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On Integer Solutions of the Ternary Quadratic Equation $3a^2+3r^2-2ar=332n^2$

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Abstract: ***Analysis is conducted on the non-trivial different integral solution to the quadratic equation*** $3a^2+3r^2-2ar=332n^2$. ***We derive distinct integral solutions in four different patterns. There are a few intriguing connections between the solutions and unique polygonal numbers that are presented.***
Keywords: Quadratic equation, integral solutions, polygonal numbers, special numbers, square number.

I. INTRODUCTION

Number theory is a vast and fascinating field of mathematics concerned with the properties of numbers in general and integers in particular as well as the wider classes of problems that arise from their study. Number theory has fascinated and inspired both amateurs and mathematicians for over two millennia. A sound and fundamental body of knowledge, it has been developed by the untiring pursuits of mathematicians all over the world. The study of Number theory is very important because all other branches depend upon this branch for their final results. The older term for number theory is "arithmetic". During the seventeenth century. The term "Number theory" was coined by the French mathematician Pierre Fermat who is consider as the "Father of modern number theory". The first scientific approach to the study of integers, that is the true origin of the theory of numbers, is generally attribution to the Greeks. Around 600BC Pythagoras and his disciples made rather thorough studies of this integer. A Greek mathematician, Diaphantus of Alexandria was able to solve equations with two or three unknowns. These equations are called Diophantine equation the study of these is known as "Diophantine analysis". The basic problem is representation of an integers n by the quadratic form with the integral values x and y.

A linear Diophantine equation is an equation between two sums of monomial of degree zero (or) one. In 628 AD, Brahmagupta an Indian mathematician gave the first explicit solution of the quadratic equation. The word quadratic is derived from the Latin word quadrates for square. The quadratic equation is a second-order polynomial equation in a single variable x.

There are several different ternary quadratic equations. To comprehend something in more detail is [1-4] visible. For the non-trivial integral answers to the ternary quadratic equation [5-7] has been researched. For numerous ternary quadratic equation [8-10] has been cited. In this article, we investigate another intriguing ternary quadratic equation $3a^2+3r^2-2ar=332n^2$ and obtain several non-trivial integral patterns. A few intriguing connections between the solutions and unique polygons, rhombic, centered and Gnomonic number are displayed.

A. Notations

$T_{m,n}$ = Polygonal number of rank n with size m

RD_n = Rhombic dodecagonal number of rank n

P_n^5 = Pentagonal Pyramidal number of rank n

TO_n = Truncated octahedral number of rank n

P_n^4 = Square Pyramidal number of rank n

CC_n = Centered cube number of rank n

SO_n = Stella octangula number of rank n

Gno_n = Gnomonic number of rank n

II. METHOD OF ANALYSIS

In order to find a non-zero distinct integral solution to the ternary quadratic equation

$$3a^2 + 3r^2 - 2ar = 332n^2 \quad (1)$$

On substitution of the linear transformations,

$$a = f + g \text{ , } r = f - g \quad (2)$$

in (1) leads to

$$f^2 + 2g^2 = 83n^2 \quad (3)$$

The following list offers four patterns for solution (3). The appropriate values of a and r are derived after the vales of f and g are known by using (2).

A. Pattern: I

In (3), $f^2 + 2g^2 = 83n^2$

Assume that

$$n = h^2 + 2s^2, h, s \neq 0 \quad (4)$$

Write $83 = (9 + i\sqrt{2})(9 - i\sqrt{2}) \quad (5)$

Applying the factorization method and substituting (4) and (5) in (3),

$$(f + i\sqrt{2}g)(f - i\sqrt{2}g) = (9 + i\sqrt{2})(9 - i\sqrt{2})(h + i\sqrt{2}s)^2(h - i\sqrt{2}s)^2 \quad (6)$$

The values of f and g are determined by equating the real and imaginary parts

$$f = f(h,s) = 9h^2 - 18s^2 - 4hs \quad (7)$$

$$g = g(h,s) = h^2 - 2s^2 + 18hs \quad (8)$$

Substituting the above values of f and g in equation (2), we get

$$a(h,s) = 10h^2 + 14hs - 20s^2$$

$$r(h,s) = 8h^2 - 22hs - 16s^2$$

$$n(h,s) = h^2 + 2s^2$$

Properties

1. $8a(h,h) - 5r(h,h) + 27n(h,h)$ *is a perfect square.*
2. $ha(h,h+1) - RD_h + 4T_{12,h} + 20Gno_h \equiv 0 \pmod{19}$.
3. $a(h,1) + r(h,1) - 2n(h,1) - 8T_{6,h} \equiv 0 \pmod{40}$
4. $hn(h,1) + a(h,1) - 2P_h^5 - 3T_{8,h} - 11Gno_h \equiv 0 \pmod{9}$
5. $18n(1,1)$ *is a nasty number*

