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# Proper Colorings in R-Regular Randic Index Graphs

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**Abstract:** The new idea of proper colourings in the r-regular Randic index graph has been proposed in this paper. New Chromatic number inequalities connected to the Randic index are established in this paper.

**Keywords:** Regular graph; Proper Colouring; Randic index; Chromatic number.

## I. INTRODUCTION

In this article, we simply take into consideration finite, undirected, simple graphs. The vertex set and edge set of a graph G are denoted by the letters V(G) and E(G). The order of G, denoted by p, is the cardinality of the vertex set. A (p,q) graph is defined as the cardinality of the edge set denoted by q edges. If G is a r – regular graph, then  $R(G) = \frac{nr}{r}$ . Proper colourings in r-regular Randic Index graph is expanded by the result of proper colourings in the magic and anti-magic graphs[11,16]. Several results and theorems have been verified by the Randic Index[1,4,5,6]. This research can be expanded to include domination which is based on Domatic numbers and Randic Index[8,9,10]. This work can also be expanded upon in the context of Automata theory[13,14,15] which has a numerous applications. There are numerous application for graph labeling in both undirected[12,17,18,22,23] and directed graphs[19,20,21].

## II. MAIN RESULT

1) Definition 2.1

The Randic indices are respectively defined as  $R(G) = \sum_{rs \in E(G)} \frac{1}{\sqrt{d(r) \cdot d(s)}}$ , where d(s) is the degree of the vertex s in G.

2) Theorem 2.1 :

If G is a r-regular Randic index graph then the chromatic number satisfies the inequality  $\left| \frac{k+r}{(v+E)^2} \right| \leq \psi(G) \leq \frac{1}{2} nr, r \geq 2$ .

Proof :

a) Case (i)

Let G be a graph of cycle  $C_n$ , 'n' be an odd integer.

Let  $C_n$  be r-regular with n vertices and m edges then  $R(G) = \frac{nr}{r}$

Let  $C_n$  be a cycle graph with Randic index, then the vertices in the cycle graphs are coloured with different colours, by proper colouring and the number used for colouring the cycle graph is 3. Therefore,  $\psi(G) = 3$ .

The following inequality is satisfied. Since K is the index number, r is the regular graph, V is the number of vertices and E is the number of edges in graph G.

The following inequality is obtained.

$$\left| \frac{k+r}{(v+E)^2} \right| \leq \psi(G) \dots\dots\dots (1)$$

The general condition of r-regular graph is denoted by as  $\frac{1}{2} nr$ .

Therefore  $\psi(G) \leq \frac{1}{2} nr$ . ..... (2)

From the equations (1) and (2) it is easy to verify that  $\left| \frac{k+r}{(v+E)^2} \right| \leq \psi(G) \leq \frac{1}{2} nr$

Hence the odd cycle satisfies  $\left| \frac{k+r}{(v+E)^2} \right| \leq \psi(G) \leq \frac{1}{2} nr$  for 2 - regular graph.

For example if  $n = 5$ , the corresponding graph is shown in Fig 2.1

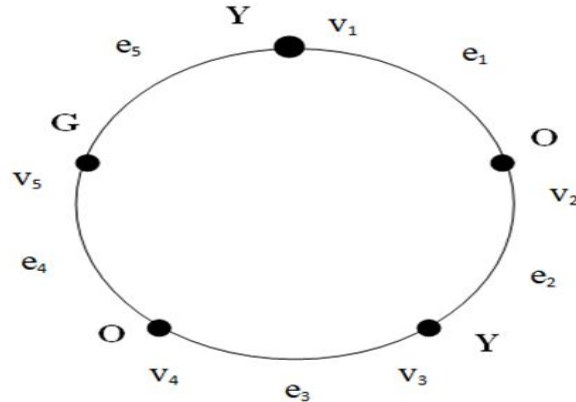


Fig 2.1 Randic Index Number For Odd Cycle

$$K = R(C_5) = 5 \left( \frac{1}{\sqrt{2.5}} \right) = 2.5$$

$$\left| \frac{k+r}{(v+E)^2} \right| \leq \psi(G) \leq \frac{1}{2} nr$$

$$0.045 \leq 3 \leq 5.$$

b) Case (ii)

Let  $G$  be a graph of cycle  $C_n$ , 'n' be an even integer.

