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Observations on Ternary Quadratic Diophantine

Equation $8(x^2 + y^2) - 15xy + 7x + 7y + 49 = 40z^2$

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Abstract: The Ternary Quadratic Diophantine Equation is given by $8(x^2 + y^2) - 15xy + 7x + 7y + 49 = 40z^2$. Number of non-zero distinct integer solutions. Some interesting relations among the solutions and the way of factorization, along with the illusion of non-zero distinct integer solutions to the above equations are obtained.

Keywords: Diophantine equation, Integral solutions, Quadratic equation with three unknowns, Ternary equation.

I. INTRODUCTION

Diophantine equation have provoked the interest of different mathematicians. It is simple to solve Diophantine equations with a degree greater than three by reducing them to equations degree 2 or 3. In [1-3], hypothesis of numbers is talked about. Diophantine quadratic equations are discussed in [5,6, and 12]. In [4,7-11] cubic, biquadratic and higher order equations are considered for its integer solutions

This correspondence deals with one more attracting ternary quadratic equation addressing a non-homogeneous cone for its infinitely non-zero integer points. In like, manner, a couple of fascinating relations among the arrangements are correlated.

II. NOTATIONS

$T_{8,n}$ = Octagonal number of rank 'n'.

$T_{10,n}$ = Decagonal number of rank 'n'.

$T_{18,n}$ = Octadecagonal number of rank 'n'.

$T_{22,n}$ = Icosidigonal number of rank 'n'.

$T_{26,n}$ = Icosihexagonal number of rank 'n'.

$T_{28,n}$ = Icosioctagonal number of rank 'n'

Gno_n = Gnomonic number of rank 'n'.

III. METHOD OF ANALYSIS

The ternary quadratic Diophantine equation for its non-zero integral solution is given by

$$8(x^2 + y^2) - 15xy + 7x + 7y + 49 = 40z^2 \quad (1)$$

The replacement of linear transformations $x = u + v$ and $y = u - v$ (2)

in (1) we get,

$$(u + 7)^2 + 31v^2 = 40z^2 \quad (3)$$

Below are four examples of distinct non-zero integer solutions to (1).

PATTERN: 1

Assume $z = z(a, b) = a^2 + 31b^2$ (4)

wherein (a, b) is a pair of non-zero integers.

$$40 = (3 + i\sqrt{31})(3 - i\sqrt{31}) \quad (5)$$

Using (4) and (5) in (3), by the method of factorization,

$$\left((u + 7) + i\sqrt{31}v \right) \left((u + 7) - i\sqrt{31}v \right) = (3 + i\sqrt{31})(3 - i\sqrt{31}) \left[(a + i\sqrt{31}b)^2 (a - i\sqrt{31}b)^2 \right] \quad (6)$$

When we compare real and imaginary components and put like terms together, we get

$$u = u(a, b) = 3a^2 - 93b^2 - 62ab - 7$$

$$v = v(a, b) = a^2 - 31b^2 + 6ab$$

The corresponding integer solutions of equation (1) are given by using the values of u & v in equation (2).

$$x = x(a, b) = 4a^2 - 124b^2 - 56ab - 7$$

$$y = y(a, b) = 2a^2 - 62b^2 - 68ab - 7$$

$$z = z(a, b) = a^2 + 31b^2$$

OBSERVATIONS:

1. $x(a, a) - y(a, a) + z(a, a) + 4T_{10,a} \equiv 0 \pmod{12}$
2. $y(1,1) - x(1,1) - z(1,1)$ is a perfect square.
3. $x(a,1) - y(a,1) + z(a,1) - T_{8,a} - 7Gno_a \equiv 0 \pmod{24}$
4. $x(a, a) - y(a, a) + 4T_{26,a} \equiv 0 \pmod{44}$
5. $x(a, a) - y(a, a) - z(a, a) + 10T_{18,a} \equiv 0 \pmod{70}$

PATTERN:2

Consider (5), we write 40 as

$$40 = \frac{(19 + i3\sqrt{31})(19 - i3\sqrt{31})}{16} \quad (7)$$

The corresponding integer solutions of equation (1) are given by using the values of u & v in equation (2).

$$x = x(A, B) = 88A^2 - 2728B^2 - 592AB - 7$$

$$y = y(A, B) = 64A^2 - 1984B^2 - 896AB - 7$$

$$z = z(A, B) = 16A^2 + 496B^2$$

OBSERVATIONS:

1. $x(1,1) - y(1,1) + z(1,1)$ is a nasty number
2. $y(A, A) - x(A, A) - 32T_{28,A} \equiv 0 \pmod{384}$

3. $x(A, A) - y(A, A) + z(A, A) - 8T_{26,A} \equiv 0 \pmod{88}$
4. $x(A, A) - y(A, A) - z(A, A) + 116T_{18,A} \equiv 0 \pmod{812}$
5. $x(A,1) - y(A,1) - 2T_{26,A} - 163Gno_A \equiv 0 \pmod{581}$

PATTERN: 3

Consider(5),we write 40 as,

$$40 = \frac{(7 + i9\sqrt{31})(7 - i9\sqrt{31})}{64} \tag{8}$$

The corresponding integer solutions of equation (1) are given by using the values of u & v in equation (2)

$$\begin{aligned} x &= x(A, B) = 128A^2 - 3968B^2 - 4352AB - 7 \\ y &= y(A, B) = -16A^2 + 496B^2 - 4576AB - 7 \\ z &= z(A, B) = 64A^2 + 1984B^2 \end{aligned}$$

OBSERVATIONS:

1. $y(1,1) - x(1,1)$ is a perfect square.
2. $x(A, A) - y(A, A) + z(A, A) + 256T_{18,A} \equiv 0 \pmod{1792}$
3. $x(A,1) - y(A,1) - 12T_{26,A} - 178Gno_A \equiv 0 \pmod{4286}$
4. $x(A, A) - y(A, A) + 1024T_{10,A} \equiv 0 \pmod{3072}$
5. $x(A, A) - y(A, A) - z(A, A) + 512T_{26,A} \equiv 0 \pmod{5632}$

PATTERN: 4

Consider(5),we write 40 as

$$40 = \frac{(63 + i\sqrt{31})(63 - i\sqrt{31})}{100} \tag{9}$$

The corresponding integer solutions of equation (1) are given by using the values of u & v in equation (2).

$$\begin{aligned} x &= x(A, B) = 640A^2 - 19840B^2 + 640AB - 7 \\ y &= y(A, B) = 620A^2 - 19220B^2 - 1880AB - 7 \\ z &= z(A, B) = 100A^2 + 3100B^2 \end{aligned}$$

OBSERVATIONS:

1. $x(A, A) - y(A, A) + z(A, A) - 640T_{18,A} \equiv 0 \pmod{4480}$
2. $x(A, A) - y(A, A) - 160T_{26,A} \equiv 0 \pmod{1760}$
3. $x(A,1) - y(A,1) - z(A,1) + 10T_{18,A} - 1225Gno_A \equiv 0 \pmod{2495}$
4. $x(A, A) - y(A, A) - z(A, A) + 128T_{22,A} \equiv 0 \pmod{1152}$

$$5. \quad x(A,1) - z(A,1) - 54T_{22,A} - 563Gno_A \equiv 0 \pmod{22384}$$

IV. CONCLUSION

Four distinct patterns of non-zero distinct integer solutions to the non-homogeneous cone given by are presented in this paper. To finish up, one might look for different examples of non-zero number unmistakable arrangements and their relating properties for different decisions of ternary quadratic Diophantine conditions.

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