B. Pattern: II

In (3), $f^2 + 2g^2 = 83n^2$

Take the linear transformations

$$\begin{aligned} n &= A + 2T \\ g &= A + 83T \end{aligned} \tag{9}$$

$$\begin{aligned} n &= A - 2T \\ g &= A - 83T \end{aligned} \tag{10}$$

Substituting (9) or (10) in (3), we get

$$f^2 = 81(A^2 - 166T^2) \tag{11}$$

Write $f = 9F$ (12)

Substituting (12) in (11), we get

$$F^2 = A^2 - 166T^2$$

$$A^2 = F^2 + 166T^2 \tag{13}$$

This is the standard form of Pell equation $x^2 = Dy^2 + z^2$

The solutions to (13) that correspond to this are

$$\begin{aligned} T &= 2hs \\ F &= 166h^2 - s^2 \\ A &= 166h^2 + s^2 \end{aligned} \tag{14}$$

Substituting (14) in (9) and (12), we get

$$\begin{aligned} n &= 166h^2 + 4hs + s^2 \\ f &= 1494h^2 - 9s^2 \\ g &= 166h^2 + 166hs + s^2 \end{aligned} \tag{15}$$

Substituting (15) in (2), we get

$$\begin{aligned} a(h,s) &= 1660h^2 + 166hs - 8s^2 \\ r(h,s) &= 1328h^2 - 166hs - 10s^2 \\ n(h,s) &= 166h^2 + 4hs + s^2 \end{aligned} \tag{16}$$

Properties:

1. $a(h,h) + r(h,h) + n(h,h) - 5T_{4,h}$ *is a perfect square.*
2. $n(h,1) + hr(h,1) - 83TO_h - 913T_{8,h} + 8Gno_h \equiv 0(\text{mod}\,413)$
3. $hr(h,h) + hn(h,h) + 405hT_{4,h}$ *is a perfect cube.*
4. $a(1,1)$ *is palindrome number.*
5. $r(h,1) - n(h,1) - 166T_{16,h} - 413Gno_h \equiv 0(\text{mod}\,402)$.

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C. Pattern: III

Consider equation (3) as

$$f^2 - 81n^2 = 2n^2 - 2g^2 \quad (17)$$

Factorizing (17) we have,

$$(f+9n)(f-9n) = 2(n-g)(n+g)$$

$$\frac{f+9n}{2(n+g)} = \frac{n-g}{f-9n} = \frac{H}{S}, S \neq 0 \quad (18)$$

Expressing this as a system of simultaneous equation

$$Sf - 2Hg + n(9S - 2H) = 0 \quad (19)$$

$$Hf + Sg - n(9H + S) = 0 \quad (20)$$

Using the cross multiplication method, we get

$$n = n(H,S) = 2H^2 + S^2 \quad (21)$$

$$f = f(H,S) = 18H^2 + 4HS - 9S^2$$

$$g = g(H,S) = -2H^2 + 18HS + S^2 \quad (22)$$

Substituting (22) in (2), we get

$$a(H,S) = 16H^2 + 22HS - 8S^2$$

$$r(H,S) = 20H^2 - 14HS - 10S^2 \quad (23)$$

$$n(H,S) = 2H^2 + S^2$$

Properties

1. $a(H,H) - r(H,H) + n(H,H) + 12T_{4,H}$ *is a perfect square*
2. $a(H,1) + r(H,1) - 4T_{20,H} - 20Gno_H \equiv 0(\mathrm{mod}\, 2)$
3. $H(n(H+1,H)) + a(H,1) - 6P_H^4 - 2T_{19,H} + 6T_{17,H} - 18T_{7,H} + 3T_{22,H} - 30T_{4,H} \equiv 0(\mathrm{mod}\, 8)$
4. $a(1,1)$ *is woodall number.*
5. $2(a(1,1)) - r(1,1)$ *is a perfect cube.*

D. Pattern: 4

In addition to (18), (3) can be expressed as

$$\frac{f-9n}{2(n-g)} = \frac{n+g}{f+9n} = \frac{H}{S}, S \neq 0 \quad (24)$$

Expressing this as a system of simultaneous equation

$$Sf + 2Hg - n(9S + 2M) = 0 \quad (25)$$

$$Hf - Sg + n(9H - S) = 0 \quad (26)$$

Using the cross multiplication method, we get

$$n = n(H,S) = -(2H^2 + S^2) \quad (27)$$

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$$f = f(H,S) = 18H^2 - 4HS - 9S^2$$
$$g = g(H,S) = -2H^2 - 18HS + S^2 \qquad (28)$$

Substituting (28) in (2), we get

$$a(H,S) = 16H^2 - 22HS - 8S^2$$
$$r(H,S) = 20H^2 + 14HS - 10S \qquad (29)$$
$$n(H,S) = -(2H^2 + S^2)$$