Let  $C_n$  be  $r$ -regular with  $n$  vertices and  $m$  edges then  $R(G) = \frac{m}{r}$

Let  $C_n$  be a cycle graph with Randic index, then the vertices in the cycle graphs are coloured with different colours, by proper colouring and the number used for colouring the cycle graph is 2. Therefore,  $\psi(G) = 2$ .

The following inequality is satisfied. Since  $K$  is the index number,  $r$  is the regular graph,  $V$  is the number of vertices and  $E$  is the number of edges in graph  $G$ .

The following inequality is obtained

$$\left| \frac{k+r}{(v+E)^2} \right| \leq \psi(G) \text{ ..... (3)}$$

The general condition of  $r$ -regular graph is denoted by as  $\frac{1}{2} nr$ .

Therefore  $\psi(G) \leq \frac{1}{2} nr$ . ..... (4)

From the equations (3) and (4) it is easy to verify  $\left| \frac{k+r}{(v+E)^2} \right| \leq \psi(G) \leq \frac{1}{2} nr$

Hence the even cycle satisfies  $\left| \frac{k+r}{(v+E)^2} \right| \leq \psi(G) \leq \frac{1}{2} nr$  for 2 - regular graph.

For example if  $n = 4$ , the corresponding graph is shown in Fig 2.2

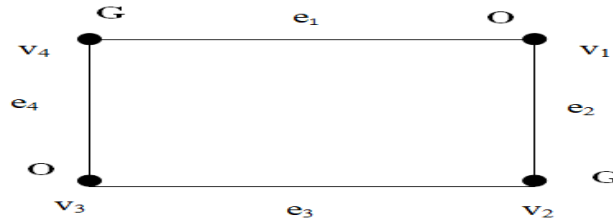


Fig 2.2 Randic Index Number For Even Cycle

$$K = R(C_4) = 4 \left( \frac{1}{\sqrt{2^2}} \right) = 2$$

$$\left| \frac{k+r}{(v+E)^2} \right| \leq \psi(G) \leq \frac{1}{2} nr$$

$$0.063 \leq 2 \leq 4.$$

c) Case (iii)

Let the graph  $G$  be Generalized Petersen Graph, here ' $n$ ' is an even integer.

Let  $V(p) = \{v_1, v_2, \dots, v_{10}\}$  be the vertices and  $E(p) = \{e_1, e_2, \dots, e_{15}\}$  be the edges of  $P(n, m)$  then  $R(G) = \frac{nr}{r}$

Let  $P(n, m)$  be a Generalized Petersen Graph with Randic index, then the vertices are coloured with different colours by proper colouring and the number of colours used for colouring this graph is 3. Therefore,  $\psi(P) = 3$ .

The following inequality is satisfied. Since  $K$  is the index number,  $r$  is the regular graph,  $V$  is the number of vertices and  $E$  is the number of edges in graph  $G$ .

The following inequality is obtained.

$$\left| \frac{k+r}{(v+E)^2} \right| \leq \psi(G) \tag{5}$$

The general condition of  $r$ -regular graph is denoted by as  $\frac{1}{2} nr$ .

$$\text{Therefore } \psi(G) \leq \frac{1}{2} nr. \tag{6}$$

From the equations (5) and (6) it is easy to verify  $\left| \frac{k+r}{(v+E)^2} \right| \leq \psi(G) \leq \frac{1}{2} nr$

Hence the Generalized Petersen Graph satisfies  $\left| \frac{k+r}{(v+E)^2} \right| \leq \psi(G) \leq \frac{1}{2} nr$  for 3-regular graphs.

