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# Observations on Ternary Quadratic Diophantine

## Equation $12(x^2 + y^2) - 23xy + 2x + 2y + 4 = 56z^2$

C. Saranya<sup>1</sup>, M. Janani<sup>2</sup>

<sup>1</sup>Assistant Professor, <sup>2</sup>PG Student, PG and Research Department of Mathematics, Cauvery College for Women (Autonomous) (Affiliated to Bharathidasan University), Tiruchirappalli, Tamil Nadu, India

**Abstract:** The Ternary Quadratic Diophantine Equation  $12(x^2 + y^2) - 23xy + 2x + 2y + 4 = 56z^2$  is analyzed for its infinite number of non-zero integral solutions. Four interesting patterns satisfying the cone are identified. There are a few interesting connections between the solutions and some unique number patterns.

**Keywords:** Diophantine equation, Integral solutions, Quadratic equation with three unknowns, Ternary equation.

### I. INTRODUCTION

Diophantine equations have stimulated the interest of various mathematicians. Diophantine equations with higher degree greater than three can be reduced in to equations of degree 2 or 3 and it can be easily solved. In [1-3], theory of numbers is discussed. In [5,6 & 12], quadratic Diophantine equations are discussed. In [4, 7-11], cubic, biquadratic and higher order equations are considered for its integral solutions.

This communication concerns with yet another interesting ternary quadratic equation  $12(x^2 + y^2) - 23xy + 2x + 2y + 4 = 56z^2$  representing a non-homogeneous cone is determined for its infinitely many non-zero integral points. Likewise, a few interesting relations among the solutions are analyzed.

### II. NOTATIONS

- $T_{6,n}$  = Hexagonal number of rank 'n'.
- $T_{8,n}$  = Octogonal number of rank 'n'.
- $T_{10,n}$  = Decagonal number of rank 'n'.
- $T_{14,n}$  = Tetradecagonal number of rank 'n'.
- $T_{16,n}$  = Hexadecagonal number of rank 'n'.
- $T_{18,n}$  = Octadecagonal number of rank 'n'.
- $T_{22,n}$  = Icosidigonal number of rank 'n'.
- $T_{24,n}$  = Icositetragonal number of rank 'n'.
- $T_{26,n}$  = Icosihexagonal number of rank 'n'.
- $Gno_n$  = Gnomonic number rank 'n'.

### III. METHOD OF ANALYSIS

The ternary quadratic diophantine equation to be tackled for its non-zero integral solution is

$$12(x^2 + y^2) - 23xy + 2x + 2y + 4 = 56z^2 \tag{1}$$

using the linear transformations

$$x = u + v \quad \text{and} \quad y = u - v \tag{2}$$

in (1) leads to,  $(u + 2)^2 + 47v^2 = 56z^2 \tag{3}$

We illustrate below four different patterns of non-zero distinct integer solutions to (1)

**A. Pattern : 1**

Assume  $z = z(a, b) = a^2 + 47b^2$  (4)

where a and b are non-zero integers.

and write  $56 = (3 + i\sqrt{47})(3 - i\sqrt{47})$  (5)

Using (4) and (5) in (3), and using factorization method,

$$\left( (u + 2) + i\sqrt{47}v \right) \left( (u + 2) - i\sqrt{47}v \right) = (3 + i\sqrt{47})(3 - i\sqrt{47}) \left[ (a + i\sqrt{47}b)^2 (a - i\sqrt{47}b)^2 \right] \tag{6}$$

Equating the like terms and comparing real and imaginary parts, we get

$$u = u(a, b) = 3a^2 - 141b^2 - 94ab - 2$$

$$v = v(a, b) = a^2 - 47b^2 + 60ab$$

Substituting the above values of u & v in equation (2), the corresponding integer solutions of (1) are given by

$$x = x(a, b) = 4a^2 - 188b^2 - 88ab - 2$$

$$y = y(a, b) = 2a^2 - 94b^2 - 100ab - 2$$

$$z = z(a, b) = a^2 + 47b^2$$

**Observations**

1.  $x(a, a) - y(a, a) + z(a, a) + 4T_{18,a} \equiv 0 \pmod{28}$
2.  $2y(1,1) - 2x(1,1) - 2z(1,1)$  is a perfect square.
3.  $y(a, a) + z(a, a) - x(a, a) - 32T_{10,a} \equiv 0 \pmod{96}$
4.  $y(a, a) - x(a, a) - 8T_{22,a} \equiv 0 \pmod{72}$
5.  $x(a,1) - y(a,1) + z(a,1) - T_{8,a} - 7Gno_a \equiv 0 \pmod{40}$
6.  $y(1,1) - x(1,1)$  is a duck number

