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On Some Theoretic Aspects of Fuzzy Subsets

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***Abstract:** This article proposes some important theoretic aspects of L-fuzzy power set of a finite set. Here, we take a non-empty finite set E and an ordered subset M of the closed interval $[0,1]$, then the set of mappings from E to M denoted by F is defined as L-fuzzy power set. Considering the disjunctive union '$(\oplus)$' operations between fuzzy subsets of F, the structure $(F,\oplus)$ forms a groupoid, which is defined as special fuzzy groupoid. Later, we try to introduce the product of special fuzzy groupoids and their properties.*

***Keywords:** L-fuzzy power set, disjunctive union, special fuzzy groupoid, product of special fuzzy groupoids*

I. INTRODUCTION

The concept of fuzzy sets was introduced by Zadeh[1] is applied to many mathematical branches. This concept was adapted by Goguen[2] and Sarchez[3] to define and study fuzzy relations. L-fuzzy sets are the generalization of fuzzy sets was first considered in 1967 by Joseph Goguen. Since then studies on L-fuzzy sets have been carried out by many researchers. Rosenfeld[4] inspired the fuzzification of algebraic structures and introduced the notion of fuzzy subgroups.

Algebraic structures play an important role in mathematics with wide ranging applications in many disciplines such as computer science, information technology, coding theory and so on. There are many researches which had given emphasis on the algebraic structures of the fuzzy sets[5,6,7,8].

Again, the concept of an L-set creates applications in many disciplines like mathematics, computer science etc. In real processes there are often situations when the object is a set of "Washed Out" boundaries in the sense that a given element may belong to a given element may belong to a given set to a greater or lower degree.

This provides sufficient motivation to researchers to review various concepts and results from $L-$ valued fuzzy sets and formations of algebra from them. In this article we represent L-fuzzy power set as special fuzzy groupoid with the help of disjunctive union operation of fuzzy subsets. Throughout the article, different properties of the special fuzzy groupoid are discussed.

II. PRELIMINARIES

This section lists some basic definitions and results which will be used in this article:

1) *Fuzzy Subsets*: Let E be a space of objects and x be a generic element of E and A be a classical set. A fuzzy subset, A of E is characterized by a membership function $\mu_A : E \to [0,1] = I$ such that the number $\mu_A(x)$ in the unit interval I is interpreted as the degree of membership of element x to the fuzzy subset A [9,10]. Fuzzy logic is used in target tracking, pattern recognition, robotics, power systems, controller design, chemical engineering, biomedical engineering, vehicular technology, economy management and decision making, aerospace applications, communications and networking, electronic engineering, and civil engineering. In many chemical engineering systems, the classification of product quality characteristics is performed by human experts, due to the absence of measuring devices. The development of mathematical models for such systems is a rather difficult task, since no equations based on first principles can be written. Chemical engineering has employed fuzzy logic in the detection of chemical agents as well as gas recognition [12.]
2) *Groupoid*: A non-empty set of elements G is said to form a groupoid if in G is defined a binary operation called the product denoted by $*$ such that $a*b \in G$ for all $a,b \in G$. It is important to mention have that the binary operation $*$ defined on the set G need not be associative [11].

III. THE $L-$ FUZZY POWER SET AND SPECIAL FUZZY GROUPOID

The set of all fuzzy subsets of E is called the fuzzy power set of EA denoted by F. Since there is infinite numbers of values in $[0,1]$, the fuzzy power set also have infinite numbers of fuzzy subsets or elements. That is why, in this article we take a non-empty finite set E and an ordered subset M of $[0,1]$ such that $M=\{0,h,2h,...,(m-1)h=1\}$, where m is any positive integer greater than 1. Then the set of all fuzzy subsets obtained from the mappings from E to M is called the L**-fuzzy power set** denoted by 'F'. The total numbers of elements or fuzzy subsets in $F=|M|^{|E|}=m^n$. For better identification we denote the fuzzy subsets of F as $0,1,2\ldots m^{n-1}$ as follows:

$F=[\{(x_0,0),(x_1,0),...,(x_{n-2},0),(x_{n-1},0)\}=0,$

$\{(x_0,0),(x_1,0),...,(x_{n-2},0),(x_{n-1},h)\}=1,$

$\{(x_0,0),(x_1,0),...,(x_{n-2},0),(x_{n-1},2h)\}=2,$

....,

$\{(x_0,0),(x_1,0),...,(x_{n-2},0),(x_{n-1},(m-1)h)\}=m-1,$

$\{(x_0,0),(x_1,0),...,(x_{n-2},h),(x_{n-1},0)\}=m,$

....,

$\{(x_0,(m-1)h),(x_1,(m-1)h),...,(x_{n-2},(m-1)h),(x_{n-1},0)\}=m^n-2$

$\{(x_0,(m-1)h),(x_1,(m-1)h),...,(x_{n-2},(m-1)h),(x_{n-1},(m-1)h)\}=m^n-1=G]$

Hence the L-fuzzy power set, $F=\{0,1,2,...m^n-1\}$, where the element '0' is the empty fuzzy subset which is also the infimum and m^n-1 or G is the universal fuzzy subset which is also the supremum in F.

