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Pellian Equation Involving Factorian Number

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Abstract: The positive Pellian binary quadratic equation $145X^2+1=Y^2$ is studied for its distinct integer solutions, along with an analysis of several interesting relationships among them. Furthermore, by utilizing the solutions various hyperbolas, parabolas, and special Pythagorean triangles are generated.
Key Word: Pell equation, Integer solutions, Factorion number.

I. INTRODUCTION

The Pell equation is a Diophantine equation of the form $Dx^2 \pm 1 = y^2$, where D is a given positive square-free integer. As one of the oldest, Diophantine equations has captivated mathematicians across the world since ancient times. J.L Lagrange established a criterion for its solvability, proving that for any positive non-square integer D, the equation always possesses integer solutions.

The Pell equation is a fundamental concept in number theory, playing a crucial role in solving Diophantine equations that require integer solutions. It has enormous applications in areas like cryptography, algebraic number theory, and the study of quadratic forms. This equation demonstrates how ancient mathematical challenges continue to provide meaningful insights into modern mathematics.

A factorion number is a special type of number in which the sum of the factorials of its digits is equal to the number itself. A number n is a factorion number if $n=\sum_{i=1}^{k}$ digit i! Examples of Factorion number are 1,2,145 and 40585.

In this study, we investigate the positive Pell equation and establish the existence of infinitely many integer solutions. Furthermore, we explore various interesting relationships among the solutions.

II. METHOD OF ANALYSIS

The positive Pellian equation representing hyperbola under consideration is,

$$145x^2+1=y^2 \tag{1}$$

The smallest positive integer solution for (1) is,

$x_0=1, y_0=13$

To obtain the other solutions of (1), consider the Pell equation

$$145x^2+1=y^2$$

Here $(\tilde{x}_0,\tilde{y}_0)=(24,289)$, where D =145,

Applying Brahmagupta lemma between $(\tilde{x}_0,\tilde{y}_0)$ and (x_0,y_0),the other integer solutions of (1) are given by,

$$\tilde{x}_n=\frac{1}{2\sqrt{145}}g_n \quad \tilde{y}_n=\frac{1}{2}f_n$$

where

$$f_n=\left(289+24\sqrt{145}\right)^{n+1}+\left(289-24\sqrt{145}\right)^{n+1}$$

$$g_n=\left(289+24\sqrt{145}\right)^{n+1}-\left(289-24\sqrt{145}\right)^{n+1}$$

$$290x_{n+1}=145f_n+13\sqrt{145}g_n$$

$$2y_{n+1}=13f_n+\sqrt{145}g_n$$

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The recurrence relations satisfied by x and y are given by,

$$x_{n+3} = 578\,x_{n+2} - x_{n+1}$$

$$y_{n+3} = 578\,y_{n+2} - y_{n+1}$$

Some numerical examples of x and y satisfying (1) is given in the following table:

n	x_{n+1}	y_{n+1}
0	1	13
1	601	7237
2	347377	4182973
3	200783305	2417751157

We observe some interesting relations among the solutions which are presented below;

- Each of the following expressions is a perfect square.
  1. $\frac{1}{8352}\left[377x_{2n+3} - 209873x_{2n+2} + 16704\right]$
  2. $\frac{1}{4827456}\left[377x_{2n+4} - 121306217x_{2n+2} + 9654912\right]$
  3. $\frac{1}{12}\left[13y_{2n+2} - 145x_{2n+2} + 24\right]$
  4. $\frac{1}{3468}\left[13y_{2n+3} - 87145x_{2n+2} + 6936\right]$
  5. $\frac{1}{2004492}\left[13y_{2n+4} - 50369665x_{2n+2} + 4008984\right]$
- Each of the following expression is a nasty number
  1. $\frac{6}{8352}\left[377x_{2n+3} - 209873x_{2n+2} + 16704\right]$
  2. $\frac{6}{4827456}\left[377x_{2n+4} - 121306217x_{2n+2} + 9654912\right]$
  3. $\frac{6}{12}\left[13y_{2n+2} - 145x_{2n+2} + 24\right]$
  4. $\frac{6}{3468}\left[13y_{2n+3} - 87145x_{2n+2} + 6936\right]$
  5. $\frac{6}{2004492}\left[13y_{2n+4} - 50369665x_{2n+2} + 4008984\right]$
- Each of the following expressions is a cubical integer
  1. $\frac{1}{8352}\left[377x_{3n+4} - 209873x_{3n+3} + 1131x_{n+2} - 629619x_{n+1}\right]$
  2. $\frac{1}{4827456}\left[377x_{3n+5} - 121306217x_{3n+3} + 1131x_{n+3} - 363918651x_{n+1}\right]$
  3. $\frac{1}{12}\left[13y_{3n+3} - 145x_{3n+3} + 39y_{n+1} - 435x_{n+1}\right]$

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4. $\frac{1}{3468}\left[13y_{3n+4} - 87145x_{3n+3} + 39y_{n+2} - 261435x_{n+1}\right]$

