracy functions which are new to the field of Linguistic Intuitionistic Fuzzy MAGDM problems are observed to be one of the effective measures in ranking of the alternatives as well as in producing weight vectors for the aggregation process in the MAGDM problems.
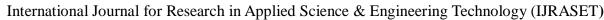
II. LINGUISTIC INTUITIONISTIC FUZZY SETS

Definition 2.1 [11] Let $\tilde{\sigma}_1 = <l_{\theta(\sigma_1)}, (\alpha(\sigma_1), \gamma(\sigma_1))>$ and $\tilde{\sigma}_2 = <l_{\theta(\sigma_2)}, (\alpha(\sigma_2), \gamma(\sigma_2))>$ be two LIFNs and $\lambda \geq 0$. Then the operations of LIFNs are defined as:

$$\tilde{\sigma}_1 + \tilde{\sigma}_2 = <l_{\theta(\sigma_1)+\theta(\sigma_2)}, (\alpha(\sigma_1)+\alpha(\sigma_2)-\alpha(\sigma_1)\alpha(\sigma_2), \gamma(\sigma_1)\gamma(\sigma_2))>,$$

$$\tilde{\sigma}_1 \otimes \tilde{\sigma}_2 = <l_{\theta(\sigma_1)\times\theta(\sigma_2)}, (\alpha(\sigma_1)\alpha(\sigma_2), \gamma(\sigma_1)+\gamma(\sigma_2)-\gamma(\sigma_1)\gamma(\sigma_2))>,$$

$$\lambda\tilde{\sigma}_1 = <l_{\lambda\times\theta(\sigma_1)}, (1-(1-\alpha(\sigma_1))^\lambda, (\gamma(\sigma_1))^\lambda)>, \text{ and } \tilde{\sigma}_1^\lambda = <l_{\theta(\sigma_1)^\lambda}, (\alpha(\sigma_1))^\lambda, 1-(1-(\gamma(\sigma_1))^\lambda)>.$$

III. SCORE FUNCTIONS FOR LINGUISTIC INTUITIONISTIC FUZZY SETS

1) Definition 3.1 Let $\tilde{\sigma}_j = <l_{\theta(\sigma_j)}, (\alpha(\sigma_j), \gamma(\sigma_j))>$ for j=1,2,…,n be a collection of Linguistic Intuitionistic Fuzzy numbers, where $l_{\theta(\sigma_j)}$ denotes the linguistic degree and, $\alpha(\sigma_j)$ denotes the membership degree and $\gamma(\sigma_j)$ denotes the non-membership degree.

The proposed LIF-Score functions are as follows:

- $S_i(\tilde{\sigma}_j) = \theta_j(\alpha(\sigma_j) - \gamma(\sigma_j)), \ S_i(\tilde{\sigma}_j) \in [-1,1]$

- $S_{ii}(\tilde{\sigma}_j) = \theta_j.(\alpha(\sigma_j) - (\gamma(\sigma_j) + \pi(\sigma_j))), \ S_{ii}(\tilde{\sigma}_j) \in [-1,1],$

- where $\pi(\sigma_j) = 1 - (\alpha(\sigma_j) + (\gamma(\sigma_j))$ is the hesitation degree.

- $S_{iii}(\tilde{\sigma}_j) = \theta_j.\left(\alpha(\sigma_j) - \left(\dfrac{\gamma(\sigma_j) + \pi(\sigma_j)}{2}\right)\right), S_{iii}(\tilde{\sigma}_j) \in [-1,1].$

- $S_{iv}(\tilde{\sigma}_j) = \theta_j.\left(\left(\dfrac{\alpha(\sigma_j) + \gamma(\sigma_j)}{2}\right) - \pi(\sigma_j)\right), S_{iv}(\tilde{\sigma}_j) \in [-1,0.5].$

- $S_v(\tilde{\sigma}_j) = \theta_j(\lambda.\alpha(\sigma_j) + (1-\lambda)(1-\gamma(\sigma_j))), \lambda \in [0,1], S_v(\tilde{\sigma}_j) \in [0,1].$

- $S_{vi}(\tilde{\sigma}_j) = \theta_j.(2\alpha(\sigma_j) + \gamma(\sigma_j) - 1), S_{vi}(\tilde{\sigma}_j) \in [0,1].$

- $S_{vii}(\sigma_j) = \theta_j(\alpha(\sigma_j) + \alpha(\sigma_j)(1 - \alpha(\sigma_j) - \gamma(\sigma_j))), S_{vii}(\sigma_j) \in [0,1].$

- $S_{viii}(\sigma_j) = \theta_j.\left(\dfrac{3\alpha(\sigma_j) - \pi(\sigma_j) - 1}{2}\right), S_{viii}(\sigma_j) \in [-1,1].$

2) Definition 3.2 Let $L_1 = \langle S_{\theta_1}, (\alpha_1, \gamma_1) \rangle$ and $L_2 = \langle S_{\theta_2}, (\alpha_2, \gamma_2) \rangle$ be two linguistic intuitionistic fuzzy numbers, where $S_{\theta_1}, S_{\theta_2}, \alpha_1, \gamma_1, \alpha_2, \gamma_2 \in [0,1]$, $0 \le \alpha_1 + \gamma_1 \le 1$ *and* $0 \le \alpha_2 + \gamma_2 \le 1.$ Then,

- If $\theta_1.\alpha_1 \ge \theta_2.\alpha_2$ and $\theta_1.\gamma_1 < \theta_2.\gamma_2$, then $L_1 > L_2.$

- If $\theta_1.\alpha_1 < \theta_2.\alpha_2$ and $\theta_1.\gamma_1 \ge \theta_2.\gamma_2$, then $L_1 < L_2.$

- If $\theta_1.\alpha_1 = \theta_2.\alpha_2$ and $\theta_1.\gamma_1 = \theta_2.\gamma_2$, then $L_1 = L_2.$

a) Theorem 3.1 Let $\tilde{\sigma}_j, (j = 1,2,\ldots.,n),$ be a collection of Linguistic Intuitionistic Fuzzy Numbers. Then the Score function $S_i(\tilde{\sigma}_j) = \theta_j(\alpha(\sigma_j) - \gamma(\sigma_j)), \ S_i(\tilde{\sigma}_j) \in [-1,1]$ is

(i) Bounded and (ii) Monotonic.