A. Disjunctive Union Operation of Fuzzy Subsets

The disjunctive union of two fuzzy subsets A and B of F denoted by '$\oplus$' and defined as:

$$A\oplus B=(A\cap B^c)\cup(A^c\cap B)$$

1) Proposition 1: The L-fuzzy power set is closed under the disjoint union operation.

Proof Let, $F=\{0,1,2,...m^n-1\}$ be the L-fuzzy power set. Let, $a=\{(x_i,\mu_a(x_i))\}$ and $b=\{(x_i,\mu_b(x_i))\}$ be any two elements of F; where $i=0,1,...n-1$.

$a\oplus b=(a\cap b')\cup(a'\cap b)$

$\Rightarrow \mu_{a\oplus b}=\max\{\min[\mu_a(x_i),1-\mu_b(x_i)],\min[1-\mu_a(x_i),\mu_b(x_i)]\}$

since, $\mu_a(x_i)\in M,\mu_b(x_i)\in M$

$\Rightarrow \max\{\min[\mu_a(x_i),1-\mu_b(x_i)],\min[1-\mu_a(x_i),\mu_b(x_i)]\}\in M$

Hence, F is closed under $\oplus$.

2) Proposition 2: The special fuzzy groupoid is commutative with respect to the disjunctive union $\oplus$ operation.

Proof: Let, $a=\{(x_i,\mu_a(x_i))\}$ and $b=\{(x_i,\mu_b(x_i))\}$ for $i=0,1,...,n-1$ be any two elements of F.

Then,

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$a \oplus b = (a \cap b') \cup (a' \cap b)$
$= \max\{\min[\mu_a(x_i), 1-\mu_b(x_i)], \min[1-\mu_a(x_i), \mu_b(x_i)]\}$
$= \max\{\min[1-\mu_a(x_i), \mu_b(x_i)], \min[\mu_a(x_i), 1-\mu_b(x_i)]\}$
$= \max\{\min[\mu_b(x_i),]1-\mu_a(x_i), \min[1-\mu_b(x_i), \mu_a(x_i)]\}$
$= b \oplus a$

Since, L-fuzzy power set is closed under $\oplus$ which is proved in proposition 1. Hence F with the operation $\oplus$ is a groupoid and it is defined as the special fuzzy groupoid.

IV. PRODUCT OF SPECIAL FUZZY GROUPOIDS

The product P of two special fuzzy groupoids A and B is defined by the Cartesian product of the fuzzy subsets of A and B. The product P of A and B consists of the pair (a,b); such that $'a'$ is an element of A and $'b'$ is an element of B. Therefore, if the special fuzzy groupoid A consists of n-fuzzy subsets and B consists of m-fuzzy subsets then their product P will consists of $n \times m$ numbers of such pairs of fuzzy subsets.

The meet (fuzzy intersection) and join (fuzzy union) of two pairs in P is formed as:

$(a,b) \wedge (c,d) = (a \wedge c, b \wedge d)$, and

$(a,b) \vee (c,d) = (a \vee c, b \vee d)$.

Here, $a \wedge c$ and $a \vee c$ are the fuzzy intersection and fuzzy union in A, while $b \wedge d$ and $b \vee d$ are the fuzzy intersection and fuzzy union in B.

The complement of a pair (a,b) in P is formed as:

$(a,b)' = (a', b')$;

where a' and b' are the complements of a and b in the special fuzzy groupoid A and B respectively. Under the operation $\oplus$, the product P also forms a special fuzzy groupoid The fuzzy subsets of the product P of special fuzzy groupoids A and B satisfies some properties. i.e if $(a_1,b_1),(a_2,b_2)$ and (a_3,b_3) be three element (in this case three pairs) of P then:

1) *The laws of forming Complement*
 a) $(\phi^A, \phi^B)' = (G^A, G^B)$ b) $(G^A, G^B)' = (\phi^A, \phi^B)$
2) *Commutative Laws*
 a) $(a_1,b_1) \wedge (a_2,b_2) = (a_2,b_2) \wedge (a_1,b_1)$ b) $(a_1,b_1) \vee (a_2,b_2) = (a_2,b_2) \vee (a_1,b_1)$
3) *Distributive Laws*
 a) $(a_1,b_1) \wedge \{(a_2,b_2) \vee (a_3,b_3)\} = \{(a_1,b_1) \wedge (a_2,b_2)\} \vee \{(a_2,b_2) \wedge (a_3,b_3)\}$ b) $(a_1,b_1) \vee \{(a_2,b_2) \wedge (a_3,b_3)\} = \{(a_1,b_1) \vee (a_2,b_2)\} \wedge \{(a_2,b_2) \vee (a_3,b_3)\}$
4) *Identity Laws*
 a) $(a_1,b_1) \vee (\phi^A, \phi^B) = (a_1,b_1)$ b) $(a_1,b_1) \wedge (G^A, G^B) = (a_1,b_1)$
5) *Complement laws:*
 a) $(a_1,b_1) \vee (a_1,b_1)' = (G^A, G^B)$ b) $(a_1,b_1) \wedge (a_1,b_1)' = (\phi^A, \phi^B)$

6) *Involution Laws*

$$\left((a_1,b_1)'\right)' = (a_1,b_1)$$

For instance, here is the proof that the commutative laws and distributive laws are shown below:

Proof of commutative laws: Commutative law for join holds by the fuzzy subsets in P:

$$(a_1,b_1)\vee(a_2,b_2) = \{(a_1\vee a_2),(b_1\vee b_2)\} \text{ [by the definition of join in } P\text{]}$$
$$= (a_2\vee a_1),(b_2\vee b_1) \text{ [} \because \text{ commutative law for join is valid in } A \text{ and } B\text{]}$$
$$= (a_2,b_2)\vee(a_1,b_1)$$

Similarly, the commutative law for meet hold in P:

$$(a_1,b_1)\vee(a_2,b_2) = \{(a_1\vee a_2),(b_1\vee b_2)\} \text{ [by definition of join in } P\text{]}$$
$$= (a_2\vee a_1),(b_2\vee b_1) \text{ [} \because \text{ commutative law for meet is valid in } A \text{ and } B\text{]}$$
$$= (a_2,b_2)\vee(a_1,b_1)$$

Proof of Distributive law:

$$\begin{aligned}(a_1,b_2)\wedge\{(a_2,b_2)\vee(a_3,b_3)\} &= (a_1,b_1)\wedge\{(a_2\vee a_3),(b_2\vee b_3)\}\\ &= \{a_1\wedge(a_2\vee a_3),b_1\wedge(b_2\vee b_3)\}\\ &= \{(a_1\wedge a_2)\vee(a_1\wedge a_3),(b_1\wedge b_2)\vee(b_1\wedge b_3)\}\\ &= \{(a_1\wedge a_2,b_1\wedge b_2)\vee(a_1\wedge a_3,b_1\wedge b_3)\}\\ &= \{(a_1,b_1)\wedge(a_2,b_2)\}\vee\{(a_1,b_1)\wedge(a_3,b_3)\}\end{aligned}$$

Similarly, the distributive law for join can also be proved.

Proposition 3: The product P of two special fuzzy groupoids is closed under the disjoint union operation.
Theorem 4.1:
The product of two special fuzzy groupoids is also a special fuzzy groupoid.
Proof: The product P with operation disjunctive union is also a special fuzzy groupoid since it is closed.
Proposition 4: The product P of two special fuzzy groupoids is commutative with respect to the disjunctive union $\oplus$ operation.

Example 1: Let, $E_1 = \{x_0, x_1, x_2\}$ be a finite set and $M_1 = \{0, h, 2h, 3h = 1\}$ and $M_2 = \{0, h, 2h = 1\}$ are the sets of membership values, Then the mappings from E into M_1 are the fuzzy subsets and we can obtain 4^3=64 elements or fuzzy subsets. Similarly, the mappings from E into M_2 are the fuzzy subsets and we can obtain 3^3=27 elements or fuzzy subsets as shown below:

$A = [0 = \{(x_0,0),(x_1,0),(x_2,0)\}$,
$1 = \{(x_0,0),(x_1,0),(x_2,h)\}$,
$2 = \{(x_0,0),(x_1,0),(x_2,2h)\}$,
…………………………………., and
62=$\{(x_0,3h),(x_1,3h),(x_2,2h)\}$,
63=$\{(x_0,3h),(x_1,3h),(x_2,3h)\}]$

$A = [0 = \{(x_0, 0), (x_1, 0), (x_2, 0)\},$

$1 = \{(x_0, 0), (x_1, 0), (x_2, h)\},$

$2 = \{(x_0, 0), (x_1, 0), (x_2, 2h)\},$

……………………………….,

$25 = \{(x_0, 2h), (x_1, 2h), (x_2, h)\},$

$26 = \{(x_0, 2h), (x_1, 2h), (x_2, 2h)\}]$

So, A and B are two special fuzzy groupoids with respect to the fuzzy disjunctive operation. Hence, there product P is also a special fuzzy groupoid that will contain $64 \times 27 = 1728$ elements as follows:

$P =$

$[(0,0), (0,1) \ldots\ldots\ldots (0,26)$

$(1,0), (1,1) \ldots\ldots\ldots (1,26)$

…………………………….,

$(63,26), (63,26) \ldots\ldots\ldots (63,26)]$

It also a special fuzzy groupoid with the operation $\oplus$.