The Generalized Petersen Graph is shown in Fig 2.3

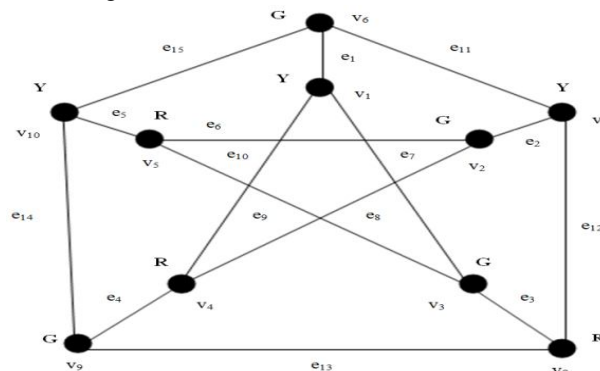


Fig 2.3 Randic Index Number For Generalized Petersen Graph

$$K = R(P_{15}) = 15 \left( \frac{1}{\sqrt{3.3}} \right) = 5$$

$$\left| \frac{k+r}{(v+E)^2} \right| \leq \psi(G) \leq \frac{1}{2} nr$$

The inequality of Randic index for generalized Petersen graph is  $0.0632 \leq 3 \leq 15$ .

d) Case (iv)

Let  $G$  be a complete graph, 'n' be an any integer.

Let  $V = \{v_1, v_2, \dots, v_n\}$  be the vertices and  $E = \{e_1, e_2, \dots, e_n\}$  be the edges of  $k_n$ , then  $R(G) = \frac{nr}{r}$

Let  $k_n$  be a complete Graph with Randic index, then the vertices are coloured with different colours by proper colouring and the number of colours used for colouring this graph is  $n$ . Therefore,  $\psi(k_n) = n$ .

The following inequality is satisfied. Since  $K$  is the index number,  $r$  is the regular graph,  $V$  is the number of vertices and  $E$  is the number of edges in graph  $G$ .

The following inequality is obtained.

$$\left| \frac{k+r}{(v+E)^2} \right| \leq \psi(G) \tag{7}$$

The general condition of  $r$ -regular graph is denoted by as  $\frac{1}{2} nr$ .

$$\text{Therefore } \psi(G) \leq \frac{1}{2} nr. \tag{8}$$

From the equations (7) and (8) it is easy to verify  $\left| \frac{k+r}{(v+E)^2} \right| \leq \psi(G) \leq \frac{1}{2} nr$

Hence the complete Graph satisfies  $\left| \frac{k+r}{(v+E)^2} \right| \leq \psi(G) \leq \frac{1}{2} nr$  for  $n$ -regular graphs.

For example if  $n = 5$ , the corresponding graph is shown in Fig 2.4

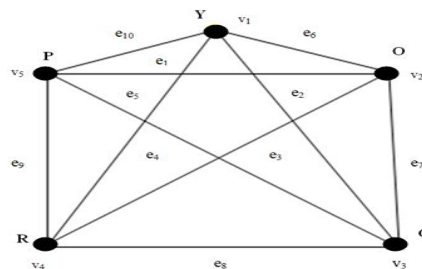


Fig 2.4 Randic Index Number For Complete Graph

$$K = R(K_5) = 10 \left( \frac{1}{\sqrt{4.4}} \right) = 2.5$$

$$\left| \frac{k+r}{(v+E)^2} \right| \leq \psi(G) \leq \frac{1}{2} nr$$

The inequality of Randic index for complete graph is  $0.029 \leq 5 \leq 10$ .

### III. CONCLUSION

In this paper, new inequality has been established. Further, it has been verified for Randic Index. Finally, we conclude that new inequalities in chromatic numbers are related to RandicIndex.

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