



IJRASET

International Journal For Research in
Applied Science and Engineering Technology



INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 13 **Issue:** I **Month of publication:** January 2025

DOI: <https://doi.org/10.22214/ijraset.2025.66702>

www.ijraset.com

Call:  08813907089

E-mail ID: ijraset@gmail.com

Python Programming on Fibonacci Graceful Anti - Magic Labeling for Star Graphs

D.Amuthavalli¹, Dr.S.Murugesan²

¹Assistant Professor, Department of Mathematics, Dhanalakshmi Srinivasan University, Perambalur.

²Gandhi Arts and Science College, Sathyamangalam

Abstract: A graph $G = (V, E, \phi)$ with p vertices and q edges is called Fibonacci Graceful Anti magic Graph [FGAMG] if a function ϕ is defined as $\phi: V \rightarrow \{0, 1, 2, \dots, F_{q+1}\}$ and the induced edge labeling $\phi^*: E \rightarrow \{F_2, F_3, F_4, \dots, F_{q+1}\}$ is defined by $\phi^*(uv) = |\phi(u) - \phi(v)|$ is bijective. In addition all the vertex sums are pairwise distinct and all the edges are unique. In this article the concept of Fibonacci Graceful Anti- Magic Labeling [FGAML] is introduced and investigate some star related graphs which are Fibonacci Graceful Anti magic Graph [FGAMG].

AMS Classification : 05C78

Keywords: Crown graph, Bi-star, Double star, Bi-shell, FGAML, FGAMG.

I. INTRODUCTION

Fibonacci graceful labeling idea was introduced by E.Barkaukas et.al [3]. Hartsfield and Ringel introduced the concept of Anti-Magic labeling. The concept of Fibonacci mean Anti-magic labeling was introduced by Ameenabibi and T.Ranjani[1]. Now this article is based on connected, undirected and Star related Graphs. Python is the most popular language in world wide. It was introduced by GuioVan Rossum on February 20,1991.Fibonacci Graceful Anti- Magic Labeling [FGAML]concept is introduced here. While investigating some star based graphs it is found that they are Fibonacci Graceful Anti magic Graph [FGAMG].Here Python coding is generated for the Fibonacci Graceful Anti-Magic Labeling.

II. DEFINITIONS

- 1) Definition 2.1: The Fibonacci numbers F_1, F_2, F_3, \dots are defined by $F_0 = 0, F_1 = 1, F_2 = 1, \dots$ and $F_{n+1} = F_n + F_{n-1}, n \geq 1$. The Fibonacci sequence is 1,1,2,3,5...
- 2) Definition 2.2: The function $\phi: V(G) \rightarrow \{0, 1, 2, \dots, F_{q+1}\}$ (here F_q is the q^{th} Fibonacci number) is called FGAML if the induced edge labeling $\phi^*: E(G) \rightarrow \{F_2, F_3, F_4, \dots, F_{q+1}\}$ defined by $\phi^*(uv) = |\phi(u) - \phi(v)|$ is bijective. In addition all the vertex sums are pairwise distinct and all the edges are unique.
- 3) Definition 2.3 : Crown graph is obtained by joining a single pendant edge to each vertex of the cycle C_n . It is denoted by $C_n \odot K_1$.
- 4) Definition 2.4: Double star $K_{1,n,n}$ is a tree obtained from the star $K_{1,n}$ by adding a new pendant edge to each of the existing n pendant vertices.
- 5) Definition 2.5 : A Bi-star $B_{n,n}$ graph is obtained by adding the central vertices of two copies of Star.
- 6) Definition 2.6 : A Bi-Shell $BS_{n,n}$ graph is obtained by adding the central vertices of two copies of Shell.
- 7) Definition 2.7 : The Vertex sum at one vertex is the sum of labels of all edges incident to such a vertex.

III. RESULTS

1) Theorem

Crown graph $C_n \odot K_1$ is FGAMG for all n .

