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Order Reduction using Basic Characteristics and Factor Division Method

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Abstract: The authors propose a mixed method for reducing the order of the high order dynamic systems. In this method, the denominator polynomial of the reduced order model is obtained by using the basic characteristics of the higher order system which are maintained in the reduced model while the coefficients of the numerator are obtained by using factor division method. This method is fundamentally simple and generates stable reduced models if the original high-order system is stable. The proposed method is illustrated with the help of the numerical example taken from the literature.
Keywords: Basic Characteristics, Order reduction, Factor Division, Stability, Transfer function

I. INTRODUCTION

THE approximation of linear systems have an important role in many engineering applications, especially in control system design, where the engineer is faced with controlling a physical system for which an analytic model is represented as a high order linear system. In many practical situations a fairly complex and high order system is not only tedious but also not cost effective for on-line implementation. It is therefore desirable that a high system be replaced by a low order system such that it retains the main qualitative properties of the original system. Several order reduction techniques for linear dynamic systems in the frequency domain are available the literature [1-4]. Further, some methods have also been suggested by combining the features of two different methods [5-7]. The Pade approximation method was originally introduced by Pade [8]. This method is computationally simple and fits initial time moments and matches the steady state values. The disadvantage of this Method is that the reduced model may be unstable even though the original system is stable. Sumit Mondal [9] utilizing the basic characteristics of original system and pade approximation to get reduced order system. Lucas [10] gives a very useful algorithm for reduction the order of high order system based on factor division method. It avoids finding the time moments and solving the Pade equation, whilst the reduced models still retains the initial time moments of full systems.

The Factor division algorithm can be used to determine the reduced numerator. In the proposed method , the denominator polynomial of reduced model is obtained using basic characteristics such as undamped natural frequency of oscillations (ω_n) ,damping ratio(ε),settling time(T_s), peak overshoot(M_p) and peak time(t_p) while the coefficient of numerator is obtained using factor division technique.

II. STATEMENT OF THE PROBLEM

Let the transfer function of high order original system of the order ‘ n ’ be

$$G_n(s) = \frac{N(s)}{D(s)} = \frac{g_0+g_1s+g_2s^2+\cdots\ldots\ldots+g_{n-1}s^{n-1}}{h_0+h_1s+h_2s^2+\cdots\ldots\ldots+h_ns^n} \qquad (1)$$

where g_i; $0\le i \le n-1$ and h_i; $0\le i \le n$ known scalar constants.

Let the transfer function of the reduced model of the order ‘ k ’ be

$$R_k(s) = \frac{N_k(s)}{D_k(s)} = \frac{c_0+c_1s+c_2s^2+\cdots\ldots\ldots+c_{k-1}s^{k-1}}{d_0+d_1s+d_2s^2+\cdots\ldots\ldots+d_ks^k} \qquad (2)$$

where ; c_j; $0\le j \le k-1$ and d_j; $0\le j \le k$ are unknown scalar constants.

The aim of this paper is to realize the k^{th} order reduced model in the form of (2) from the original system (1) such that it retains the important features of the original high -order system.

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III. REDUCTION METHOD

The reduction procedure for getting the k^{th}-order reduced models consists of the following two steps:

1) *Step-1:* Determination of the denominator polynomial for the k^{th} -order reduced model using basic characteristics of original system by the following procedure

- Firstly determine the basic characteristics of original system
- Then assume damping ratio (ε)=0.99 for an aperiodic or almost periodic system, and number oscillations before the system settles=1
- Determine the natural frequency(ω_n) using

$$\omega_n = \frac{4}{\varepsilon * Ts}$$

- Obtain the reduced order denominator as

$$D_2(s) = s^2 + 2 * \varepsilon * \omega_n + \omega_n^2$$

2) *Step-2*

Determination of the numerator of k^{th} order reduced model using Factor Division algorithm [11]

After obtaining the reduced denominator, the numerator of the reduced model is determined as follows

$$N_k(s) = \frac{N(s)}{D(s)} \times D_{k(s)} = \frac{N(s)}{D(s)/D_{k(s)}}$$

Where $D_k(s)$ is reduced order denominator

There are two approaches for determining of numerator of reduced order model.

(i) By performing the product of N(s) and $D_k(s)$ as the first row of factor division algorithm and D(s) as the second row up to s^{k-1} terms are needed in both rows.

(ii) By expressing $N(s)D_k(s)/D(s)$ as $N(s)/[D(s)/D_k(s)]$ and using factor division algorithm twice; the first time to find the term up to s^{k-1} in the expansion of $D(s)/D_k(s)$(i.e. put D(s) in the first row and $D_k(s)$ in the second row, using only terms up to s^{k-1}), and second time with N(s) in the first row and the expansion $[D(s)/D_k(s)]$ in the second row.

Therefore the numerator $N_k(s)$ of the reduced order model ($R_k(s)$) in eq.(2) will be the series expansion of

$$\frac{N(s)}{\frac{D(s)}{D_{k(s)}}} = \frac{\sum_{i=0}^{k-1} c_i s^i}{\sum_{i=0}^{k} d_i s^i}$$

About s=0 up to term of order s^{k-1}.

