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Order Statistics of Doubly Truncated Additive Uniform Exponential Distribution

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Abstract: In this paper we truncated the Additive Uniform Exponential Distribution (AUED) proposed by Venkata Subbarao Uppu (2010).The probability density functions of rth order Statistics, lth moment of the rth order Statistic, minimum, maximum order statistics, mean of the maximum and minimum order statistics, the joint density function of two order statistics of the truncated distribution were calculated and discussed in detailed .
Keywords: Additive Uniform Exponential Distribution, Truncation, moments, minimum order statistic, Maximum order statistic, joint density of the order Statistics, complete length of service

I. INTRODUCTION

To introduce the phenomenon of converting an infinite range into a finite range of the random variate we utilize the concept of truncation .The range of the Additive Uniform Exponential distribution is (0, ∞). But in many practical situations arising at places like quality control, agricultural experiments, reliability studies, etc., the variate under study will have a finite range. For example in manpower modeling the complete length of service of an employee will have a finite range. Similarly in other areas like the incontrol times of processes, productions will have minimum and maximum limits. So, for these sorts of situations the random variable under study is to be considered as doubly truncated. Hence, in this paper we develop and analyze a Doubly Truncated Additive Uniform Exponential Distribution.

We discuss the probability density function of Doubly Truncated Additive Uniform Exponential Distribution as follows:

The probability density function of additive uniform exponential distribution (AUED) is

$$g_X(x) = \frac{1}{a}\left[1-e^{-\theta x}\right];\ 0 \le X \le a$$

$$= \frac{e^{-\theta x}}{a}\left[e^{a\theta}-1\right];\ a \le X < \infty \qquad (1)$$

$$'a'\ and\ '\theta'\ are\ the\ parameters\ of\ the\ distribution,\ a > 0\ and\ \theta > 0$$

Consider that the range of the random variable is finite say (L,U). Then the probability density function of the Doubly Truncated Additive Uniform Exponential Distribution (DTAUED) is

$$f_X(x) = \frac{g_X(x)}{G(U)-G(L)} \qquad L \le X \le U \qquad (2)$$

Where $g_X(x)$ is given in equation (1)

And $G(U)-G(L) = \int_L^a f(x)dx + \int_a^U f(x)dx = \int_L^a \frac{1}{a}\left[1-e^{-\theta x}\right]dx + \int_a^U \frac{e^{-\theta x}}{a}\left[e^{a\theta}-1\right]dx$

On simplification $G(U)-G(L) = \frac{1}{a}\left[(a-L)+\frac{1}{\theta}(1-e^{-L\theta}+e^{-U\theta}-e^{\theta(a-U)}\right]$

Let the value of $G(U)-G(L) = A$

Therefore the probability density function of Doubly Truncated Additive Uniform Exponential distribution (DTAUED) is

$$f_X(x) = \frac{1}{A}\left[1-e^{-\theta x}\right];\quad L \le X \le a$$

$$= \frac{e^{-\theta x}}{A}\left[e^{a\theta}-1\right];\quad a \le X < U \qquad (3)$$

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The lower and upper truncation points are L and U respectively. The degrees of truncation are G (L) and 1 - G (U). If L is replaced by 0 or U is replaced by ∞ the distribution is singly truncated from below or above respectively.
Distribution function is defined as

$$F_X(x) = \int_L^x \frac{1}{A}\left[1 - e^{-\theta t}\right] dt = \frac{1}{A}\left[(x - L) + \frac{1}{\theta}\left(e^{-\theta x} - e^{-\theta L}\right)\right] \; for \; L \le X \le a \qquad (4)$$

And

$$F_X(x) = \int_L^a f(t)dt + \int_a^x f(t)dt = \int_L^a \frac{1}{A}\left[1 - e^{-\theta t}\right] dt + \int_a^x \frac{e^{-\theta t}}{A}\left[e^{a\theta} - 1\right]dt$$

$$= \frac{1}{A}\left[(a - L) + \frac{1}{\theta}\left[1 + e^{-\theta x} - e^{-L\theta} - e^{(a-x)\theta}\right]\right] \; ;\text{for } a \le X \le U \qquad (5)$$

II. ORDER STATISTICS OF DOUBLY TRUNCATED ADDITIVE UNIFORM EXPONENTIAL DISTRIBUTION

We derived the distribution of extreme order statistics and the joint distribution of the order statistics and some properties. The distribution of extreme order statistics are very important for studying the inferences related to the maximum, minimum and median of the data sets. For example in manpower modeling the complete length of service of an employee is a random variable and studying its dynamics is very important for several operating policies regarding welfare and pensionable benefits to find the probability distribution of the maximum duration of a state of an employee in an organization can be derived through order statistics.

