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# Properties of the Ternary Cubic Equation

$$5x^2 - 3y^2 = z^3 \sum$$

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**Abstract:** To identify its different integral non-zero solutions, the ternary cubic equation  $5x^2 - 3y^2 = z^3$  is taken into consideration. Different integral solution patterns to the ternary cubic equation under consideration are obtained in each pattern by using the linear transformation and the method of factorization; interesting relationships between the solutions and some polygonal numbers, such as pyramidal and central pyramidal numbers, are also displayed.

**Keywords:** Diophantine equations, Ternary equation, Cubic Equation with Three Unknowns, Integral Solutions

## I. INTRODUCTION

Number theory, which is used to explain anything that can be quantified, is the language of patterns and relationships. A polynomial equation called a Diophantine equation can only have integers as solutions. Theories of numbers were featured in [1–3]. A unique Pythagorean triangle problem and its integral solutions are featured in [4, 5]. Higher order equations are taken into account for integral solutions in [6–10].

The non-homogeneous cubic equation with three unknowns represented by the equation is discussed in this communication, and in particular, a few intriguing relationships between the solutions are highlighted.

### A. Notations

$T_{m,a}$ : Polygonal Number of rank  $a$  with side  $m$

$Gno_a$ : Gnomonic Number of rank  $a$

$Star_a$ : Star Number of rank  $a$

$O_a$ : Octahedral Number of rank  $a$

$P_a^m$ : Pyramidal Number of rank  $a$  with sides  $m$

$SO_a$ : Stella Octangula Number of rank  $a$

$CC_a$ : Centered Cube Number of rank  $a$

$CS_a$ : Centered Square Number of rank  $a$

$RD_a$ : Rhombic Dodecagonal Number of rank  $a$

$TO_a$ : Truncated Octahedral Number of rank  $a$

## II. METHOD OF ANALYSIS

A non-zero integral solution to the Cubic equation can be found by

$$5x^2 - 3y^2 = z^3 \tag{1}$$

Upon switching the transformations,

$$x = X + 3T, y = X + 5T, z = 2Z \tag{2}$$

$$\text{in (1) leads to, } X^2 - 15T^2 = 4Z^3 \tag{3}$$

Below, we present illustration of distinct integer non-zero patterns (1)

### A. Pattern: 1

$$\text{Assume } Z = z(a,b) = a^2 - 15b^2 \tag{4}$$

Where a and b are positive integers.

$$\text{And write } 4 = (8 + 2\sqrt{15})(8 - 2\sqrt{15}) \tag{5}$$

Using the factorization approach, replacing (3) with (4) and (5),

$$(X + \sqrt{15}T)(X - \sqrt{15}T) = (8 + 2\sqrt{15})(8 - 2\sqrt{15})(a + \sqrt{15}b)^3(a - \sqrt{15}b)^3$$

Comparing real and imaginary elements while equating similar phrases,  $X = 8a^3 + 360ab^2 + 90a^2b + 450b^3$

$$T = 2a^3 + 90ab^2 + 24a^2b + 120b^3$$

The appropriate integer solutions of equation (1) are provided by substituting the above mentioned values of X and T into equation

$$x = x(a, b) = 14a^3 + 630ab^2 + 162a^2b + 810b^3$$

$$(2) y = y(a, b) = 18a^3 + 810ab^2 + 210a^2b + 1050b^3$$

$$z = z(a, b) = 2a^2 - 30b^2$$

Properties:

1.  $-7z(a, a)$  and  $-28z(a, a)$  are Perfect Squares
2.  $-x(1,1) + y(1,1) - z(1,1)$  is a Harshad Number
3.  $y(a,1) - x(a,1) - 6P_a^6 - star_a - 2T_{29,a} - T_{26,a} \equiv 239(\text{mod}123)$
4.  $y(a,1) - x(a,1) - 124z(a,1) - 2SO_a - 91Gno_a \equiv 0(\text{mod}4051)$
5.  $-y(a,1) - z(a,1) + 18P_a^3 + 4Star_a \equiv 1076(\text{mod}852)$
6.  $x(a,1) + y(a,1) - 16z(a,1) - 2TO_a - 29T_{30,a} \equiv 2352(\text{mod}1769)$
7.  $-x(a,1) + y(a,1) - 23z(a,1) + 12P_a^5 + T_{18,a} \equiv 930(\text{mod}187)$