Proof:

Let $C_n \odot K_1$ be the crown. The order of the crown is $p = 2n$ and the size is $q = 2n$. Let $V(G) = \{a_1, a_2, \dots, a_n, b_1, b_2, \dots, b_n\}$ here a_1, a_2, \dots, a_n are the vertices of the cycle C_n and b_i 's are the pendant vertices adjacent to a_i 's of the cycle C_n . The edge set $E(G) = \{e_{ii}, e_{ij}\}$ where $e_{ii} = (a_i b_i)$ and $e_{ij} = (a_i a_j)$.

Define $\phi: V \rightarrow \{0, 1, 2, \dots, F_{q+1}\}$

$$\begin{aligned} \phi(a_1) &= 0 \\ \phi(a_{i+1}) &= F_{2i+2} \quad \text{for } i = 1, 2, 3, \dots, n-1 \\ \phi(b_{i+1}) &= F_{2i+1} \quad \text{for } i = 1, 3, \dots, n-1 \\ \phi(b_3) &= 6 \\ \phi(b_1) &= F_{2n+1} \end{aligned}$$

The edge labels are

$$\begin{aligned} e_{ii} &= \phi^*(a_i b_i) = |\phi(a_i) - \phi(b_i)| \quad \text{for } i = 1, 2, 3, \dots, n \\ e_{ij} &= \phi^*(a_i a_j) = |\phi(a_i) - \phi(a_j)| \quad \text{for } i = 1, 2, 3, \dots, n \text{ and for } j = i + 1 \end{aligned}$$

Here all the vertex sums are pairwise distinct and all the edges are unique.

Thus ϕ admits *FGAML*.

Hence Crown $C_n \odot K_1$ is *FGAMG*.

Python coding and output is enclosed in the Appendix .

Illustration 3.1

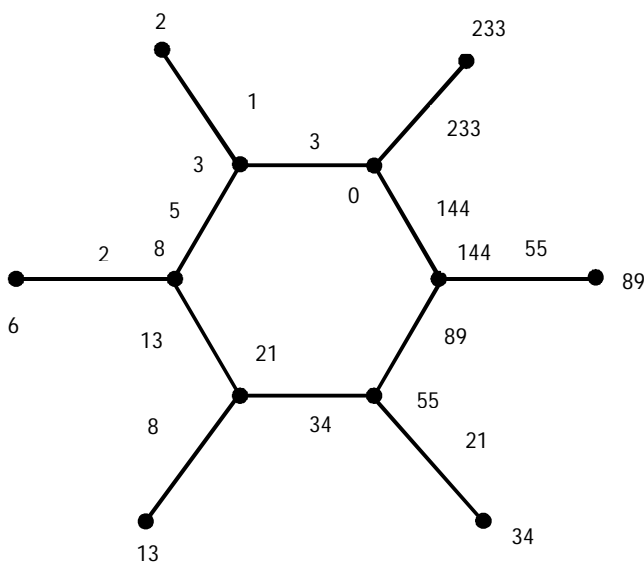


Figure 1. The Crown $C_6 \odot K_1$ is *FGAMG*

2) Theorem

Double star graph is *FGAMG* for all n .

Proof :

Let $K_{1,n,n}$ be the double star. The order of $K_{1,n,n}$ is $p = 2n + 1$ and the size $q = 2n$. By the definition of $K_{1,n,n}$ the vertex set $V(G) = \{a, b_1, b_2, \dots, b_n, c_1, c_2, \dots, c_n\}$. Let a be the central vertex, and b_1, b_2, \dots, b_n be the vertices of the star and c_1, c_2, \dots, c_n be the vertices adding new pendent edge to each existing of the n pendent edges. The edge set $E(G) = \{e_i, e_{ii}\}$ here $e_i = (ab_i)$ and $e_{ii} = (b_i c_i)$.