Let $X_{1:n} \le X_{2:n} \le X_{3:n} \ldots\ldots\ldots \le X_{m:n}$ be the order statistics obtained from a random sample of size n from DTAUED having the density function as given in (3)

The probability density function of the r^{th} order statistics is given by (David 1981) is

$$f_{r:n}(u) = D_{r:n}[F(u)]^{r-1}[1 - F(u)]^{n-r} f(u); \; -\infty < u < \infty \qquad (6)$$

Where $D_{r:n} = \frac{n!}{(r-1)!(n-r)!}$ (7)

Substituting the values from equations (3) (4) and (5)

$$f_{r:n}(x) = \frac{D_{r:n}}{A^n}\left[\left(\theta(x - L) + (e^{-\theta x} - e^{-L\theta})\right]^{r-1}\left[(A - x + L)\theta - (e^{-\theta x} - e^{-L\theta}\right]^{n-r}\left(1 - e^{-\theta u}\right)$$

For $L \le X \le a$ (8)

$$f_{r:n}(u) = \frac{D_{r:n}}{A^n\theta^{n-1}}\left[\theta(a - L) + 1 + e^{-\theta x} - e^{-L\theta} - e^{\theta(a-x)})\right]^{r-1}[\theta(A - a + L)]^{n-r}(1 - e^{-\theta x}) \quad \text{For } a \le X < U$$

(9)

Where $D_{r:n}$ is given in equation (7)

The l^{th} moment of the r^{th} order statistic is

$$E(X^l{}_{r:n}) = \int_L^a x^l \frac{D_{r:n}}{A^n}\left[\left(\theta(x - L) + (e^{-\theta x} - e^{-L\theta})\right]^{r-1}\left[(A - x + L)\theta - (e^{-\theta x} - e^{-L\theta}\right]^{n-r}$$

$$\left(1 - e^{-\theta u}\right) d + \frac{D_{r:n}}{A^n\theta^{n-1}}\int_a^U u^l\left[\theta(a - L) + 1 + e^{-\theta x} - e^{-L\theta} - e^{\theta(a-x)})\right]^{r-1}[\theta(A - a + L)]^{n-r}(1 - e^{-\theta x})dx$$

And on simplification

$$\sum_{s=0}^{n-r}\sum_{k=0}^{r-1}\sum_{m=0}^{n-s-k-1}\sum_{j=0}^{s}\sum_{i=0}^{k}\sum_{q=1}^{\infty}\sum_{h=0}^{\infty}\frac{D_{r:n}}{A^n\theta^{n-1}}\binom{n-1}{s}\binom{r-1}{k}\binom{n-s-k}{m}\binom{s}{j}\binom{k}{i}\frac{\theta^{q+h}}{q!\,h!}$$

$$\frac{(-1)^{2n-r-s-m-i+q+h-1}e^{-a\theta(n-s-k-m-1)}}{(l + s - j + k + q + h + 1)}\left[a^{l+s-j+k+q+h+1} - L^{l+s-j+k+q+h+1}\right]$$

$$+\frac{D_{r:n}}{A^n\theta^{n-1}}\sum_{s=0}^{r-1}\sum_{k=0}^{n-r}\sum_{m=0}^{\infty}\binom{n-r}{k}\binom{r-1}{s}\left[\theta(a - L) + 1 - e^{-L\theta}\right]^{r-1}\left[\theta(A - a + L) + e^{-L\theta}\right]^k$$

$(e^{a\theta}-1)^{r-s}(-1)^{r+m-s-1}(1+e^{a\theta})^{n-r-k-1}$ (10)

The probability density function of the first order statistic of DTAUED can be obtained by taking r=1 in equations (5) and (6)

For $L \le X \le a$

$$f_{1:n}(x) = \frac{n}{A^n\theta^{n-1}}\left[\theta(A-x+L)+e^{-\theta x}+e^{-L\theta}\right]^{n-1}(1-e^{-\theta x}) \quad (11)$$