B. Pattern: 2

The modified form of equation (3) is

$$X^2 - 15T^2 = 4Z^3 * 1 \tag{6}$$

$$\text{Put 4 in as, } 4 = (8 + 2\sqrt{15})(8 - 2\sqrt{15}) \tag{7}$$

$$\text{and } 1 = (4 + \sqrt{15})(4 - \sqrt{15}) \tag{8}$$

When (7) and (8) are substituted in equation (6) and the process of factorization is used, as described in Pattern 1, the corresponding

$$x = x(a, b) = 110a^3 + 4950ab^2 + 1278a^2b + 6390b^3$$

integer solutions of (1) are represented by  $y = y(a, b) = 142a^3 + 6390ab^2 + 1650a^2b + 8250b^3$

$$z = z(a, b) = 2a^2 - 30b^2$$

Properties:

1.  $y(1,1) - x(1,1) + z(1,1) - 22T_{3,4}$  is a nasty number
2.  $x(1,1) + y(1,1) - 26T_{3,4}$  is a perfect square number
3.  $y(1,1) - x(1,1) + 11z(1,1) - T_{8,3}$  Is a cubic number
4.  $y(a,1) - x(a,1) - 8RD_a - 72T_{15,a} - 886Gno_a \equiv 0(\text{mod}2762)$
5.  $y(a,1) - x(a,1) - 186z(a,1) - 48O_a - 712Gno_a \equiv 0(\text{mod}8152)$

6.  $y(a,1) - x(a,1) - 2TO_a - 47T_{22,a} \equiv 1392 \pmod{1815}$
7.  $y(a,1) - 825x(a,1) - 213O_a \equiv 33000 \pmod{6319}$
8.  $2x(a,1) - 426z(a,1) - 110SO_a - 284T_{26,a} - 6567Gno_a \equiv 0 \pmod{6567}$

**C. Pattern: 3**

Write  $1 = (31 + 8\sqrt{15})(31 - 8\sqrt{15})$  (9)

By replacing (7) and (9) in equation (6) and factorising the result using the steps in Pattern 1, the corresponding integer solutions of

$$x = x(a,b) = 866a^3 + 38970ab^2 + 10062a^2b + 50310b^3$$

(1) are represented by  $y = y(a,b) = 1118a^3 + 50310ab^2 + 12990a^2b + 64950b^3$

$$z = z(a,b) = 2a^2 - 30b^2$$

**Properties:**

1.  $y(a,1) - 16z(a,1) - 2236P_a^5 - 1184T_{22,a} - 30483Gno_a \equiv 0 \pmod{95913}$
2.  $y(a,1) - x(a,1) + z(a,1) - 189HO_a - 827T_{10,a} - 5796Gno_a \equiv 0 \pmod{23056}$
3.  $\frac{1}{10}(x(1,1) - y(1,1))$  is a square number
4.  $x(1,b) + y(1,b) + 23z(1,b) - 138312P_b^7 - 3239Star_b - 44295Gnob \equiv 0 \pmod{42086}$
5.  $y(1,1) - x(1,1) + 24T_{3,4}$  is a nasty number
6.  $x(a,1) - 16z(a,1) - 4336CC_a - 11329CS_a + 22658T_{3,a} \equiv 39894 \pmod{71658}$
7.  $-x(a,1) + y(a,1) - 23z(a,1) - 63RD_a - 239T_{30,a} \equiv 14013 \pmod{14195}$
8.  $y(a,1) - x(a,1) + 19z(a,1) - 126SO_a - 1483T_{6,a} \equiv 14070 \pmod{12949}$

**Note:**

Additionally, 4 and 1 might be written as

$$4 = (62 + 16\sqrt{15})(62 - 16\sqrt{15})$$

$$1 = \frac{(8 + \sqrt{15})(8 - \sqrt{15})}{49}$$

In relation to these options, one may find many patterns of solutions of (1)

**III. CONCLUSION**

Three unique patterns of non-zero distinct integer solutions to the given non-homogeneous problem  $5x^2 - 3y^2 = z^3$  are shown in this paper.

For various options of cubic Diophantine equations, additional patterns of non-zero integer unique solutions and their corresponding characteristics may be found.

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