Proof:

(i) Boundedness: From definition 3.1 (a), $S_i(\tilde{\sigma}_j) = \theta_j(\alpha(\sigma_j) - \gamma(\sigma_j)), \ S_i(\tilde{\sigma}_j) \in [-1,1]$

Since $\theta_j, \alpha(\sigma_j), \gamma(\sigma_j) \le 1$ and $\alpha(\sigma_j) + \gamma(\sigma_j) \le 1$,

$\alpha(\sigma_j) - \gamma(\sigma_j) \le 1$ and $\theta_j\alpha(\sigma_j) - \theta_j\gamma(\sigma_j) \le 1$ $[\because \theta_j \le 1].$

And since $0 \le \theta_j, \alpha(\sigma_j), \gamma(\sigma_j) \le 1$ and $-1 \le \alpha(\sigma_j) - \gamma(\sigma_j)$

hence $\theta_j\alpha(\sigma_j) - \theta_j\gamma(\sigma_j) \ge -1,$ [if $\theta_j = 1$]

(ii) Monotonicity: If $\tilde{\sigma}_1 > \tilde{\sigma}_2,$ then $S_i(\tilde{\sigma}_1) > S_i(\tilde{\sigma}_2)$

$\theta_1.\mu_1 \ge \theta_2.\mu_2$ and $\theta_1.\gamma_1 < \theta_2.\gamma_2 \Rightarrow -\theta_1.\gamma_1 > -\theta_2.\gamma_2,$

$\Rightarrow \theta_1.\mu_1 - \theta_1.\gamma_1 > \theta_2.\mu_2 - \theta_2.\gamma_2$

$\Rightarrow S_i(\tilde{\sigma}_1) > S_i(\tilde{\sigma}_2).$

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b) Theorem 3.2 Let $\tilde{\sigma}_j, (j = 1,2,\ldots,n)$, be a collection of Linguistic Intuitionistic Fuzzy Numbers. Then the Score function

$$S_{ii}(\tilde{\sigma}_j) = \theta_j.(\alpha(\sigma_j)-(\gamma(\sigma_j)+\pi(\sigma_j))),\ S_{ii}(\tilde{\sigma}_j) \in [-1,1]$$ is bounded.

Proof:

(i)   Boundedness: From definition 3.1 (b),

$$S_{ii}(\tilde{\sigma}_j) = \theta_j.(\alpha(\sigma_j)-(\gamma(\sigma_j)+\pi(\sigma_j))),\ S_{ii}(\tilde{\sigma}_j) \in [-1,1].$$

Since, $\theta_j, \alpha(\sigma_j), \gamma(\sigma_j), \pi(\sigma_j) \leq 1$ and $\alpha(\sigma_j)+\gamma(\sigma_j)+\pi(\sigma_j) \leq 1$;

$\alpha(\sigma_j)-\left(\gamma(\sigma_j)+\pi(\sigma_j)\right) \leq 1$ and $\theta_j\left(\alpha(\sigma_j)-\left(\gamma(\sigma_j)+\pi(\sigma_j)\right)\right) \leq 1$ $[\because \theta_j \leq 1]$.

And since $0 \leq \theta_j, \alpha(\sigma_j), \gamma(\sigma_j), \pi(\sigma_j) \leq 1$ and $-1 \leq \alpha(\sigma_j)-\left(\gamma(\sigma_j)+\pi(\sigma_j)\right)$

hence $\theta_j.(\alpha(\sigma_j)-(\gamma(\sigma_j)+\pi(\sigma_j))) \geq -1,$ [if $\theta_j = 1$].

c) Theorem 3.3 Let $\tilde{\sigma}_j, (j = 1,2,\ldots,n)$, be a collection of Linguistic Intuitionistic Fuzzy Numbers. Then the Score function

$S_{iii}(\tilde{\sigma}_j) = \theta_j.\left(\alpha(\sigma_j) - \left(\frac{\gamma(\sigma_j)+\pi(\sigma_j)}{2}\right)\right),\ S_{iii}(\tilde{\sigma}_j) \in [-1,1]$ is bounded.

Proof:

(i)   Boundedness: From definition 3.1 (c),

$$S_{iii}(\tilde{\sigma}_j) = \theta_j.\left(\alpha(\sigma_j) - \left(\frac{\gamma(\sigma_j)+\pi(\sigma_j)}{2}\right)\right).$$

Since $\theta_j, \alpha(\sigma_j), \gamma(\sigma_j), \pi(\sigma_j) \leq 1$ and $\alpha(\sigma_j)+\gamma(\sigma_j)+\pi(\sigma_j) \leq 1$,

$\left(\alpha(\sigma_j)-\left(\frac{\gamma(\sigma_j)+\pi(\sigma_j)}{2}\right)\right) \leq 1$ and $\theta_j\left(\alpha(\sigma_j)-\left(\frac{\gamma(\sigma_j)+\pi(\sigma_j)}{2}\right)\right) \leq 1$ $[\because \theta_j \leq 1]$.