**B. Pattern : 2**

We substitute (5) with 56 as

$$56 = \frac{(29 + i5\sqrt{47})(29 - i5\sqrt{47})}{36} \tag{7}$$

Following the methodology introduced over, the relating non-zero particular whole number answers for (1) are Gotten as

$$x = x(A, B) = 204A^2 - 9588B^2 - 2472AB - 2$$

$$y = y(A, B) = 144A^2 - 6768B^2 - 3168AB - 2$$

$$z = z(A, B) = 36A^2 + 1692B^2$$

**Observations**

1.  $x(A, A) - y(A, A) + z(A, A) + 48T_{16,A} \equiv 0 \pmod{288}$
2.  $x(A,1) - y(A,1) - 6T_{22,A} - 375Gno_A \equiv 0 \pmod{2445}$
3.  $x(A, A) - y(A, A) + 172T_{26,A} \equiv 0 \pmod{1892}$
4.  $y(A,1) - x(A,1) - z(A,1) + 16T_{14,A} + 388Gno_A \equiv 0 \pmod{740}$
5. Each of the following expressions represents a Duck number

- i.  $x(1,1) - 2y(1,1)$
- ii.  $x(1,1) - 2y(1,1) - z(1,1)$
- iii.  $-y(1,1) - z(1,1)$

C. Pattern : 3

We substitute (5) with 56 as

$$56 = \frac{(67 + i\sqrt{47})(67 - i\sqrt{47})}{81} \tag{8}$$

Following the methodology introduced over, the relating non-zero particular whole number answers for (1) are gotten as

$$x = x(A, B) = 612A^2 - 28764B^2 - 360AB - 2$$

$$y = y(A, B) = 594A^2 - 27918B^2 - 2052AB - 2$$

$$z = z(A, B) = 81A^2 + 38078B^2$$

Observations

- 1.  $x(A, A) - y(A, A) + z(A, A) - 132T_{26,A} - 726Gno_A \equiv 0 \pmod{38885}$
- 2.  $y(A, A) - x(A, A) + 132T_{26,A} \equiv 0 \pmod{1452}$
- 3.  $x(A, 1) - z(A, 1) - 177T_{8,A} - 357Gno_A \equiv 0 \pmod{66487}$
- 4.  $4x(1,1) - 4y(1,1)$  is a palindromic number
- 5.  $z(1,1) - x(1,1) - y(1,1)$  is a duck number

D. Pattern : 4

We substitute (5) with 56 as

$$56 = \frac{(83 + i5\sqrt{47})(83 - i5\sqrt{47})}{144} \tag{9}$$

Following the methodology introduced over, the relating non-zero particular whole number answers for (1) are gotten as

$$x = x(A, B) = 1056A^2 - 49632B^2 - 3648AB - 2$$

$$y = y(A, B) = 936A^2 - 43992B^2 - 7632AB - 2$$

$$z = z(A, B) = 144A^2 + 6768B^2$$

Observations

- 1.  $x(A, 1) - y(A, 1) - z(A, 1) + 2T_{26,A} - 1981Gno_A \equiv 0 \pmod{10427}$
- 2.  $x(A, 1) - y(A, 1) - 10T_{24,A} - 2047Gno_A \equiv 0 \pmod{3593}$
- 3.  $y(1,1) - x(1,1) + z(1,1)$  is a palindromic number
- 4.  $y(1,1) - x(1,1)$  is a nasty number.
- 5.  $y(1,1) - z(1,1) - 2x(1,1)$  is a duck number.

IV. CONCLUSION

This paper discusses four distinct patterns of non-zero distinct integer solutions to the non-homogeneous cone given by  $12(x^2 + y^2) - 23xy + 2x + 2y - 4 = 56z^2$ . To conclude, one may search for other patterns of solutions and their corresponding properties.



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