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Pythagorean Fuzzy Soft Nano Topological Space and it's Applications

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Abstract: Our primary goal is to introduce the conceptual framework of Pythagorean fuzzy soft nano topological space (PFSNTS) from the fuzzy topological space (FTS), nano topological space (NTS), and fuzzy soft topological space (FSTS). We also look at the Pythagorean distinctive characteristics. This article has discussed several concepts related to intuitionistic, nano, and Pythagorean. The primary intention of this work is to use programming languages to make decisions in the contemporary world.
Keywords: Pythagorean, Membership (MBS), Intuitionistic, Nano topology (NT), Nano Topological space (NTS), Fuzzy Nano Topology (FNT), Fuzzy Topological Space (FTS), Fuzzy Soft Nano Topology (FNST), Pythagorean Fuzzy Soft Nano Topology (PFSNT).

I. INTRODUCTION

Fuzzy sets are sets whose elements have degrees of membership. Lotfi A. Zadeh independently established fuzzy sets in 1965 as a growth of the conventional conception of a set. The following domains see the utilization of fuzzy sets: Manipulation of images, Artificial intelligence and video editing, Failure Mode Identification, Consequences Analysis Estimating and interpreting quantities Algorithms for control. A collection of items with grades for membership and non-membership is called IFS. IFSs are fuzzy sets that go beyond the standard definition of a set. K. T. Atanassov presented the eye-catching and pragmatic IFS theory for problem resolution. An extension of the IFS [1] is the PFS [1]. By taking into thought the MBS function and N-MBS function, it serves as a tool for managing ambiguity. The requirement condition is $v+\mu \leq 1$, and Yager and Abbasov developed the Pythagorean fuzzy set. Molodtsov introduced the soft set in 1999; A parameterized family of sets has the designation a soft set. Lellis Thivagar and Carmel Richard introduced the theory of NT. Recently; there has been an upward trend of interest regarding this branch of topology. In this section, we have introduced the new idea of the PFSNTS and established some new nomenclature. Ultimately, our goal is to choose the ideal color palette for PFSNTS to paint the house.

II. PRELIMINARIES

Definition 1. Assume that X is a non-void set. A collection of subsets of X is called a topology [3] on X if

(i) The empty set ϕ and the space X are both members of τ.
(ii) Every collection of sets in τ has a union that is contained in τ.
(iii) Every limited collection of intersection sets in τ is also a part of τ.

Both (X,τ) are typically referred to as **topological spaces**, where X is a set and is a grouping of subsets of X.

Definition 2. Let $A = (U, \mathfrak{I})$ be an approximation space, and take into consideration,
$X \subseteq U$,

(i) The lowest approximation [1] of X with regarding to $\mathfrak{I}$, and it is demonstrated by $L_{\mathfrak{I}}(X)$, $L_{\mathfrak{I}}(X) = \bigcup_{x\in U}\{\mathfrak{I}(X) : \mathfrak{I}(X) \subseteq X\}$, where $\mathfrak{I}_X$ depicts the equivalency class found by $x \in U$.
(ii) The highest approximation [1] of X with regarding to $\mathfrak{I}$, and it is demonstrated by $U_{\mathfrak{I}}(X)$, $U_{\mathfrak{I}}(X) = \bigcup_{x\in U}\{\mathfrak{I}(X) : \mathfrak{I}(X) \cap X \neq \phi\}$, where $\mathfrak{I}_X$ depicts the equivalency class found by $x \in U$.
(iii) The bordering region [1] of X with regarding to $\mathfrak{I}$, and it is demonstrated by $B_{\mathfrak{I}}(X) = U_{\mathfrak{I}}(X) - L_{\mathfrak{I}}(X)$, where $\mathfrak{I}_X$ depicts the equivalency class found by $x \in U$.