And since $0 \leq \theta_j, \alpha(\sigma_j), \gamma(\sigma_j), \pi(\sigma_j) \leq 1$ and $-1 \leq \alpha(\sigma_j)-\left(\gamma(\sigma_j)+\pi(\sigma_j)\right)$

hence $\theta_j.(\alpha(\sigma_j)-(\gamma(\sigma_j)+\pi(\sigma_j))) \geq -1,$ [if $\theta_j = 1$].

d) Theorem 3.4 Let $\tilde{\sigma}_j, (j = 1,2,\ldots,n)$, be a collection of Linguistic Intuitionistic Fuzzy Numbers. Then the Score function

$$S_{iv}(\tilde{\sigma}_j) = \theta_j.\left(\left(\frac{\alpha(\sigma_j)+\gamma(\sigma_j)}{2}\right)-\pi(\sigma_j)\right),$$

$S_{iv}(\tilde{\sigma}_j) \in [-1,0.5]$ is bounded.

Proof:

(i)   Boundedness: From definition 3.1 (d),

$$S_{iv}(\tilde{\sigma}_j) = \theta_j.\left(\left(\frac{\alpha(\sigma_j)+\gamma(\sigma_j)}{2}\right)-\pi(\sigma_j)\right),$$

Since $\theta_j, \alpha(\sigma_j), \gamma(\sigma_j), \pi(\sigma_j) \leq 1$ and $\alpha(\sigma_j)+\gamma(\sigma_j)+\pi(\sigma_j) \leq 1$,

$\left(\left(\frac{\alpha(\sigma_j)+\gamma(\sigma_j)}{2}\right)-\pi(\sigma_j)\right) \leq 0.5$ and $\theta_j\left(\left(\frac{\alpha(\sigma_j)+\gamma(\sigma_j)}{2}\right)-\pi(\sigma_j)\right) \leq 0.5$ $[\because \theta_j \leq 1]$.

And since $0 \leq \theta_j, \alpha(\sigma_j), \gamma(\sigma_j), \pi(\sigma_j) \leq 1$ and $-1 \leq \left(\left(\dfrac{\alpha(\sigma_j) + \gamma(\sigma_j)}{2} \right) - \pi(\sigma_j) \right)$

hence $\theta_j \cdot \left(\left(\dfrac{\alpha(\sigma_j) + \gamma(\sigma_j)}{2} \right) - \pi(\sigma_j) \right) \geq -1,$ [if $\theta_j = 1$].

e) Theorem 3.5 Let $\tilde{\sigma}_j, (j = 1, 2, \ldots, n),$ be a collection of Linguistic Intuitionistic Fuzzy Numbers. Then the Score function

$$S_v(\tilde{\sigma}_j) = \theta_j(\lambda \cdot \alpha(\sigma_j) + (1 - \lambda)(1 - \gamma(\sigma_j))), \lambda \in [0,1],$$

$S_v(\tilde{\sigma}_j) \in [0,1]$ is (i) Bounded, (ii) Monotonic.

   Proof:

(i)   Boundedness: From definition 3.1 (e),
$$S_v(\tilde{\sigma}_j) = \theta_j(\lambda \cdot \alpha(\sigma_j) + (1 - \lambda)(1 - \gamma(\sigma_j))), \lambda \in [0,1]$$

Since $\theta_j, \alpha(\sigma_j), \gamma(\sigma_j) \leq 1, \ 0 \leq \lambda \leq 1$ and $\alpha(\sigma_j) + \gamma(\sigma_j) \leq 1,$

$0 \leq \lambda \cdot \alpha(\sigma_j) \leq 1, \ 0 \leq (1 - \lambda) \leq 1, \ 0 \leq (1 - \gamma(\sigma_j)) \leq 1 \qquad [\because \theta_j \leq 1].$

$S_v(\tilde{\sigma}_j) = \theta_j(\lambda \cdot \alpha(\sigma_j) + (1 - \lambda)(1 - \gamma(\sigma_j))), \quad [\because \lambda \in [0,1], \theta_j \leq 1].$

$0 \leq S_v(\tilde{\sigma}_j) \leq 1.$

(ii)   Monotonicity: If $\tilde{\sigma}_1 > \tilde{\sigma}_2,$ then $S_i(\tilde{\sigma}_1) > S_i(\tilde{\sigma}_2)$

$\theta_1 \cdot \alpha(\sigma_1) \geq \theta_2 \cdot \alpha(\sigma_2)$ and $\theta_1 \cdot \gamma(\sigma_1) < \theta_2 \cdot \gamma(\sigma_2) \Rightarrow -\theta_1 \cdot \gamma(\sigma_1) > -\theta_2 \cdot \gamma(\sigma_2),$

$\Rightarrow (1 - \lambda)(\theta_1 - \theta_1 \cdot \gamma(\sigma_1)) > (1 - \lambda)(\theta_2 - \theta_2 \cdot \gamma(\sigma_2))$

$\Rightarrow \theta_1(\lambda \cdot \alpha(\sigma_1) + (1 - \lambda)(1 - \gamma(\sigma_1))) > \theta_2(\lambda \cdot \alpha(\sigma_2) + (1 - \lambda)(1 - \gamma(\sigma_2)))$

$\Rightarrow S_i(\tilde{\sigma}_1) > S_i(\tilde{\sigma}_2).$

IV.   ACCURACY FUNCTIONS FOR LINGUISTIC INTUITIONISTIC FUZZY SETS

This section proposes new Accuracy functions for Linguistic Intuitionistic Fuzzy numbers.

1) Definition 4.1   Let $\tilde{\sigma}_j = \langle l_{\theta(\sigma_j)}, (\alpha(\sigma_j), \gamma(\sigma_j)) \rangle$ for j=1,2,...,n be a collection of Linguistic Intuitionistic Fuzzy

numbers, where $l_{\theta(\sigma_j)}$ denotes the linguistic degree and, $\alpha(\sigma_j)$ denotes the membership degree and $\gamma(\sigma_j)$ denotes the non-

membership degree.