This is easily obtained by modifing the moment generating[14].which uses the familiar routh recurrence formulae to generate the third, fifth, and seventh etc rows as,

$$\begin{array}{llllllll}
\propto_0 = \frac{g_0}{h_0} < & g_0 & g_1 & g_2 & . & \dots\dots\dots\dots.. & g_{k-1} \\
 & h_0 & h_1 & h_2 & . & \dots\dots\dots\dots & h_{k-1} \\
\propto_1 = \frac{l_0}{h_0} < & l_0 & l_1 & l_2 & & \dots\dots\dots\dots. & l_{k-2} \\
 & h_0 & h_1 & h_2 & & \dots\dots\dots\dots\dots & h_{k-2} \\
\propto_2 = \frac{m_0}{h_0} < & m_0 & m_1 & m_2 & & \dots\dots\dots & m_{k-3} \\
 & h_0 & h_1 & h_2 & & \dots\dots\dots\dots\dots & h_{k-3}
\end{array}$$

....
....
....

$$\begin{array}{lll}
\propto_{k-2} = \frac{p_0}{h_0} < & p_0 & p_1 \\
 & h_0 & h_1 \\
\propto_{k-1} = \frac{q_0}{h_0} < & q_0 & \\
 & h_0 &
\end{array}$$

Where
$l_i = g_{i+1} - \propto_0 * h_{i+1}$, i=0,1,2,..........................
$m_i = l_{i+1} - \propto_1 * h_{i+1}$,i=0,1,2......................
....
....

$$p_0 = p_1 - \propto_{k-2} h_1$$

Therefore, the numerator $N_k(s)$of eq.(2) is given by
N_k(s)=$\sum_{i=0}^{k-1} \propto_i s^i$

IV. METHOD FOR COMPARISON

In order to check the quality of the proposed method the quantitative comparison in term of rise time (t_r), settling time (t_s) and maximum overshoot (M_p) with the original system has been done.

V. NUMERICAL EXAMPLE

The proposed method explains by considering numerical example, taken from the literature. The goodness of the proposed method is measured by calculating the rise time (t_r), settling time (t_s) and maximum overshoot (M_p) and compare with the original system .

Example-: Consider a 4th-order system from the literature [8]

$$G(s) = \frac{24 + 24s + 7s^2 + s^3}{24 + 50s + 35s^2 + 10s^3 + s^4}$$

1) Step-I

Determination of Denominator of reduced order
Denominator of reduced order model is determine using following basic characteristics of original system
RiseTime: 2.260

SettlingTime: 3.9307

SettlingMin: 0.9002

SettlingMax: 0.9991

Overshoot: 0

Undershoot: 0

Peak: 0.9991

Peak Time: 6.9770

ε=0.99 for an aperiodic or almost periodic system, and number oscillations before the system settles=1
since $\omega_n = \frac{4}{\varepsilon * Ts}$

Therefore ω_n=4/0.99*3.93=1.0281
The Reduced denominator is given by

$D_2(s) = s^2 + 2 * \varepsilon * \omega n * s + \omega_n^2$

$= s^2 + 2.0356s + 1.0569$

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2) Step 2

Now using the factor division method the numerator of reduced order model is given as
Consider $D_4(s)/D_2(s)$

$\alpha_0 = 22.707$ 24   50
                 1.0569   2.0356

$\alpha_1 = 3.574$ 3.7776
                 1.0569

Now Considering $N_4/D_4(s)/D_2(s)$

$\alpha_0 = 1.05694$ 24   24
                     22.707   3.574

$\alpha_1 = 0.8906$ 20.2224
                     22.707

Thus Reduced Numerator is given as
$N_2(s) = 1.05694+0.8906s$

Thus the Reduced model is given as

$$R_2(s) = \frac{1.05694+0.8906s}{1.0569+2.0356s+s^2}$$

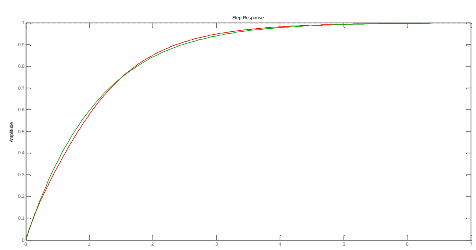

Figure 1 Step Response Comparison between original system and reduced system

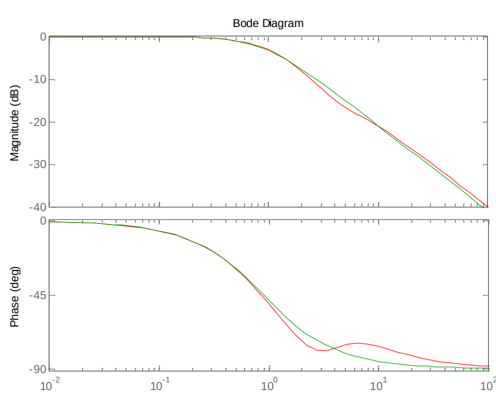

Figure 2. Bode Plots of original system and reduced system

Table- I
Qualitative Comparison With The Original System

System	Rise Time t_r (sec.)	Peak Overshoot (M_p)	Settling time (T_s)
4th order [10]	2.2602	0	3.9307
2nd order	2.3592	0	4.1170