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Example 2. Given U = {1,2,3,4,5} with $U/\Im$ = {{4},{1,2},{3,5}} and X = {1,4} then, the topology $\tau_{\Im}(X)$ = {ϕ,U, {4}, {1,2}, {1,2,4}}.

$$L_{\Im}(X) = \bigcup_{X \in U}\{\Im(X) : \Im(X) \subseteq X\}$$

⇒ Lowest Approximation= {4}

$$U_{\Im}(X) = \bigcup_{X \in U}\{\Im(X) : \Im(X) \cap X \neq \phi\}$$

⇒ Highest Approximation= {{4} ∪ {1,2}}

$B_{\Im}(X) = U_{\Im}(X) - L_{\Im}(X)$

⇒ Bordering Region= {1,2}.

Definition 3. Let R be the equivalence relation is defined on the universal set U Furthermore, τ fulfils the subsequent requirements

(i) Both the empty set ϕ and the entire set X belong to τ.

(ii) Any sub-collection of τ consists of the union of its elements.

(iii) Any finite of τ encompasses the intersection of every one of its parts.

Then the τ is typically referred to as **NT** on X, we call (X,τ) a **NTS [1]**.

Definition 4. A pair (F, E) is typically referred to be a SS [3] over U if and only if is the set of all subsets of U. This is the circumstance if U is a universal set, and E is a universal set of parameters.

Definition 5. Let X is a non-void set of a universal set U, E is a universal set of parameters and τ has a family of Fuzzy Soft Rough Sets over X if F: E ⟶ P(U) which satisfy the requirements listed below:

(i) $\tilde{\phi}_E, \tilde{I}_E \in \tau$

(ii) Let $f_A, g_B \in \tau$ then $f_A \cap g_B \in \tau$

(iii) Let $(f_A)_i \in \tau$ for all i ∈ J, then $\bigcup_{i \in J}(f_A)_i \in \tau$

τ is typically referred to as the topology of **FSNT [3]** on X and (X,τ) is called a **FSNTS [3]**.

III. PYTHAGOREAN FUZZY SOFT NANO TOPOLOGICAL SPACE

Definition 6. Let F be a non-revoke fixed set and N be a subset of F and F has a set of parameters S : F ⟶ P(U) where U is a universal set then W is said to be a fuzzy soft subset[1] and is of the form N = {⟨t,δ_0(t)⟩,t ∈ F} where $\delta_0 : F \longrightarrow [0,1]$ is the degree of membership function.

Definition 7. If S is the subset of I and I is a non-revoke fixed set, then S is an Intuitionistic fuzzy subset[1] of the shape S = {⟨t,α(t),β(t)⟩,t ∈ I} where α : I ⟶ [0,1],β : I ⟶ [0,1] is the degree of non-membership function and membership function accomplishes $\alpha(t) + \beta(t) \leq 1$.

Definition 8. If S is a subset of a non-revoke set L, the set S is a Pythagorean fuzzy soft subset[3], which can be represented in the following manner S = {⟨t,α(t),β(t)⟩,t ∈ L} in which α : I ⟶ [0,1],β : I ⟶ [0,1] is the degree of non-membership and membership function provides $\alpha^2(t) + \beta^2(t) \leq 1$ and S is the set of all parameters in the universal set U.

Definition 9. Let G be a Pythagorean element in S. The PFSNLA, PFSNUA and PFSNBR of G as demonstrated through $PFSNL_{\Im}(G)$, $PFSNU_{\Im}(G)$, $PFSNB_{\Im}(G)$ are consequently characterized as

$$PFSNL_{\Im}(G) = \{(t,\lambda_{LG}(t)\eta_{LG}(t)/t \in S)\}$$
$$PFSNU_{\Im}(G) = \{(t,\lambda_{HG}(t)\eta_{HG}(t)/t \in S)\}$$
$$PFSNB_{\Im}(G) = PFSNU_{\Im}(G) - PFSNL_{\Im}(G)$$

where λ_G is the MBS function, η_G is the N-MBS function in $\Im$. $\Im$ is the equivalence relation in the non-revoke set S.