Define $\phi: V \rightarrow \{0, 1, 2, \dots, F_{q+1}\}$ as follows

$$\begin{aligned} \phi(a) &= 0, \\ \phi(b_i) &= F_{2n+2-i} \quad i = 1, 2, \dots, n. \\ \phi(c_i) &= \phi(b_i) - F_{n+2-i} \quad i = 1, 2, \dots, n. \end{aligned}$$

The edge labels are

$$\begin{aligned} e_{ii} &= \phi^*(b_i c_i) = |\phi(b_i) - \phi(c_i)| \quad \text{for } i = 1, 2, 3, \dots, n \\ e_i &= \phi^*(ab_i) = |\phi(a) - \phi(b_i)| \quad \text{for } i = 1, 2, 3, \dots, n \end{aligned}$$

Here all the vertex sums are pairwise distinct and all the edges are unique.

Thus ϕ admits *AML*.

Hence the double star $K_{1,n,n}$ is *FGAMG*.

Python coding and output is enclosed in the Appendix .

Illustration 3.2

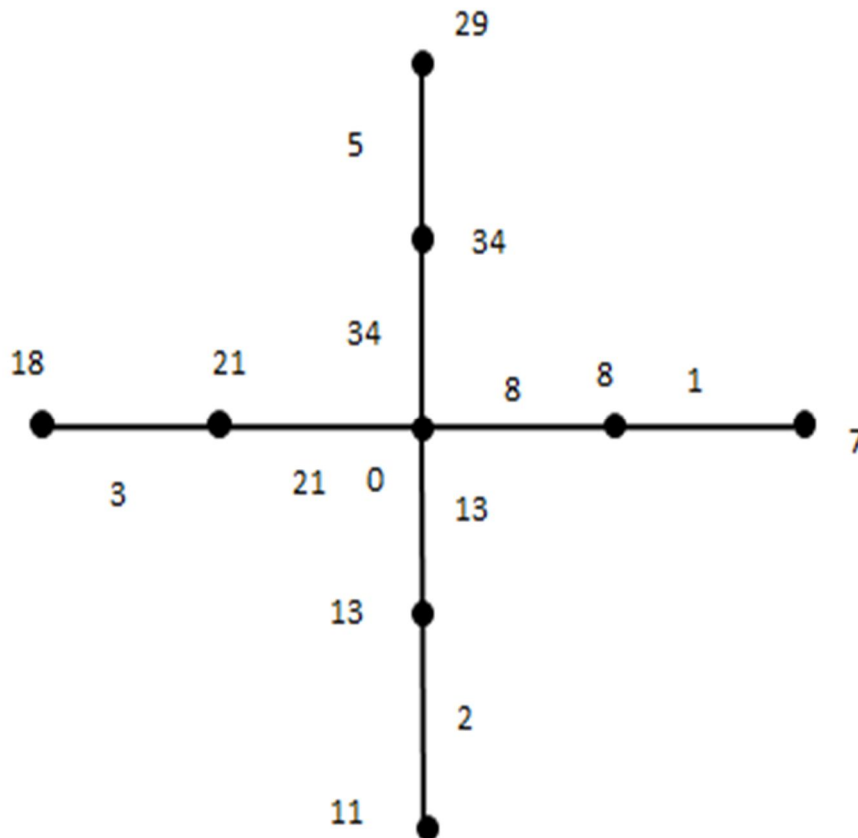


Fig 2. Double star $K_{1,4,4}$ is FGAMG

3) Theorem

Bi-star $B_{n,n}$ is FGAMG for all n.

Proof:

Let $B_{n,n}$ be Bi-star, By the definition of $B_{n,n}$, The order of the Bi-star $B_{n,n}$ is $p = 2n + 2$ and size is $q = 2n + 1$. The vertex set $V(G) = \{a_0, a_1, a_2, \dots, a_n, b_0, b_1, b_2, \dots, b_n\}$. Let a_1, a_2, \dots, a_n be the adjacent vertices of a_0 and b_1, b_2, \dots, b_n be the adjacent vertices of b_0 .

$$E(G) = \{e_1^*, e_j, e_j\} \text{ here } e_i = (b_0 b_i), g_i = (a_0 a_i), e_1^* = (a_0 b_0).$$