For $a \le X \le U$

$$f_{1:n}(u) = \frac{n}{A^n\theta^{n-1}}\left[\theta(A-a+L)-1-e^{-\theta x}+e^{-L\theta}+e^{\theta(a-x)}\right]^{n-1}(e^{a\theta}-1)e^{-\theta x} \quad (12)$$

Where $D_{1:n}$ is given in equation (7)

The mean of the first order statistic is given by

$$E(X_{(1)}) = \int_a^L x\frac{n}{A^n\theta^{n-1}}\left[\theta(A-x+L)+e^{-\theta x}+e^{-L\theta}\right]^{n-1}(1-e^{-\theta x})\,dx + \int_a^U \frac{n}{A^n\theta^{n-1}}\left[\theta(A-a+L)-1-e^{-\theta x}+e^{-L\theta}+e^{\theta(a-x)}\right]^{n-1}(e^{a\theta}-1)e^{-\theta x}\,dx$$

On simplification we get

$$E(X_{(1)}) = \frac{n}{A^n\theta^{n-1}}\sum_{r=0}^{n-1}\sum_{s=1}^{\infty}\sum_{m=0}^{n-r-1}\sum_{k=0}^{\infty}\binom{n-1}{r}\binom{n-r-1}{m}[\theta(A+L)+e^{-L\theta}]^r\frac{\theta^{s+m+k}(-1)^{n-r+s+k}}{s!}\left[\frac{a^{m+s+k+2}-L^{m+s+k+2}}{m+s+k+2}\right] + \frac{n}{A^n\theta^{n-1}}\sum_{r=0}^{n-1}\binom{n-1}{r}[\theta(A-a+L-1)+e^{-L\theta}]^r(e^{a\theta}-1)^{n-r}\left[\frac{Ue^{-\theta U(n-r)}-ae^{-a\theta(n-r)}}{(r-n)\theta}-\frac{e^{-\theta U(n-r)}+e^{-a\theta(n-r)}}{(r-n)^2\theta^2}\right]$$

(13)

The probability density function of the highest order statistic of AUED can be obtained by taking r = n in equations (5) and (6)

$$f_{n:n}(X) = = \frac{n}{A^n\theta^{n-1}}\left[(x-L)+e^{-\theta x}-e^{-L\theta}\right]^{n-1}(1-e^{-\theta x}) \ ; \text{For } L \le X \le a \quad (14)$$

And

$$f_{n:n}(X) = \frac{n}{A^n\theta^{n-1}}\left[a\theta+e^{-\theta u}(1-e^{a\theta})\right]^{n-1}e^{-\theta u}(e^{a\theta}-1); \text{For } a \le X < U \quad (15)$$

Where $D_{n:n}$ = n calculated by using equation (7)

The mean of the highest order statistic is given by

$$E(U_{(n)}) = \int_L^a \frac{n}{A^n\theta^{n-1}}\left[(x-L)+e^{-\theta x}-e^{-L\theta}\right]^{n-1}(1-e^{-\theta x})\quad dx + \int_a^U \frac{n}{A^n\theta^{n-1}}\left[a\theta+e^{-\theta u}(1-e^{a\theta})\right]^{n-1}e^{-\theta u}(e^{a\theta}-1)\,dx$$

And on simplification we get

$$\frac{n}{A^n\theta^{n-1}}\sum_{r=0}^{n-1}\sum_{s=0}^{n-r-1}\sum_{m=1}^{\infty}\sum_{k=0}^{\infty}\binom{n-1}{r}\binom{n-r-1}{s}(-1)^{r+m+k}\frac{(\theta s)^{k+m}}{k!\,m!}\left[\frac{a^{m+k+n-r-s+1}-L^{m+k+n-r-s+1}}{m+k+n-r-s+1}\right] + \frac{n(e^{a\theta}-1)}{A^n\theta^{n-1}}\sum_{r=0}^{n-1}(-1)^{n-r-1}\left(1-e^{a\theta}\right)^{n-r-1}\left[\theta(a-L)+1-e^{-L\theta}\right]^r \left[\frac{ae^{-a\theta(n-r)}-Ue^{-\theta U(n-r)}}{(n-r)\theta}-\frac{e^{-\theta U(n-r)}+e^{-a\theta(n-r)}}{(r-n)^2\theta^2}\right] \quad (16)$$