The LIF-Score functions are proposed as follows:

- $A_i(\sigma_j) = \theta_j(\alpha(\sigma_j) + \gamma(\sigma_j)), \ A_i(\sigma_j) \in [0,1]$

- $A_{ii}(\sigma_j) = \theta_j(\alpha(\sigma_j) + (\gamma(\sigma_j) + \pi(\sigma_j))), \ A_{ii}(\sigma_j) \in [0,1]$

- $A_{iii}(\sigma_j) = \theta_j \left(\alpha(\sigma_j) + \left(\dfrac{\gamma(\sigma_j) + \pi(\sigma_j)}{2} \right) \right), \ A_{iii}(\sigma_j) \in [0,1]$

- $A_{iv}(\sigma_j) = \theta_j \cdot \left(\left(\dfrac{\gamma(\sigma_j) + \alpha(\sigma_j)}{2} \right) + \pi(\sigma_j) \right), \ A_{iv}(\sigma_j) \in [0,1]$

- $A_v(\sigma_j) = \theta_j \cdot \left(\dfrac{1 - \gamma(\sigma_j)}{2 - \alpha(\sigma_j) - \gamma(\sigma_j)} \right), \ A_v(\sigma_j) \in [0,1]$

a) Theorem 4.1 Let $\tilde{\sigma}_j, (j = 1,2,\ldots,n),$ be a collection of Linguistic Intuitionistic Fuzzy numbers. Then the Score functions:

- $A_i(\sigma_j) = \theta_j(\alpha(\sigma_j) + \gamma(\sigma_j)), \ A_i(\sigma_j) \in [0,1]$

- $A_{ii}(\sigma_j) = \theta_j(\alpha(\sigma_j) + (\gamma(\sigma_j) + \pi(\sigma_j))), \ A_{ii}(\sigma_j) \in [0,1]$

- $A_{iii}(\sigma_j) = \theta_j \left(\alpha(\sigma_j) + \left(\dfrac{\gamma(\sigma_j) + \pi(\sigma_j)}{2} \right) \right), \ A_{iii}(\sigma_j) \in [0,1]$

- $A_{iv}(\sigma_j) = \theta_j \cdot \left(\left(\dfrac{\gamma(\sigma_j) + \alpha(\sigma_j)}{2} \right) + \pi(\sigma_j) \right), \ A_{iv}(\sigma_j) \in [0,1]$

- $A_v(\sigma_j) = \theta_j \cdot \left(\dfrac{1 - \gamma(\sigma_j)}{2 - \alpha(\sigma_j) - \gamma(\sigma_j)} \right), \ A_v(\sigma_j) \in [0,1]$

are bounded.

Proof:

➢ From definition 4.1 (a), $A_i(\sigma_j) = \theta_j(\alpha(\sigma_j) + \gamma(\sigma_j)), \ A_i(\sigma_j) \in [0,1]$.

Since $\theta_j, \alpha(\sigma_j), \gamma(\sigma_j) \le 1$ and $0 \le \alpha(\sigma_j) + \gamma(\sigma_j) \le 1$,

$\qquad 0 \le \theta_j\left(\alpha(\sigma_j) + \gamma(\sigma_j) \right) \le 1 \quad [\because \theta_j \le 1].$

➢ From definition 4.1 (b),

$A_{ii}(\sigma_j) = \theta_j(\alpha(\sigma_j) + (\gamma(\sigma_j) + \pi(\sigma_j))),$

Since $\theta_j, \alpha(\sigma_j), \gamma(\sigma_j) \le 1$ and $\alpha(\sigma_j) + \gamma(\sigma_j) + \pi(\sigma_j) = 1$,

$\qquad 0 \le \theta_j(\alpha(\sigma_j) + (\gamma(\sigma_j) + \pi(\sigma_j))) \le 1, \quad [\because \theta_j \le 1].$

➢ From definition 4.1 (c),

$A_{iii}(\sigma_j) = \theta_j \left(\alpha(\sigma_j) + \left(\dfrac{\gamma(\sigma_j) + \pi(\sigma_j)}{2} \right) \right),$

Since $\theta_j, \alpha(\sigma_j), \gamma(\sigma_j) \le 1$ and $\alpha(\sigma_j) + \gamma(\sigma_j) + \pi(\sigma_j) = 1$,

$\qquad \Rightarrow 0.5 \le \left(\alpha(\sigma_j) + \left(\dfrac{\gamma(\sigma_j) + \pi(\sigma_j)}{2} \right) \right) \le 1$

$\qquad \Rightarrow 0 \le \theta_j \left(\alpha(\sigma_j) + \left(\dfrac{\gamma(\sigma_j) + \pi(\sigma_j)}{2} \right) \right) \le 1, \quad [\because 0 \le \theta_j \le 1].$

➢ From definition 4.1 (d),

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$$A_{iv}(\sigma_j) = \theta_j \cdot \left(\left(\frac{\gamma(\sigma_j) + \alpha(\sigma_j)}{2} \right) + \pi(\sigma_j) \right),$$

Since $\theta_j, \alpha(\sigma_j), \gamma(\sigma_j) \le 1$ and $\alpha(\sigma_j) + \gamma(\sigma_j) + \pi(\sigma_j) = 1$,

$$\Rightarrow 0.5 \le \left(\pi(\sigma_j) + \left(\frac{\gamma(\sigma_j) + \alpha(\sigma_j)}{2} \right) \right) \le 1$$

$$\Rightarrow 0 \le \theta_j \left(\pi(\sigma_j) + \left(\frac{\gamma(\sigma_j) + \alpha(\sigma_j)}{2} \right) \right) \le 1, \quad [\because 0 \le \theta_j \le 1].$$

➢ From definition 4.1 (e),

$$A_v(\sigma_j) = \theta_j \cdot \left(\frac{1 - \gamma(\sigma_j)}{2 - \alpha(\sigma_j) - \gamma(\sigma_j)} \right),$$