Define $\phi: V \rightarrow \{0, 1, 2, \dots, F_{q+1}\}$ as follows

$$\phi(a_0) = 0$$

$$\phi(a_{i+1}) = F_{n+2+i} \quad \text{for } i = 1, 2, 3, \dots, n-1$$

$$\phi(b_{i+1}) = F_{i+2} + F_2 \quad \text{for } i = 1, 2, 3, \dots, n-1$$

$$\phi(b_0) = F_2$$

The edge labels are

$$e_i = \phi^*(b_0 b_i) = |\phi(b_0) - \phi(b_i)| \quad \text{for } i = 1, 2, 3, \dots, n$$

$$g_i = \phi^*(a_0 a_i) = |\phi(a_0) - \phi(a_i)| \quad \text{for } i = 1, 2, 3, \dots, n$$

$$e_1^* = \phi^*(a_0 b_0) = |\phi(a_0) - \phi(b_0)|$$

Here all the vertex sums are pairwise distinct and all the edges are unique.

Thus ϕ admits FGAML.

Hence the graph $B_{n,n}$ is FGAMG.

Python coding and output is enclosed in the Appendix .

Illustration 3.3

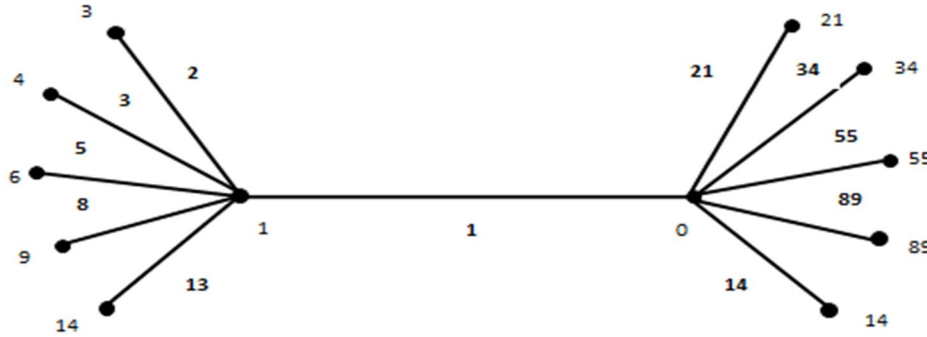


Fig 3. Bi-star $B_{5,5}$ is FGAMG

4) Theorem

Bi-shell $BS_{n,n}$ is FGAMG for all n.

Proof:

Let $BS_{n,n}$ be the Bi-shell graph, The order of $B_{n,n}$ is $p = 2n + 2$ and the size is $q = 4n - 1$. The vertex set $V(G) = \{a_0, a_1, a_2, \dots, a_n, b_0, b_1, b_2, \dots, b_n\}$. Let a_1, a_2, \dots, a_n be the adjacent vertices of a_0 and b_1, b_2, \dots, b_n be the adjacent vertices of b_0 .

$$E(G) = \{e_1^*, e_i, e_{ij}, g_i, g_{ij}\} \text{ here } e_i = (b_0 b_i), g_i = (a_0 a_i), e_1^* = (a_0 b_0), e_{ij} = (b_i b_j), g_{ij} = (a_i a_j).$$

Define $\phi: V \rightarrow \{0, 1, 2, \dots, F_{q+1}\}$

$$\phi(a_0) = F_2$$

$$\phi(a_i) = F_{2i+1} + F_2 \quad \text{for } i = 1, 2, 3, \dots, n$$

$$\phi(b_i) = F_{4n-2i+2} \quad \text{for } i = 1, 2, 3, \dots, n$$

$$\phi(b_0) = 0$$

The edge labels are

$$e_i = \phi^*(b_0 b_i) = |\phi(b_0) - \phi(b_i)| \quad \text{for } i = 1, 2, 3, \dots, n$$

$$g_i = \phi^*(a_0 a_i) = |\phi(a_0) - \phi(a_i)| \quad \text{for } i = 1, 2, 3, \dots, n$$

$$e_1^* = \phi^*(a_0 b_0) = |\phi(a_0) - \phi(b_0)|$$

$$e_{ij} = \phi^*(b_i b_j) = |\phi(b_i) - \phi(b_j)| \quad \text{for } i = 1, 2, 3, \dots, n \text{ and for } j = i + 1$$

$$g_{ij} = \phi^*(a_i a_j) = |\phi(a_i) - \phi(a_j)| \quad \text{for } i = 1, 2, 3, \dots, n \text{ and for } j = i + 1$$

Here all the vertex sums are pairwise distinct and all the edges are unique.

