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Real Gas Effect on Motion of Self-gravitating Cylindrical Detonation Waves

Pushpender Kumar Gangwar[1], Rajesh Kumar Verma[2]
[1]Department of Physics, Bareilly College, Bareilly, India
[2]Department of Physics, K.S. Saket (P.G.) College, Ayodhya, India

Abstract: The effect of real gas on the adiabatic propagation of strong cylindrical imploding detonation waves in the non-homogeneous medium having power varying density distribution. Ignoring the effect of overtaking disturbances the CCW solution has been obtained for the problem at the detonation waves are initially in Chapman-Jouguet state. The analytical expressions for the detonation velocity just behind the front along with other flow variables are derived. The expressions for the freely propagating velocity of detonation front are derived. Using the jump condition across the strong detonation front, the numerical values of the pressure and density across the front have been computed with the help of the software MATLAB. It is observed that the variation of flow parameters with convergence of detonation front is depend upon the change in Alfven Mach number, the parameters of realness of the gas and initial density distribution. Finally, it is found that change in parameter of realness of the medium plays an important role on the post flow variables. The effect of change in Alfven Mach number and density parameter is also discussed through graphs. The outcome of the present study is compared with the results for the case of ideal reacting gas.

Keywords: CCW theory, Self-gravitation, Strong detonation waves, variable density, Imploding shock.

I. INTRODUCTION

On account of the importance of study of phenomenon of shock wave propagation in different branches of science and technology, several scientists have considered the problem experimentally and analytically. The motion of detonation waves in different type of uniform and non-uniform of media having cylindrical and spherical symmetries. Welsh (1967), Nigmatulin (1967) and Teipel (1983) have consider the problem of detonation wave propagation into a uniform combustible ideal gas. Vishwakarma et al. (2018) have further studied the the problem of Nigmatulin (1967) for the case of magnetogasdynamics. In non-uniform atmosphere, Verma and Singh (1980), (1981) have explored the problem of detonation wave propagation of Teipel (1983) study. The CCW theory[Chester (1954)Chisnell (1955)Whitham (1958)] have been used by Tyl and Wlodarczyk (1983) for analytical study of concentric detonation waves in gaseous reactive mixtures. Self-gravitation effect on the adiabatic motion of weak and strong shock having cylindrical and spherical symmetries having different type of initial density distribution in pure and dusty real and ideal gases have been studied by Gangwar (2018), (2020), (2022). Recently Gangwar and Verma (2024)a, (2024)b have applied the well-known CCW method for the motion of strong detonation front in ideal gas under the effect of self-gravitation and rotation.

In this present paper, the effect of realness of the reacting gaseous atmosphere have been analyzed by using the well-known CCW method for the propagation of converging detonation waves having cylindrical symmetry under the effect of self-gravitation. The effect of the flow behind the detonation front have been neglecting in this study. It is assumed that the detonation front is travels with sonic speed relative to the burst gas, which determines the law of convergence and it is assumed as Chapman-Jouguet front. In the case of strong detonation wave propagation, the pressure and internal energy in the undisturbed gas have been neglected in comparison to their values in the disturbed fluid. The values of non-dimensional pressure, density and detonation velocity have been derived and computed numerically. The impact of realness of the medium have been depicted through figures. It is found that the the change in the parameter of realness of the gas have play significant roll on the variation of all flow variables.

II. FUNDAMENTAL EQUATIONS, BOUNDARY CONDITIONS AND ANALYTICAL EXPRESSIONS

The Fundamental governing equations for one dimensional unsteady flow of reacting gas can be written as Tyl and Wlodarczyk (1983)Yadav (1992)

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+\rho\frac{\partial p}{\partial r}+\frac{Gm}{r^2}=0 \tag{1}$$

$$\frac{\partial\rho}{\partial t}+u\frac{\partial\rho}{\partial r}+\rho a^2\left(\frac{\partial u}{\partial t}+\frac{u}{r}\right)=0 \tag{2}$$

$$\frac{\partial\varepsilon}{\partial t}+u\frac{\partial\varepsilon}{\partial r}+\frac{p}{\rho^2}\left(\frac{\partial\rho}{\partial t}+u\frac{\partial\rho}{\partial r}\right)=0 \tag{3}$$

$$\frac{\partial m}{\partial t}-2\pi r\rho=0 \tag{4}$$

where *u, p, ρ, m, G* and ε are the particle velocity, the pressure, density, mass of the radius *r*, Gravitational constant and internal energy per unit mass of the ideal gas, respectively and '*a*' is the local speed of sound in ideal gas is given by

$$a^2=\frac{\gamma p}{\rho}\left(\frac{1+2b\rho}{1+b\rho}\right)\approx\frac{\gamma p}{\rho(1-b\rho)} \tag{5}$$

where γ is the ratio of specific heats of the gas and '*b*' is the effective volume occupied by the gas molecules. The equation of state for the real gas is given by

$$p=\Gamma\rho T(1+b\rho) \tag{6}$$

where $c_p-c_v=\Gamma\left(1+\frac{b\rho^2}{1+2b\rho}\right)\approx\Gamma$ is the gas constant, and c_p and c_v are the specific heats of the gas at constant pressure and volume, respectively. *T* is the temperature of the medium. The internal energy per unit mass of the gas in this case may be given as

$$\varepsilon=\frac{p}{(\gamma-1)\rho(1+b\rho)} \tag{7}$$

The density distribution law of the gas ahead of detonation front is taken to be power varying and given as

$$\rho_a=Ar_a^{-w} \tag{8}$$

where A is the constant and the $\eta=r_a/R$ is the ratio of radius 'r_a' to the internal radius of the detonation front '*R*' and 'w'a positive integer and is known as density parameter.