Since $\theta_j, \alpha(\sigma_j), \gamma(\sigma_j) \le 1$ and $0 \le 1 - \gamma(\sigma_j) \le 1 - \gamma(\sigma_j) + 1 - \alpha(\sigma_j)$,

$$\Rightarrow 0 \le \left(\frac{1 - \gamma(\sigma_j)}{2 - \alpha(\sigma_j) - \gamma(\sigma_j)} \right) \le 1$$

$$\Rightarrow 0 \le \theta_j \left(\frac{1 - \gamma(\sigma_j)}{2 - \alpha(\sigma_j) - \gamma(\sigma_j)} \right) \le 1, \quad [\because 0 \le \theta_j \le 1].$$

V. The MAGDM with LIF-TOPSIS method using Score and Accuracy function

Let $E = \{\varepsilon_1, \varepsilon_2, \ldots \varepsilon_p\}$ be a set of experts, $L = (L_1, L_2, \ldots L_m)$ be a discrete set of alternatives, $C = (C_1, C_2, \ldots C_n)$ be the set of attributes, and $W = (\omega_1, \omega_2, \ldots \omega_n)^T$ be the weighting vector of the attributes, $\sum_{j=1}^{n} \omega_j = 1, \omega_j \ge 0$. Let $\lambda = (\lambda_1, \lambda_2, \ldots \lambda_p)$ be the expert's weighting vector, $\sum_{k=1}^{p} \lambda_k = 1$. Suppose that $\tilde{R}^k = \left[\tilde{t}_{ij}^k \right]_{mXn}$ is the decision matrix, where $\tilde{t}^k{}_{ij} = \langle a^k{}_{ij}, (\alpha_{ijk}, \gamma_{ijk}) \rangle$ takes the form of the Linguistic Intuitionistic number, given by the decision maker ε_k, for alternative L_i with respect to attribute C_j.

Rank the alternatives by using the steps below:

1) *Step 1: Normalize the given matrices and make the integrated matrix*

Integrate the matrix $\tilde{R}^k = \left[\tilde{t}_{ij}^k \right]_{mXn}$ given by decision maker ε_k into the integrated matrix $\tilde{R}^k = \left[\tilde{t}_{ij}^k \right]_{mXn} : \tilde{t}_{ij} = \sum_{k=1}^{p} \lambda_k \tilde{t}^k{}_{ij}$,

where, $\tilde{t}_{ij} = \langle a_{ij}, (\alpha_{ij}, \gamma_{ij}) \rangle$.

2) *Step 2: The integrated matrix is converted into a crisp matrix using Score function/ Accuracy function*

The score matrix is $S(\tilde{R}^k) = \left[S\tilde{t}_{ij}^k \right]_{mXn}$, where $S\tilde{t}^k{}_{ij}$ is the score value of the alternative L_i with respect to attribute C_j.

3) *Step 3: Evaluate the attribute weights*

For the attribute C_j, the deviation values of alternative L_i to all the other alternatives can be defined as $D_{ij}(\omega_j) = \sum_{l=1}^{m} (S\tilde{t}_{ij}^k) \omega_j$,

where $D_j(\omega_j) = \sum_{i=1}^{m} D_{ij}(\omega_j) = \sum_{i=1}^{m}\sum_{l=1}^{m} S\tilde{t}_{ij}^{k}\omega_j$ indicates the total deviation values of all alternatives to the other alternatives

for the attribute C_j. $D(\omega_j) = \sum_{j=1}^{n} D_j(\omega_j) = \sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{l=1}^{m} S\tilde{t}_{ij}^{k}\omega_j$ represents the deviation of all attributes to all alternatives. The

optimum model is built as follows:

$$\begin{cases} max\, D\,(\omega_j) = \sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{l=1}^{m} S\tilde{t}^{k}{}_{ij}\,\omega_j \\ s.t \sum_{j=1}^{n} \omega_j^2 = 1, \omega_j \geq 0, j = 1,2,\dots n \end{cases}$$

and: $\omega_j = \sum_{j=1}^{n} S\tilde{t}_{ij} \bigg/ \sum_{j=1}^{n}\sum_{i=1}^{m} S\tilde{t}_{ij}$.

4) *Step 4: To rank the alternatives, use the LIF-TOPSIS method.*

The fundamental principle of TOPSIS is that the chosen alternative ought to be closest to the positive ideal solution and most far away from the negative optimal solution.

a) Construct the weighted Score matrix:

$$S\tilde{P} = (\tilde{p}_{ij})_{m \times n} = \begin{bmatrix} \omega_1 S\tilde{t}_{11} & \omega_2 S\tilde{t}_{12} & \dots & \omega_n S\tilde{t}_{1n} \\ \omega_1 S\tilde{t}_{21} & \omega_2 S\tilde{t}_{22} & \dots & \omega_n S\tilde{t}_{2n} \\ \dots & \dots & \dots & \dots \\ \omega_1 S\tilde{t}_{m1} & \omega_2 S\tilde{t}_{m2} & \dots & \omega_n S\tilde{t}_{mn} \end{bmatrix},$$

b) Decide the separation Score measures $S\tilde{p}_i^{+}, S\tilde{p}_i^{-}\ for\ i = 1, 2, ..., m,$ based on Euclidean distance from the positive and negative ideal solution:

$$S\tilde{p}_i^{+} = \sqrt{\sum_{j=1}^{n}\left(1 - w_j.S\tilde{t}_{ij}\right)},$$

$$S\tilde{p}_i^{-} = \sqrt{\sum_{j=1}^{n}\left(-1 - w_j.S\tilde{t}_{ij}\right)} \quad for\ i = 1, 2, ..., m.$$

c) Compute the relative closeness coefficient as follows: $\quad C(L_i) = \dfrac{S\tilde{p}_i^{-}}{S\tilde{p}_i^{+} + S\tilde{p}_i^{-}}(i = 1, 2, ..., \text{m})$

d) To rank the alternatives, apply the relative closeness coefficient for all alternatives. The bigger $C(L_i)$, the better the alternative.