Properties

$1. a(H,H) + r(H,H) + 2n(H,H)\, is\, a\, perfect cube$

$2. -72n(1,1)\, is\, a\, perfect cube$

$3. 4r(1,1) + 12n(1,1)\, is\, a\, duck number$

$4. H(r(H,1) - a(H,1) + n(H,1)) - CC_H - 3T_{28,H} - 15Gno_H \equiv 0 (\mathrm{mod}\, 16)$

$5. a(H+1,H) - SO_H + 4T_{7,H} = 0$

III. CONCLUSION

The ternary quadratic equation $3a^2 + 3r^2 - 2ar = 332n^2$ has four different patterns of non- zero distinct integral solutions, which we described in this paper. For other quadratic equation , one can look for other patterns of non-zero integer unique solutions and their accompanying features.

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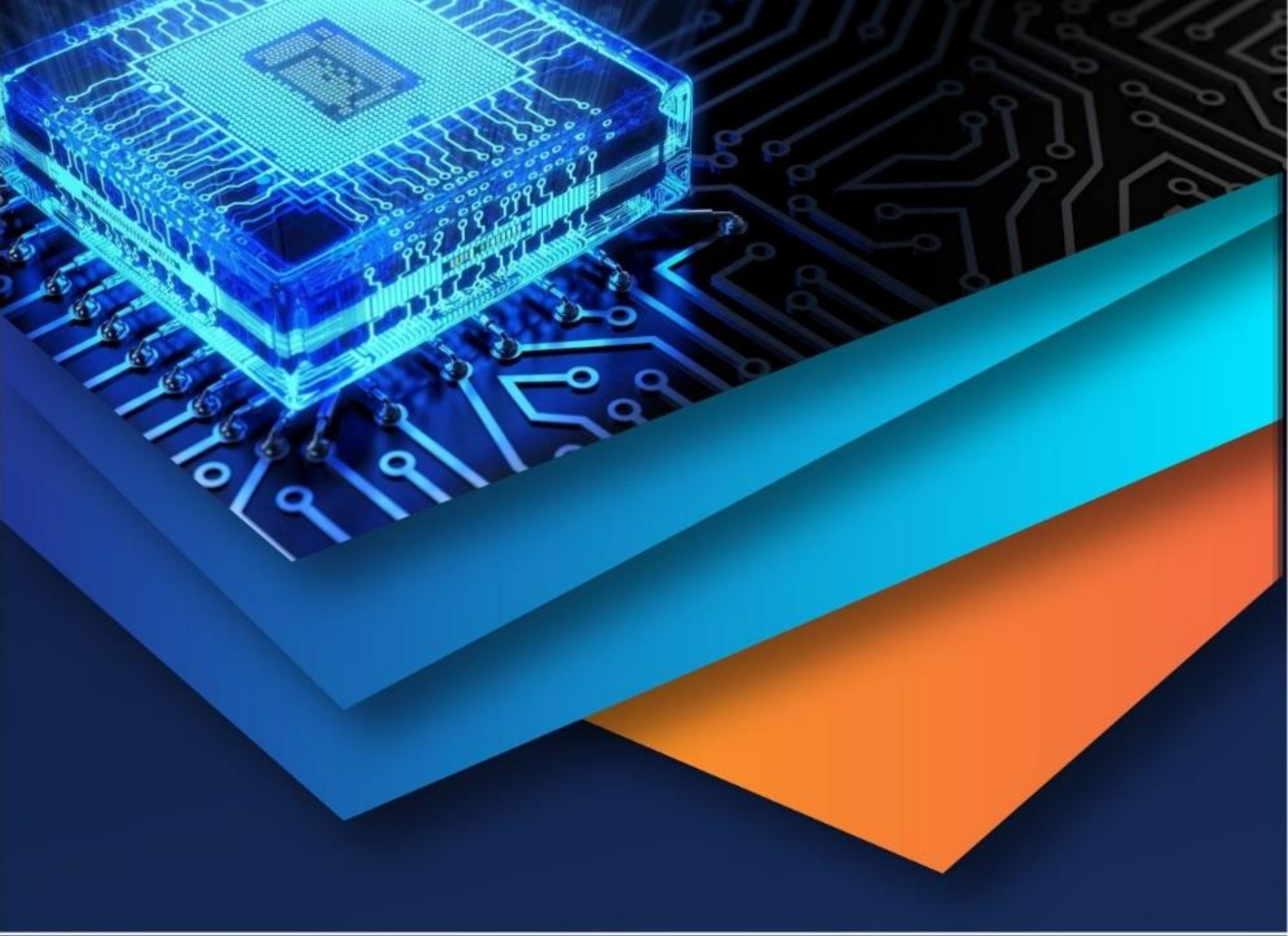

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