V. CONCLUSIONS

The proposed work presented some important properties of L -fuzzy power set. Considering the disjunctive union '$\oplus$' operations between two fuzzy subsets of the L -fuzzy power set, it forms a groupoid, which is defined as special fuzzy groupoid. Throughout this article different properties of the L -fuzzy power set and special fuzzy groupoid are studied. We hope the results established in this article would find some applications and enrich the theory of fuzzy algebra.

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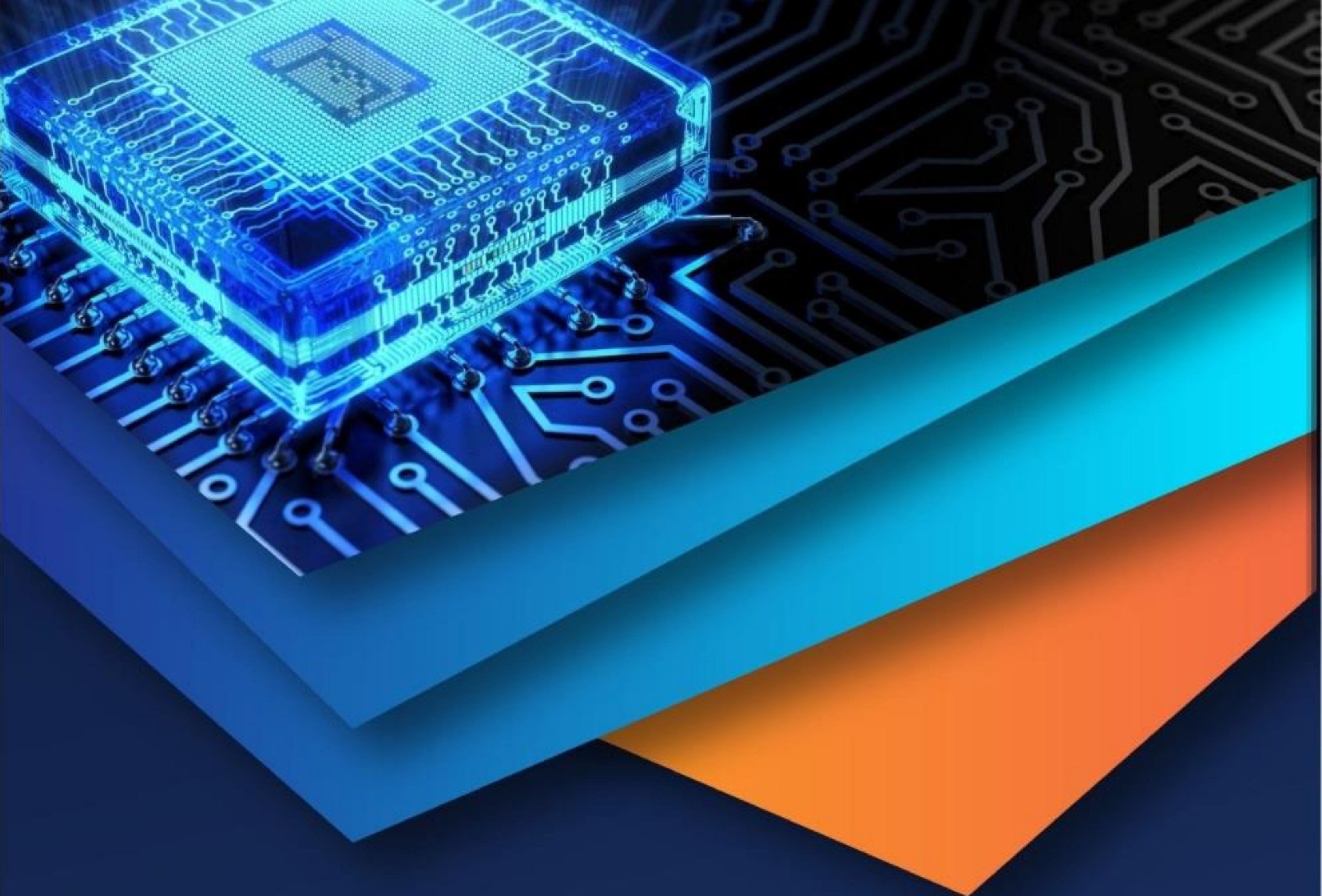

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