The joint density function of the order statistics $U_{m:n}$ and $U_{s:n}$(m< S) is given by (David 1981)

$$f(u,v)_{m,s:n} = D_{m,s:n}[F(U)]^{m-1}[F(V)-F(U)]^{s-m-1}[1-F(V)]^{n-s}f(u)f(v) \quad (17)$$

Where $D_{r:n} = \frac{n!}{(r-1)!(n-r)!}$

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Here ,We have three cases based up on the sample space of the variates they are

(i) $L \leq X_m < X_s \leq a$

(ii) $L \leq X_m \leq a; a \leq X_s < U$

(iii) $a \leq U_m < U_s < U$

For case (i) the joint probability density function of the order statistics $X_{m:n}$ and $X_{s:n}$ is

For $L \leq X_m < X_s \leq a$

$$f(X,Y)_{m,s:n} = \frac{D_{m,s:n}}{A^n\theta^{n-2}}\left[\theta(x-L)+e^{-\theta x}-e^{-\theta L}\right]^{m-1}\left[(y-x)\theta+e^{-\theta y}-e^{-\theta x}\right]^{s-m-1}\left[(A-y+L)\theta-e^{-\theta y}+e^{-\theta L}\right]^{n-s}(1-e^{-\theta x})(1-e^{-\theta y}) \quad (18)$$

Where $D_{m,s:n}$ is given in equation (4)

For case (ii)

For $L \leq X_m \leq a; a \leq X_s < U$

$$f(X,Y)_{m,s:n} = \frac{D_{m,s:n}}{A^n\theta^{n-2}}\left[\theta(x-L)+e^{-\theta x}-e^{-\theta L}\right]^{m-1}\left[\theta(a-x)+e^{-\theta y}-e^{-\theta x}-e^{-\theta(a-y)}+1\right]^{s-m-1}\left[(A-a+L)\theta+e^{-\theta y}-e^{-\theta L}-e^{-\theta(a-y)}+1\right]^{n-s}(1-e^{-\theta x})e^{-\theta x}(e^{a\theta}-1) \quad (19)$$

Where $D_{m,s:n}$ is given in equation (4)

For case (iii)

$a \leq U_m < U_s < U$

$$f(X,Y)_{m,s:n} = \frac{D_{m,s:n}}{A^n\theta^{n-2}}\left[\theta(a-L)+e^{-\theta x}-e^{-\theta L}-e^{\theta(a-x)}+1\right]^{m-1}\left[\left(e^{-\theta y}-e^{-\theta x}-e^{-\theta(a-y)}+e^{\theta(a-x)}\right)\right]^{s-m-1}\left[A-\theta(a-L)+e^{-\theta y}-e^{-\theta L}-e^{\theta(a-y)}-1\right]^{n-s}e^{-2\theta x}(e^{a\theta}-1)^2 \quad (20)$$

Where $D_{m,s:n}$ is given in equation (4)

The joint density function of the smallest and highest order statistics can be obtained by taking m=1 and S = n in the equations (15),(16),(17)

Therefore the joint probability density function of the extreme order statistics $X_{1:n}$ and $X_{n:n}$

Then for case (i)

$$f(X,Y)_{1,n:n} = \frac{D_{1,n:n}}{A^n\theta^{n-2}}[(y-x)\theta+(e^{-\theta y}-e^{-\theta x}]^{n-2}(1-e^{-\theta x})(1-e^{-\theta y}) \quad (21)$$

For case (ii)

$$f(X,Y)_{1,n:n} = \frac{D_{1,n:n}}{A^n\theta^{n-2}}[(a-x)\theta+1+e^{-\theta y}-e^{-\theta x}-e^{(a-y)\theta}]^{n-2}(1-e^{-\theta x})e^{-\theta x}(e^{a\theta}-1) \quad (22)$$

For case (iii)

$$f(X,Y)_{1,n:n} = \frac{D_{1,n:n}}{A^n\theta^{n-2}}\left[e^{-\theta y}-e^{-\theta x}-e^{(a-y)\theta}+e^{(a-x)\theta}\right]^{n-2}e^{-2\theta x}(e^{a\theta}-1)^2 \quad (23)$$

III. CONCLUSIONS

The above order statistics are very useful in manpower planning models, especially the minimum and maximum order statistics are used to calculate the pensionable benefits of an employee in an organization by treating the complete length of service as a random variable which is additive in nature. Order statistics were employed in many ways in acceptance sampling. First-order statistics are used to improve the robustness of sampling plans by variables. In life testing, these are much useful to shorten testing times to produce many lifetime distributions. In actuarial sciences, these have tremendous potential in joint life insurance aspects to calculate the distribution of life span and insurance risk. Order statistics are concerned with the ranks as well as the magnitude of the observations we can use them in the grouping of continuous data into frequency classification.