VI.   NUMERICAL ILLUSTRATION

Assume there are four industries (alternatives) $\{L_1, L_2, L_3, L_4\}$ to be weighed against certain criteria. Evaluate industries in terms of their technological innovation capability, evaluating 'factors' such as resource ability for digitalization (C_1), organizational innovation (C_2), Innovation Centers (C_3), and Innovative products (C_4). Consider a group of experts whose weights are given as $\lambda = (0.4, 0.32, 0.28)$. The Experts assessment of the four industries are listed in Tables 1, 2, and 3.

Table 1: Decision Matrix I

Industries	Digitalization C_1	Organizational innovation C_2	Innovation Centers C_3	Innovative products C_4
L_1	$\langle l_5, (0.2, 0.7) \rangle$	$\langle l_2, (0.4, 0.6) \rangle$	$\langle l_5, (0.5, 0.5) \rangle$	$\langle l_3, (0.2, 0.6) \rangle$
L_2	$\langle l_4, (0.4, 0.6) \rangle$	$\langle l_5, (0.4, 0.5) \rangle$	$\langle l_3, (0.1, 0.8) \rangle$	$\langle l_4, (0.5, 0.5) \rangle$
L_3	$\langle l_3, (0.2, 0.7) \rangle$	$\langle l_4, (0.2, 0.7) \rangle$	$\langle l_4, (0.3, 0.7) \rangle$	$\langle l_5, (0.2, 0.7) \rangle$
L_4	$\langle l_6, (0.5, 0.4) \rangle$	$\langle l_2, (0.2, 0.8) \rangle$	$\langle l_3, (0.2, 0.6) \rangle$	$\langle l_3, (0.3, 0.6) \rangle$

Table 2: Decision Matrix II

Industries	Digitalization C_1	Organizational innovation C_2	Innovation Centers C_3	Innovative products C_4
L_1	$\langle l_4, (0.1, 0.7) \rangle$	$\langle l_3, (0.2, 0.7) \rangle$	$\langle l_3, (0.2, 0.8) \rangle$	$\langle l_6, (0.4, 0.5) \rangle$
L_2	$\langle l_5, (0.4, 0.5) \rangle$	$\langle l_3, (0.3, 0.6) \rangle$	$\langle l_4, (0.2, 0.6) \rangle$	$\langle l_3, (0.2, 0.7) \rangle$
L_3	$\langle l_4, (0.2, 0.6) \rangle$	$\langle l_4, (0.2, 0.7) \rangle$	$\langle l_2, (0.4, 0.6) \rangle$	$\langle l_3, (0.3, 0.7) \rangle$
L_4	$\langle l_5, (0.3, 0.6) \rangle$	$\langle l_4, (0.4, 0.5) \rangle$	$\langle l_2, (0.3, 0.6) \rangle$	$\langle l_4, (0.2, 0.6) \rangle$

Table 3: Decision Matrix III

Industries	Digitalization C_1	Organizational innovation C_2	Innovation Centers C_3	Innovative products C_4
L_1	$\langle l_5, (0.2, 0.6) \rangle$	$\langle l_3, (0.3, 0.7) \rangle$	$\langle l_4, (0.4, 0.5) \rangle$	$\langle l_4, (0.2, 0.7) \rangle$
L_2	$\langle l_4, (0.3, 0.7) \rangle$	$\langle l_5, (0.3, 0.6) \rangle$	$\langle l_2, (0.1, 0.8) \rangle$	$\langle l_3, (0.4, 0.6) \rangle$
L_3	$\langle l_4, (0.2, 0.7) \rangle$	$\langle l_5, (0.3, 0.6) \rangle$	$\langle l_1, (0.1, 0.8) \rangle$	$\langle l_4, (0.2, 0.7) \rangle$
L_4	$\langle l_3, (0.2, 0.7) \rangle$	$\langle l_3, (0.1, 0.7) \rangle$	$\langle l_4, (0.3, 0.6) \rangle$	$\langle l_5, (0.4, 0.5) \rangle$

1) *Step 1: Normalize the given matrices and make the integrated matrix:*
The following matrices are obtained by following the normalization of the linguistic values for the above mentioned decision matrices.

Table 4: Normalized Decision Matrix I

Industries	Digitalization C_1	Organizational innovation C_2	Innovation Centers C_3	Innovative products C_4
L_1	$\langle l_{0.833}, (0.2, 0.7) \rangle$	$\langle l_{0.333}, (0.4, 0.6) \rangle$	$\langle l_{0.833}, (0.5, 0.5) \rangle$	$\langle l_{0.5}, (0.2, 0.6) \rangle$
L_2	$\langle l_{0.667}, (0.4, 0.6) \rangle$	$\langle l_{0.833}, (0.4, 0.5) \rangle$	$\langle l_{0.5}, (0.1, 0.8) \rangle$	$\langle l_{0.667}, (0.5, 0.5) \rangle$
L_3	$\langle l_{0.5}, (0.2, 0.7) \rangle$	$\langle l_{0.667}, (0.2, 0.7) \rangle$	$\langle l_{0.667}, (0.3, 0.7) \rangle$	$\langle l_{0.833}, (0.2, 0.7) \rangle$
L_4	$\langle l_1, (0.5, 0.4) \rangle$	$\langle l_{0.333}, (0.2, 0.8) \rangle$	$\langle l_{0.5}, (0.2, 0.6) \rangle$	$\langle l_{0.5}, (0.3, 0.6) \rangle$