5. $\frac{1}{2004492}\left[13y_{3n+5} - 50369665x_{3n+3} + 39y_{n+3} - 151108995x_{n+1}\right]$

III. REMARKABLE OBSERVATIONS

Employing linear combinations among the solution of (1), one may generate integer solution for other choices of hyperbola which are presented in the table:

HYPERBOLA

S.No	Hyperbola
1	$x^2 - 145y^2 = 2790236160$
2	$x^2 - 145y^2 = 9321732572\ 0000$
3	$145x^2 - y^2 = 334080$
4	$145x^2 - y^2 = 2790269568$
5	$145x^2 - y^2 = 9321732572$

PARABOLA

S.No	Parabola
1	$576x - y^2 = 192430080$
2	$332928x - y^2 = 642878108400$
3	$3480x - y^2 = 334080$
4	$1005720x - y^2 = 2790269568$
5	$5813026799x - y^2 = 9321732572$

IV. CONCLUSION

This paper examines integer solutions to the positive Pell equation involving Factorian numbers. From the diverse nature of Diophantine equations, future research can build on this approach by investigating various types of Factorian numbers within different mathematical contexts.

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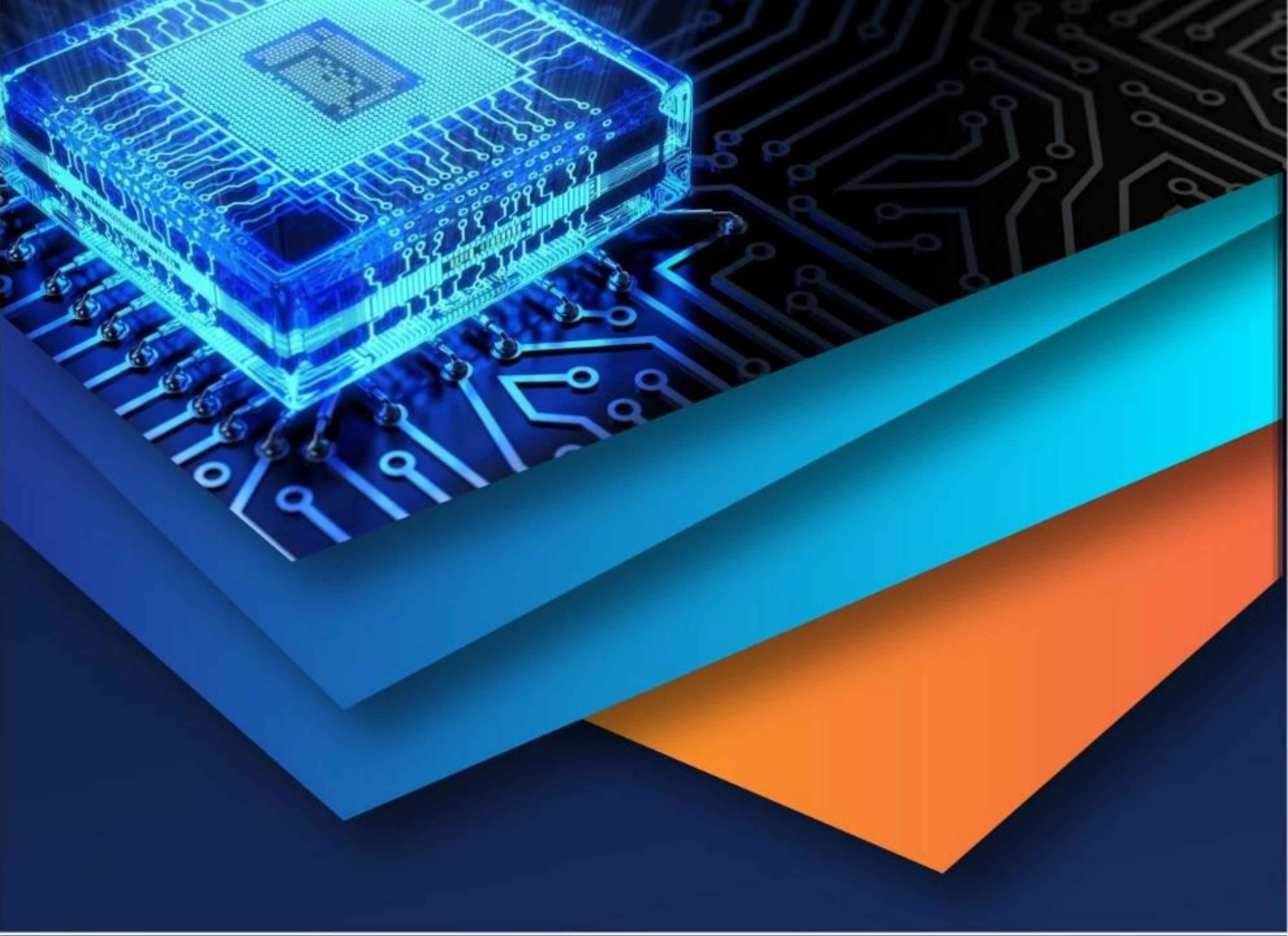

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