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Definition 10. If S is the equivalence relation defined on the universal set S and if $\tau_3(X)$ consists of the empty set, universal set, Pythagorean Fuzzy Soft Nano Lowest Approximation, Pythagorean Fuzzy Soft Nano Highest Approximation and Pythagorean Fuzzy Soft Nano Bordering Region accomplish the subsequent axioms:

(i) The empty set and the universal set belong to $\tau_3(X)$

(ii) The union $(^S H_i)$ of all elements belongs to $\tau_3(X)$ if (H_i) belongs to $\tau_3(X)$

(iii) The intersection $(^T H_i)$ of all elements belongs to $\tau_3(X)$ if (H_i) belongs to $\tau_3(X)$.

Then $\tau_3(X)$ is typically referred to as the **PFSNT** and we have $(S,\tau_3(X))$ as a **PFSNTS.**

Theorem 1. Let S be a universal set which is non-revoke,

(a) If $PFSNL_3(X)$ is an empty set and $PFSNU_3(X)$ is a universal set then we get $\tau_3(X)$ = {emptyset,universalset} is the imprecise PFSNT on S.

(b) If $PFSNL_3(X)$ is equal to Y, $PFSNU_3(X)$ is also equal to Y then $\tau_3(X)$ = {emptyset,universalset,Pythagorean Fuzzy Soft Nano Lowest Approximation} is a PFSNT.

(c) If $PFSNL_3(X)$ is an empty set and $PFSNU_3(X)$ is not a universal set, then $\tau_3(X)$ = {emptyset,universalset,Pythagorean Fuzzy Soft Nano Highest Approximation} is a PFSNT.

(d) If $PFSNL_3(X)$ is not empty and $PFSNU_3(X)$ is a universal set, then $\tau_3(X)$ = {emptyset,universalset,Pythagorean Fuzzy Soft Nano Lowest Approximation, Pythagorean Fuzzy Soft Nano Bordering region} is a PFSNT.

$\tau_3(X)$ = {emptyset,universal set,Pythagorean Fuzzy Soft Nano Lowest Approximation,
Pythagorean Fuzzy Soft Nano Highest Approximation, Pythagorean Fuzzy Soft Nano Bordering region} is the unique PFSNT on S.

Theorem 2. Consider two fuzzy soft sets as x_1 and x_2. Let $x_1 \subset x_2$, $x_1 = (F_1,A)$ and $x_2 = (F_2,A)$ Proof.

- Membership function $\mu_x(u)$
- Non-membership function $\nu_x(u)$

$$\mu_{x1}(u) = \min(F_1(a)(x), 1 - F_2(a)(x))\ a \in A, x \in U$$
$$\mu_{x2}(u) = \min(F_2(a)(x), 1 - F_1(a)(x))\ a \in A, x \in U$$
$$\nu_{x1}(u) = \max(F_1(a)(x), 1 - F_2(a)(x))\ a \in A, x \in U$$
$$\nu_{x2}(u) = \max(F_2(a)(x), 1 - F_1(a)(x))\ a \in A, x \in U$$

$\therefore \mu_{x1}(u) \leq \mu_{x2}(u)$ [membership function]

$\therefore \nu_{x1}(u) \leq \nu_{x2}(u)$ [non-membership function]

$\therefore$ x is an Intuitionistic fuzzy subset.

Similarly,

$$\mu_x^2(u) + \nu_x^2(u) \leq 1$$

Uncertainty holds.

Pythagorean fuzzy soft subsets automatically satisfy the intuitionistic fuzzy subset.

$\therefore$ x is a Pythagorean fuzzy soft subset.