Table 5: Normalized Decision Matrix II

Industries	Digitalization C_1	Organizational innovation C_2	Innovation Centers C_3	Innovative products C_4
L_1	$\langle l_{0.667},(0.1,0.7)\rangle$	$\langle l_{0.5},(0.2,0.7)\rangle$	$\langle l_{0.5},(0.2,0.8)\rangle$	$\langle l_1,(0.4,0.5)\rangle$
L_2	$\langle l_{0.833},(0.4,0.5)\rangle$	$\langle l_{0.5},(0.3,0.6)\rangle$	$\langle l_{0.667},(0.2,0.6)\rangle$	$\langle l_{0.5},(0.2,0.7)\rangle$
L_3	$\langle l_{0.667},(0.2,0.6)\rangle$	$\langle l_{0.667},(0.2,0.7)\rangle$	$\langle l_{0.333},(0.4,0.6)\rangle$	$\langle l_{0.5},(0.3,0.7)\rangle$
L_4	$\langle l_{0.833},(0.3,0.6)\rangle$	$\langle l_{0.667},(0.4,0.5)\rangle$	$\langle l_{0.333},(0.3,0.6)\rangle$	$\langle l_{0.667},(0.2,0.6)\rangle$

Table 6: Normalized Decision Matrix III

Industries	Digitalization C_1	Organizational innovation C_2	Innovation Centers C_3	Innovative products C_4
L_1	$\langle l_{0.833},(0.2,0.6)\rangle$	$\langle l_{0.5},(0.3,0.7)\rangle$	$\langle l_{0.667},(0.4,0.5)\rangle$	$\langle l_{0.667},(0.2,0.7)\rangle$
L_2	$\langle l_{0.667},(0.3,0.7)\rangle$	$\langle l_{0.833},(0.3,0.6)\rangle$	$\langle l_{0.333},(0.1,0.8)\rangle$	$\langle l_{0.5},(0.4,0.6)\rangle$
L_3	$\langle l_{0.667},(0.2,0.7)\rangle$	$\langle l_{0.833},(0.3,0.6)\rangle$	$\langle l_{0.167},(0.1,0.8)\rangle$	$\langle l_{0.667},(0.2,0.7)\rangle$
L_4	$\langle l_{0.5},(0.2,0.7)\rangle$	$\langle l_{0.5},(0.1,0.7)\rangle$	$\langle l_{0.667},(0.3,0.6)\rangle$	$\langle l_{0.833},(0.4,0.5)\rangle$

For the first value, the calculation is

$$0.4*\langle l_{0.833},(0.2,0.7)\rangle + 0.32*\langle l_{0.667},(0.1,0.7)\rangle + 0.28*\langle l_{0.833},(0.2,0.6)\rangle$$

$$= \langle l_{0.333},(0.915,0.867)\rangle + \langle l_{0.213},(0.967,0.892)\rangle + \langle l_{0.233},(0.939,0.867)\rangle$$

$$= \langle l_{0.78},(0.169,0.670)\rangle$$

Similarly, calculations can be done for all the elements in the matrices, and the collective matrix is given as follows:

$$\tilde{R} = \begin{bmatrix} \langle l_{0.780},(0.169,0.670)\rangle & \langle l_{0.433},(0.313,0.658)\rangle & \langle l_{0.680},(0.388,0.581)\rangle & \langle l_{0.707},(0.270,0.591)\rangle \\ \langle l_{0.720},(0.374,0.591)\rangle & \langle l_{0.727},(0.342,0.558)\rangle & \langle l_{0.507},(0.133,0.730)\rangle & \langle l_{0.567},(0.388,0.586)\rangle \\ \langle l_{0.600},(0.200,0.666)\rangle & \langle l_{0.713},(0.229,0.670)\rangle & \langle l_{0.420},(0.285,0.692)\rangle & \langle l_{0.680},(0.233,0.700)\rangle \\ \langle l_{0.807},(0.365,0.533)\rangle & \langle l_{0.487},(0.246,0.663)\rangle & \langle l_{0.493},(0.262,0.600)\rangle & \langle l_{0.647},(0.300,0.570)\rangle \end{bmatrix}$$

2) *Step 2: Using score function of definition 3.1 (a), the score matrix is obtained as follows:*

Score value of $\langle l_{0.780},(0.169,0.670)\rangle$ is $0.780 \times (0.169 - 0.670) = -0.131$.

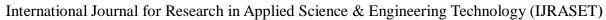Similarly, all Score values are computed and listed below.

$$S\tilde{R} = \begin{bmatrix} -0.391 & -0.150 & -0.131 & -0.227 \\ -0.157 & -0.157 & -0.302 & -0.112 \\ -0.380 & -0.315 & -0.171 & -0.317 \\ -0.135 & -0.203 & -0.167 & -0.174 \end{bmatrix}$$

Calculate the attribute weights using $\omega_j = \sum_{j=1}^{n} S\tilde{t}_{ij} \Big/ \sum_{j=1}^{n}\sum_{i=1}^{m} S\tilde{t}_{ij}$.

$$\omega_1 = \frac{[-0.391 - 0.157 - 0.380 - 0.135]}{\begin{bmatrix} -0.391 - 0.157 - 0.380 - 0.135 - 0.150 - 0.157 - 0.315 - 0.203 \\ -0.131 - 0.302 - 0.171 - 0.167 - 0.227 - 0.112 - 0.317 - 0.174 \end{bmatrix}} = \frac{-0.963}{-3.389} = 0.284$$

$\omega_1 = 0.284$. Similarly, all the remaining weights are computed.

Hence $\omega = (0.284, 0.243, 0.228, 0.245)^T$.

To rank the alternatives, use the TOPSIS method:

a) Make the weighted score matrix:

Here, $\omega_1 \tilde{t}_{11} = 0.284 \times -0.391 = -0.111$.