Thus ϕ admits FGAML.

Hence the graph $BS_{n,n}$ is FGAMG.

Python coding and output is enclosed in the Appendix.

Illustration 3.4

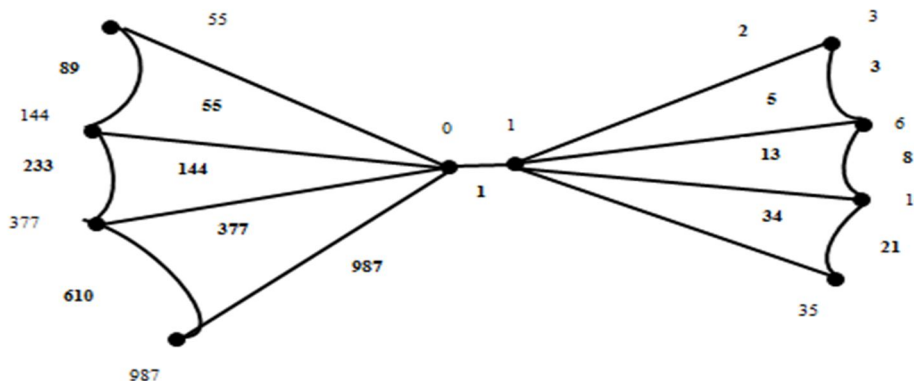


Fig.4. Bi-Shell $BS_{4,4}$ is FGAMG

IV. CONCLUSION

In this article the concept of Fibonacci Graceful Anti-Magic Labeling and demonstrated some Star related graphs are Fibonacci Graceful Anti – Magic Graphs. Python codings are generated for the Fibonacci Graceful Anti-Magic Labeling for the Star related Graphs. In future different concept of labeling will be developed.

V. APPENDIX

1) Python Coding and output for Crown graph $C_n \odot K_1$.

```

+ Code + Text
n=int(input("Enter the n Value: "))
subscript = str.maketrans("0123456789", "0123456789")
print(f"φ(a{1}) = 0".translate(subscript))
print(f"φ(b{3}) = 6".translate(subscript))
d = 2*n+1
print(f"φ(b{1}) = F{d}".translate(subscript))

for i in range(n):
    i+=1
    b=(2*i)+2
    print(f"φ(a{i+1}) = F{b}".translate(subscript))
    if i%2:
        c=(2*i)+1
        print(f"φ(b{i+1}) = F{c}".translate(subscript))
    else:
        None
    print("")

```

Enter the n Value: 5
φ(a₁) = 0
φ(b₃) = 6
φ(b₁) = F₁₁
φ(a₂) = F₄
φ(b₂) = F₃

φ(a₃) = F₆

φ(a₄) = F₈
φ(b₄) = F₇

φ(a₅) = F₁₀
φ(b₅) = F₉

3s completed at 12:43 PM

2) Python Coding and output for double star $K_{1,n,n}$

```

+ Code + Text
n=int(input("Enter the n Value: "))
subscript = str.maketrans("0123456789", "0123456789")

for i in range(n):
    i+=1
    print(f"φ(a) = F{0}".translate(subscript))

    b=(2*n)+2-1
    print(f"φ(b{i}) = F{b}".translate(subscript))

    d = (n+2-1)
    print(f"φ(c{i}) =F{b} - F{d}".translate(subscript))

    print("")