VI. CONCLUSIONS

The authors presented an order reduction method for the linear dynamic system of high order systems. The basic characteristics of original system are utilized for determination of denominator polynomial of the reduced model while Factor division algorithm is used for calculation of the numerator coefficients. The advantages of proposed method are stable, simplicity, efficient and computer oriented. The proposed method has been explained with an example taken from the literature. The step responses and Bode plots of the original and reduced system of second order are shown in the Figure-1 and Figure-2 respectively. A quantitative comparison of reduced order model obtain by proposed method with the original system is shown in the Table-I from which we can conclude that proposed method is comparable in quality.

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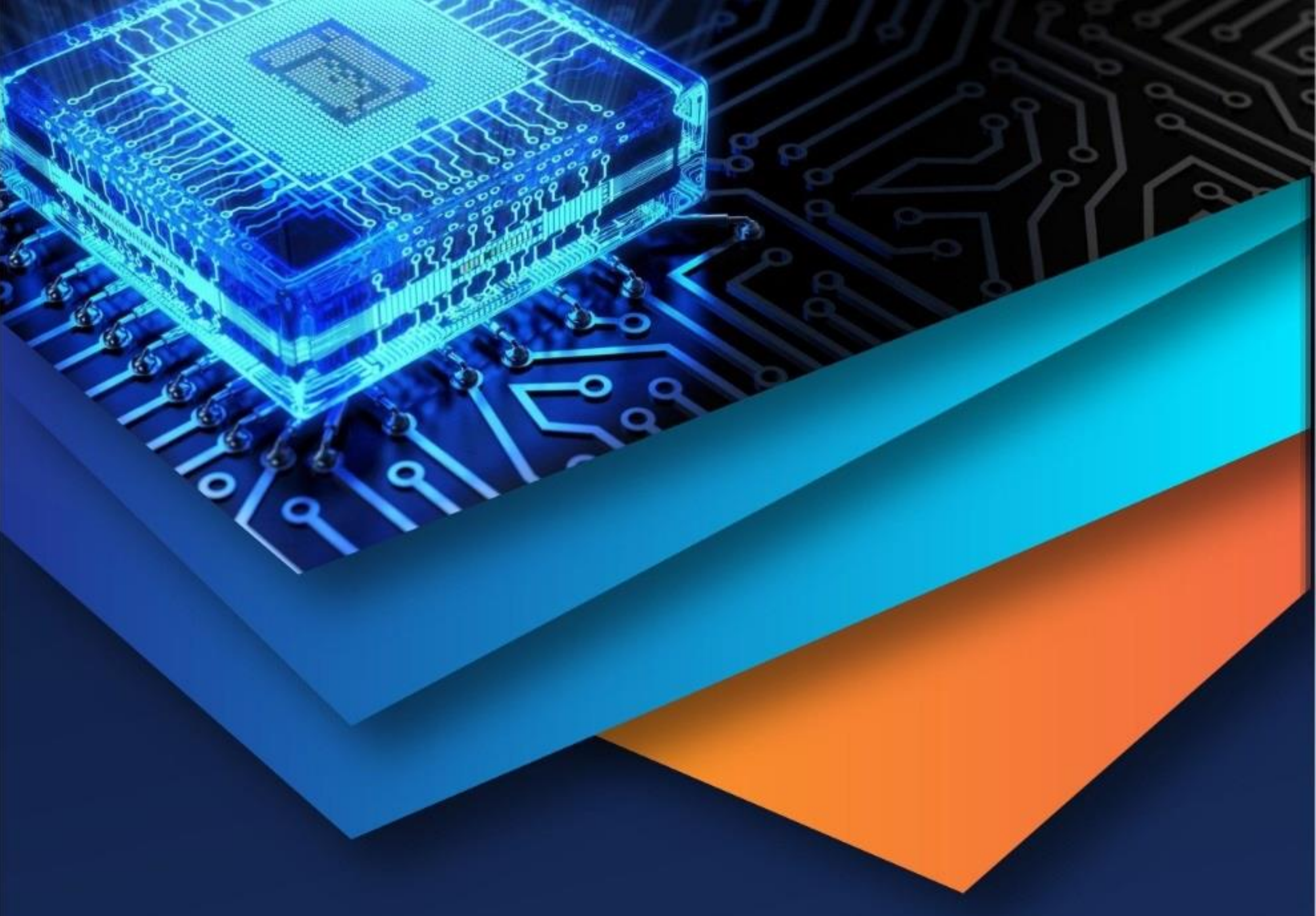

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