Let x be a Pythagorean element in U proving this by the intuitionistic fuzzy conditions.

$$PFSN_{LA}(x_1) = \{u \in U: \mu_{x1}(u) > \nu_{x1}(u)\}$$
$$PFSN_{LA}(x_2) = \{u \in U: \mu_{x2}(u) > \nu_{x2}(u)\}$$

By IFS,

$$\mu_{x1}(u) \leq \mu_{x2}(u),\ \nu_{x1}(u) \geq \nu_{x2}(u)$$
$$PFSN_{HA}(x_1) = \{u \in U: \mu_{x1}(u) \geq \nu_{x1}(u)\}$$
$$PFSN_{HA}(x_2) = \{u \in U: \mu_{x2}(u) \geq \nu_{x2}(u)\}$$

By IFS,

$$\mu_{x1}(u) \leq \mu_{x2}(u),\ \nu_{x1}(u) \geq \nu_{x2}(u)$$
$$PFSNB(x) = \{\mu_{x1}(u) = \nu_{x1}(u)\}$$

$\therefore$ The membership function holds for the equivalence relation.

To Prove: PFSNTS:

1) Universal set and empty set

$PFSN_{LA}(U) = U,\ PFSN_{HA}(U) = U$

$PFSN_{LA}(\phi) = \phi\ PFSN_{HA}(\phi) = \phi$

2) Closed under union

$\mu \cup_i x_i(u) = \sup\{\mu x_i(u) = i\}$

$\nu \cup_i x_i(u) = \inf\{\nu x_i(u) = i\}$

3) Closed under Intersection

$\mu \cap^n{}_i x_i(u) = \inf\{\mu x_i(u) : 1 \le i \le n\}$

$\nu \cap^n{}_i x_i(u) = \sup\{\nu x_i(u) : 1 \le i \le n\}$

$PFSN_{BORDER}(x) = \{u \in U: \mu_x(u) = \nu_x(u)\}$

$\therefore x_1$ and x_2 are PFSNTS.

A. *Applications of PFSNTS*

The following variety of colors are available in the paint showroom: Blue(B), White(W), Violet(V), Red(R), Yellow(Y). Which color applies best for the painting of the house?

	B	W	BEST COLOUR
C1	0.115	0.526	0.2899
C2	0.33	0.24	0.1665
C3	0.1	0.501	0.2610
C4	0.456	0.3	0.2979
C5	0.19	0.22	0.0845
C6	0.39	0.08	0.1585
C7	0.2	0.5678	0.3624
C8	0.54	0.11	0.3037

Table 2.

Here we choose the colors (B, W), (W, C), (V, R), (R, Y) are the set of parameters such as (α,β). By the definition of Intuitionistic fuzzy subset [1], $\alpha(t)+\beta(t) \le 1$ we get, $0.115 + 0.526 \le 1$.

All the values are calculated in the same way. By the definition of Pythagorean fuzzy soft subset[1], $\alpha^2(t) + \beta^2(t) \le 1$, we get, $((0.115)^2 + (0.526)^2) \le 1$.

All the values are calculated in the same manner.

The Table 2 contains a pair of colors (B, W) and the colors are satisfied the above condition.

Next, we consider an alternative pair of colors (R, Y) as shown in Table 3.

R	Y	BEST COLOUR
1	0.772	1.5960
0.98	0.1	0.9704
0.87	0.902	1.5705
0.901	0.543	1.1067
0.67	0.7	0.9388
0.77	0.84	1.2985
1	0.728	1.5300
0.712	0.8	1.1469

Table 3.

Contains a pair of colors (B, W) and the colors do not satisfy the necessities.

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B. To Identify the Best Combination for Various Skin Types

The following ingredients are used in the product:

Rice flour (R), Green gram (G), Rose petal powder (Ro), Orange Peel powder (O), Chickpea powder (C), Multani Meti (M), and Vetiver (V).

Which Combination suits the Following Skin types?

Skin Type	R	G	Best Combination
Normal	0.696	0.281	0.5474
Oily	0.451	0.511	0.4645
Dry	0.352	0.454	0.330
Combination	0.405	0.553	0.4698
Sensitive	0.429	0.528	0.4628

Table 4.