```

Enter the n Value: 5
φ(a) = F₀
φ(b₁) = F₁₁
φ(c₁) =F₁₁ - F₆

φ(a) = F₀
φ(b₂) = F₁₀
φ(c₂) =F₁₀ - F₅

φ(a) = F₀
φ(b₃) = F₉
φ(c₃) =F₉ - F₄

φ(a) = F₀
φ(b₄) = F₈
φ(c₄) =F₈ - F₃

φ(a) = F₀
φ(b₅) = F₇
φ(c₅) =F₇ - F₂

5s completed at 12:18 PM

3) Python Coding and output for Bi-star $B_{n,n}$:

```

+ Code + Text
n=int(input("Enter the n Value: "))

subscript = str.maketrans("0123456789", "0123456789")

print(f"φ(a{0}) = F{0}".translate(subscript))
print(f"φ(b{0}) = F{2}".translate(subscript))

for i in range (n):
    i+=1
    b= n + 2 +1
    print(f"φ(a{i+1}) = F{b}".translate(subscript))
    print(f"φ(b{i+1}) = F{i+2} + F{2}".translate(subscript))
    print("")

Enter the n Value: 4
φ(a0) = F0
φ(b0) = F2
φ(a2) = F7
φ(b2) = F3 + F2

φ(a3) = F8
φ(b3) = F4 + F2

φ(a4) = F9
φ(b4) = F5 + F2

φ(a5) = F10
φ(b5) = F6 + F2
    
```

7s completed at 12:55 PM

4) Python Coding and output for Bi-shell $BS_{n,n}$

```

+ Code + Text
n=int(input("Enter the n Value: "))

subscript = str.maketrans("0123456789", "0123456789")

for i in range (n):
    i+=1
    print(f"φ(a{0}) = F{2}".translate(subscript))

    b= 2*i + 1
    print(f"φ(a{i}) = F{b} + F{2}".translate(subscript))

    c = 4*n - 2*i + 2
    print(f"φ(b{i}) = F{c}".translate(subscript))

    print(f"φ(b{0}) = 0".translate(subscript))
    print("")

Enter the n Value: 4
φ(a0) = F2
φ(a1) = F3 + F2
φ(b1) = F44
φ(b0) = 0

φ(a0) = F2
φ(a2) = F3 + F2
φ(b2) = F44
φ(b0) = 0

φ(a0) = F2
φ(a3) = F7 + F2
φ(b3) = F44
φ(b0) = 0

φ(a0) = F2
    
```

3s completed at 3:46 AM

REFERENCES

- [1] Ameenal Bibi.K and T.Ranjani "Fibonacci mean Anti-magic labeling of some graphs" Kong.Res.J. ISSN : 2349-2694
- [2] Bondy J. A. and Murty U.S. R., Graph Theory with applications, Newyork Macmill Ltd. Press (1976).
- [3] David .W and Anthony .E. Barakaukas, "Fibonacci graceful graphs".
- [4] D.Amuthavalli and M.Sangeetha "Super Fibonacci Gracefulness of shell related graphs". International Journal of Mathematics Trends and Technology volume 53-No.6.January 2018.
- [5] D.Amuthavalli and M.Premkumar "Super Fibonacci Graceful labeling of cycle related graphs". International Journal of Mathematics Trends and Technology volume 38-No.3.October 2016.
- [6] Joseph A. Gallian, "A Dynamic survey of graph labeling", the electronic journal of combinatorics,(2022).
- [7] Sivaranjani.B and R.Kala "Fibonacci mean Anti-magic labeling of some graphs" Advances in Mathematics: Scientific Journal 9 (2020), no.5 2561-2572.ISSN: 1857-8365.
- [8] Solomon Golomb "Graph Theory and Computing"(Academic Press: 1972).
- [9] S.Somasundaram & T.Nicholas "on (a,d) -anti magic special trees, unicyclic graphs and complete bipartite graphs ars.com, 70,2004 207-220.
- [10] Tao-Ming wang and Cheng-Chih Hsiao "On anti-magic labeling for graphs products", the ScienceDirect, Discrete Mathematics 308(2008) 3624-3633.



10.22214/IJRASET



45.98



IMPACT FACTOR:
7.129



IMPACT FACTOR:
7.429



INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Call : 08813907089  (24*7 Support on Whatsapp)