Similarly, all the elements of the Weighted Score matrix can be computed.

$$\tilde{P} = \begin{bmatrix} -0.111 & -0.036 & -0.030 & -0.056 \\ -0.044 & -0.038 & -0.069 & -0.027 \\ -0.080 & -0.077 & -0.039 & -0.078 \\ -0.038 & -0.049 & -0.038 & -0.043 \end{bmatrix}$$

b) Decide the separation measure,

$$S\tilde{p}_i^+ = \sqrt{\sum_{j=1}^{n} \left(1 - w_j . S\tilde{t}_{ij}\right)^2},$$

$$S\tilde{p}_i^- = \sqrt{\sum_{j=1}^{n} \left(-1 - w_j . S\tilde{t}_{ij}\right)^2} \quad for \ i = 1, 2, ..., m.$$

$$S\tilde{p}_1^+ = \sqrt{\left(1 + 0.111\right)^2 + \left(1 + 0.044\right)^2 + \left(1 + 0.080\right)^2 + \left(1 + 0.038\right)^2}$$

$$= \sqrt{1.234 + 1.091 + 1.165 + 1.078}$$

,

$$S\tilde{p}_1^+ = 2.138$$

$$S\tilde{p}_1^- = \sqrt{\left(-1 + 0.111\right)^2 + \left(-1 + 0.044\right)^2 + \left(-1 + 0.080\right)^2 + \left(-1 + 0.038\right)^2}$$

$$= \sqrt{0.790 + 0.913 + 0.847 + 0.925}$$

$$S\tilde{p}_1^- = 1.864$$

Similarly, all values are calculated and are listed below:

$$S\tilde{p}_2^+ = 2.100, S\tilde{p}_3^+ = 2.088, S\tilde{p}_4^+ = 2.102.$$

$$S\tilde{p}_2^- = 1.900, S\tilde{p}_3^- = 1.913, S\tilde{p}_4^- = 1.899.$$

c) The relative closeness coefficient is calculated as follows:

$$C(L_1) = \frac{S\tilde{p}_1^-}{S\tilde{p}_1^+ + S\tilde{p}_1^-} (i = 1, 2, ..., m)$$

$$C(L_1) = \frac{1.864}{2.138 + 1.864} = 0.466$$

Similarly, closeness coefficient is calculated for all the alternatives as:

$$C(L_2) = 0.475, \ C(L_3) = 0.478, \ C(L_4) = 0.475.$$

Hence, the ranking of the best alternative is $C(L_3) > C(L_2) = C(L_4) > C(L_1)$. Based on the order of ranking, $L_3 > L_2 = L_4 > L_1$, L_3 is observed to be the best alternative.

Table 6.1: Weight Vector obtained from different Score and accuracy functions and the ranking of alternatives for TOPSIS.

S.No:	Proposed Score and accuracy function	Weights using Score and Accuracy function	Ranking of Alternatives
1	$S_i(\tilde{\sigma}_j)$	$\omega_1 = 0.284; \omega_2 = 0.243; \omega_3 = 0.228;$ $\omega_4 = 0.245.$	$L_3 > L_2 = L_4 > L_1.$
2	$S_{ii}(\tilde{\sigma}_j)$	$\omega_1 = 0.295; \omega_2 = 0.238; \omega_3 = 0.218;$ $\omega_4 = 0.249.$	$L_3 > L_2 > L_4 > L_1.$
3	$S_{iii}(\tilde{\sigma}_j)$	$\omega_1 = 0.310; \omega_2 = 0.241; \omega_3 = 0.242;$ $\omega_4 = 0.207.$	$L_4 > L_2 > L_3 > L_1$
4	$S_{iv}(\tilde{\sigma}_j)$	$\omega_1 = 0.273; \omega_2 = 0.244; \omega_3 = 0.222;$ $\omega_4 = 0.261.$	$L_1 > L_4 > L_2 > L_3$
5	$S_v(\tilde{\sigma}_j)$	$\omega_1 = 0.296; \omega_2 = 0.233; \omega_3 = 0.202;$ $\omega_4 = 0.269.$	$L_1 > L_4 > L_2 > L_3$
6	$A_i(\tilde{\sigma}_j)$	$\omega_1 = 0.287; \omega_2 = 0.239; \omega_3 = 0.213;$ $\omega_4 = 0.261.$	$L_1 > L_4 > L_2 > L_3$
7	$A_{ii}(\tilde{\sigma}_j)$	$\omega_1 = 0.291; \omega_2 = 0.237; \omega_3 = 0.211;$ $\omega_4 = 0.261.$	$L_1 > L_4 > L_2 > L_3$
8	$A_{iii}(\tilde{\sigma}_j)$	$\omega_1 = 0.290; \omega_2 = 0.237; \omega_3 = 0.210;$ $\omega_4 = 0.263.$	$L_1 > L_4 > L_2 > L_3$
9	$A_{iv}(\tilde{\sigma}_j)$	$\omega_1 = 0.296; \omega_2 = 0.235; \omega_3 = 0.208;$ $\omega_4 = 0.261.$	$L_1 > L_4 > L_2 > L_3$
10	$A_v(\tilde{\sigma}_j)$	$\omega_1 = 0.298; \omega_2 = 0.233; \omega_3 = 0.202;$ $\omega_4 = 0.267.$	$L_1 > L_4 > L_2 > L_3$

From the above table, it can be observed that L_1 is the best alternative using many of the proposed functions.

VII.   RESULTS

Various score and accuracy functions are proposed in this research work which are in turn used for the data interpretation in this study. Various theorems which proved the properties of the proposed functions are given in detail. The weights required for aggregation in the TOPSIS technique are mostly determined by these suggested Score and accuracy functions. A Decision Support System (DSS) called TOPSIS is used in decision systems by which the attribute-based differences are determined and finally, the best option in the MAGDM involved is obtained.

VIII.   CONCLUSIONS

In this paper, New Score functions for LIFNs are proposed and in the theorems, it is proved that the proposed score functions satisfy the conditions namely Boundedness and Monotonic properties. New Accuracy functions for LIFNs are proposed and in the theorems, it is proved that the proposed Accuracy functions satisfy the conditions namely Boundedness. LIF-TOPSIS algorithm using the proposed Score and Accuracy function is constructed. Every Score function is applied in the proposed LIF-TOPSIS for computing weights and in the final ranking of the alternatives. Numerical illustration is given to prove the consistency of the proposed Score and Accuracy functions applied in the TOPSIS method of solving MAGDM problems.

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