Here we choose (R, G), (Ro, R), (O, C), (G, M), and (V, Ro) are the set of parameters such as (α,β)

By the definition of the Intuitionistic fuzzy subset, $\alpha(t)+\beta(t) \leq 1$. We get, $0.696+0.281 \leq 1$ All these values are calculated in the same manner. $\alpha^2(t) + \beta^2(t) \leq 1$ we get, $((0.696)^2 + (0.281)^2) \leq 1$

All the values are calculated in the same manner.

Table 4 satisfies the above condition, The highest effectiveness for a specific skin type is achieved by combining Rice Flour (R) and Green Gram (G).

Skin Type	Ro	G	Best Combination
Normal	0.413	0.443	0.3668
Oily	0.632	0.321	0.502
Dry	0.661	0.329	0.5451
Combination	0.312	0.511	0.3584
Sensitive	0.592	0.403	0.512

Table 5.

Table 5 satisfies the above condition, The highest effectiveness for a specific skin type is achieved by combining Rose Petal Powder (Ro) and Rice Flour (R).

Skin Type	O	C	Best Combination
Normal	0.525	0.411	0.444546
Oily	0.605	0.391	0.518906
Dry	0.484	0.384	0.381712
Combination	0.545	0.395	0.45305
Sensitive	0.432	0.398	0.345028

Table 6.

Table 6 satisfies the above condition, The highest effectiveness for a specific skin type is achieved by combining Orange Peel Powder (O) and Chickpea Flour (C).

Skin Type	G	M	Best Combination
Normal	0.592	0.367	0.485153
Oily	0.587	0.353	0.469178
Dry	0.572	0.311	0.423905
Combination	0.655	0.342	0.545989
Sensitive	0.502	0.351	0.375205

Table 7.

Table 7 satisfies the above condition, The highest effectiveness for a specific skin type is achieved by combining Green Gram (G) and Multani Meti (M).

Skin Type	G	M	Best Combination
Normal	0.486	0.395	0.392221
Oily	0.439	0.515	0.457946
Dry	0.572	0.345	0.446209
Combination	0.528	0.431	0.464545
Sensitive	0.685	0.314	0.567821

Table 8.

Table 8 satisfies the above condition, The highest effectiveness for a specific skin type is achieved by combining Vetiver (V) and Rose Petal Powder (Ro).

C. Algorithm

Step1: Create a table with the determined appropriate values in it.
Step2: To find the values of the Pythagorean parameters, install the necessary packages and load them.
Step3: For calculating the value of the hypotenuse, enter the following values for the triangle sides, side a (height) and side b (base).
Step4: Verify whether the values meet the requirement of being positive or not, and whether they are numerical or not.
Step5: Verify that the value is a vector.
Step6: After that, print the hypotenuse after calculating the square root of the nourished sides.
We determine that the colors (B, W) are the best choices for the house painting based on the computation above and the algorithm.

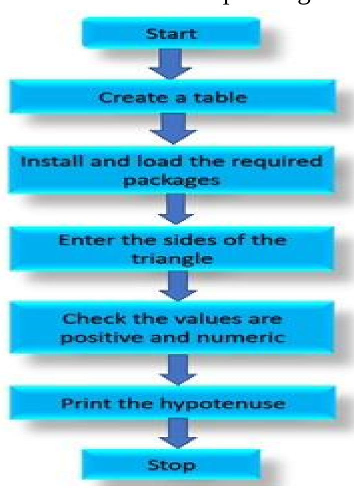

Figure 1. FLOW CHART

IV. CONCLUSION

In this paper, we introduced and explored the concept of Pythagorean Fuzzy Soft Nano Topological Space (PFSNTS) by integrating fuzzy topology, nano topology, and soft topology. We established key definitions, theorems, and properties of this novel framework. Furthermore, we demonstrated its practical applicability in decision-making scenarios, particularly in selecting optimal choices such as color combinations for house painting and ingredient blends for skin care. Our findings highlight the effectiveness of PFSNTS in handling uncertainty and imprecision in real-world problems. Future research can extend this work by applying PFSNTS to other domains, such as medical diagnosis, risk assessment, and artificial intelligence.