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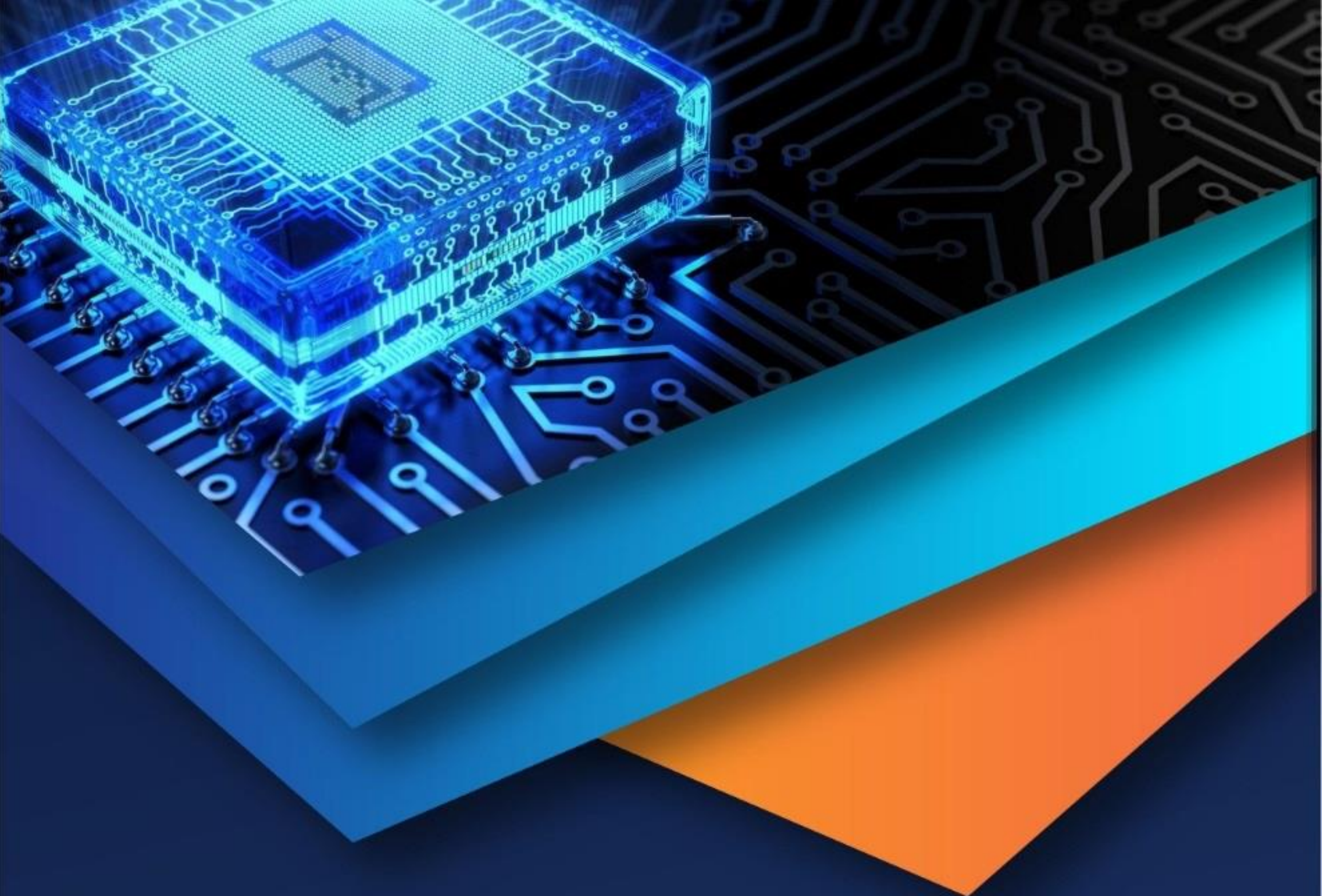

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