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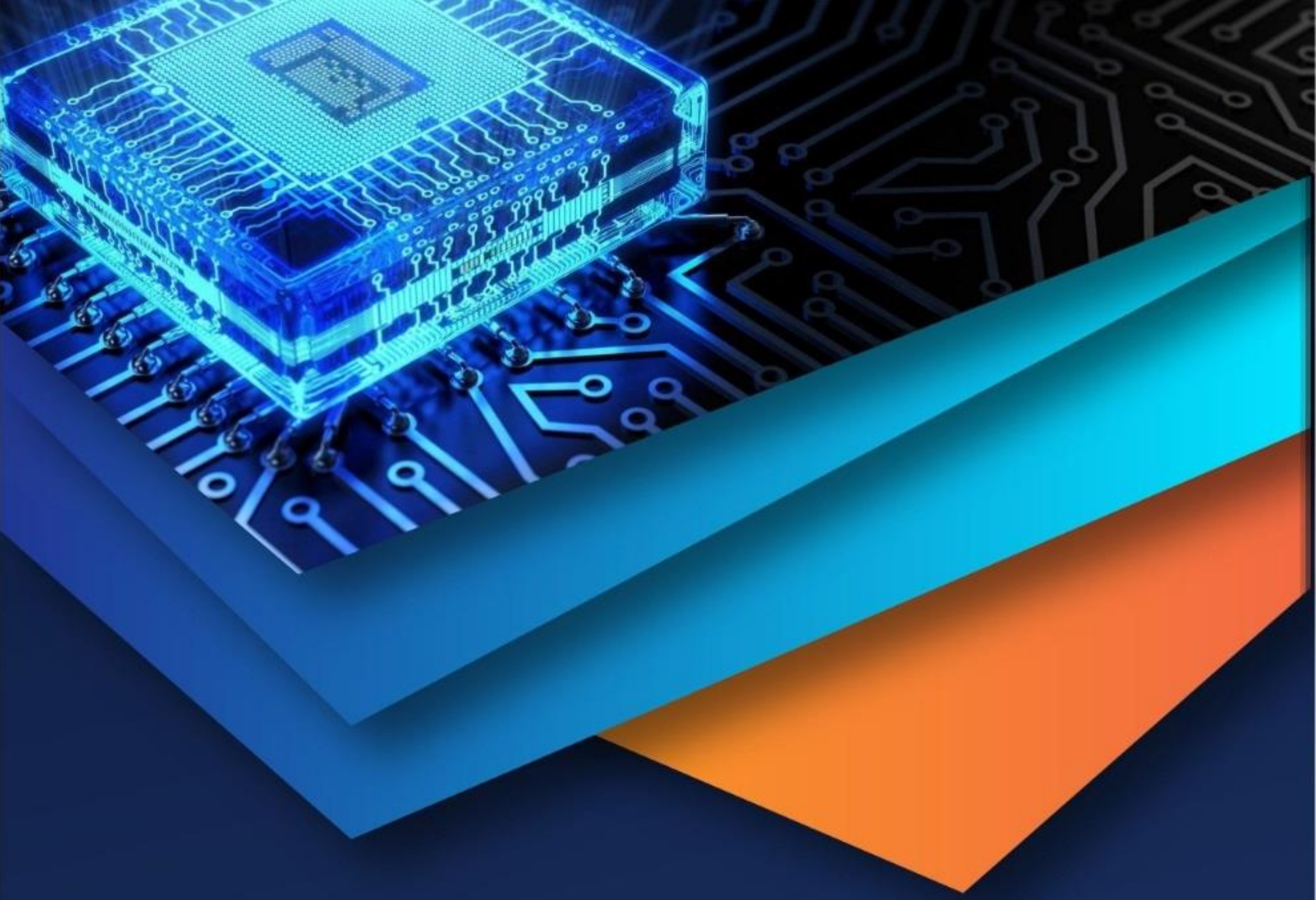

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