Let u_a,ρ_a,p_a and E_a represent the undisturbed values of flow velocity, density, pressure, and internal energy per unit mass of real reacting gas just ahead of the shock front, and u,ρ,p, and ε be the modified values of respective quantities at immediately behind the front. In the case of detonation wave propagation, the jump conditions across the detonation front may be given by Whitham (1958), Tyl and Wlodarczyk (1983), Vishwakarma et al. (2018)

$$\rho(U_D-u)=\rho_a U_D \tag{9}$$

$$p=p_a+\rho_a U_D u_a \tag{10}$$

$$\rho(U_D-u)=\rho_a U_D \tag{11}$$

$$\varepsilon=\varepsilon_a+\frac{1}{2}(p+p_a)\left(\frac{1}{\rho_a}-\frac{1}{\rho}\right)+q \tag{12}$$

where, the indices 'a' refers to the sates just ahead the detonation front, *q* is the heat energy released per unit mass, and U_D represent the velocity of detonation front.

The detonation front is assumed to be in Chapman-Jouguet state. Chapman-Jouguet condition requires that the flow ahead of the shock front will be in sonic state and, in the shock-fixed coordinates, i.e.

$$\left|U_{D_{CJ}}-u_{CJ}\right|=a_{CJ} \tag{13}$$

where the indices '$_{CJ}$' denote the Chapman-Jouguet state.

The boundary conditions across the detonation front in this case are

$$p_{CJ}=\frac{1-\phi}{\gamma+1}\rho_a U_{D_{CJ}}^2 \tag{14}$$

$$u_{CJ}=\frac{1-\phi}{\gamma+1}U_{D_{CJ}} \tag{15}$$

$$\rho_{CJ}=\frac{\gamma+1}{\gamma+\phi}\rho_a \tag{16}$$

$$a_{CJ}=\frac{\gamma+\phi}{\gamma+1}\left|U_{D_{CJ}}\right| \tag{17}$$

$$U_{D_{CJ}}=\frac{\sqrt{2q(\gamma^2-1)}}{1-\phi} \tag{18}$$

where $\phi = b\rho_a$ is the parameter of realness of the gas. Using the equation (8) and (14)-(18), the boundary conditions across the strong detonation front having power decreasing density distribution may be written in the term of velocity $(\xi = u_n/u_{CJ})$ of burnt gas

$$\frac{U_D}{U_{D_{CJ}}} = \frac{\xi^2 - 1}{2\xi} \tag{19}$$

$$\frac{p}{p_{CJ}} = \frac{\xi^2 + 1}{2}\eta^{-w} \tag{20}$$

$$\frac{\rho}{\rho_{CJ}} = \frac{(\gamma+\phi)(\xi^2+1)}{(\gamma+1)+(\gamma-1+2\phi)\xi^2}\eta^{-w} \tag{21}$$

$$\frac{a}{a_{CJ}} = \left[\frac{\left\{(\gamma+1)+(\gamma-1+2\phi)\xi^2\right\}^2(1-\phi)}{2(\gamma+2\phi)\left[\left\{(\gamma+1)+(\gamma-1+2\phi)\xi^2\right\} - \phi(\gamma+1)(\xi^2+1)\right]}\right]^{1/2} \tag{22}$$

where $\xi = u_n/u_{CJ}$ and $\eta = r_a/R$,

Under the CJ-condition Li et al. (1997)Li and Ben-Dor (1998) the detonation velocity in terms of Alfven ah number M_{CJ} is given by

$$U_D = U_{D_{CJ}} \text{ and } M_{CJ} = U_{D_{CJ}}/a_a \tag{23}$$

where $a_a^2 = \dfrac{\gamma p_a}{\rho_a(1+b\rho_a)}$

From equation(4), the mass inside the cylinder of radius r is given by

$$m = \frac{2\pi A r^{2-w}}{2-w} \tag{24}$$

At the equilibrium state, of the gas is assumed to be specified by the condition, $u = 0 = \partial/\partial t, p = p_a$ Therefore, from equation(1), the equilibrium condition prevailing the front of the shock can be written as

$$\frac{1}{\rho_a}\frac{dp_a}{dr} = -\frac{Gm}{r^2} \tag{25}$$

On solving the equation(25) and putting the value of mass m from equation(24), we have

$$p_a = \frac{\pi G A^2}{(2-w)(w-1)}\eta^{2(1-w)} \tag{26}$$

From the above expression (26) it is observed that for the positive initial pressure the value of density parameter is of the range $2 > w > 1$ Yadav and Gangwar (2003). The local speed of sound in the real gas having self-gravitation effect with power decreasing density distribution can be given by using the equations(5) and (26) we have:

$$a_a^2 = \frac{\pi\gamma G A}{(2-w)(w-1)(1-\phi)}\eta^{(2-w)} \tag{27}$$

Using the equations(15),(18), and (23), we get

$$u_{CJ}^2 = \frac{\pi\gamma G A(1-\phi)M_{CJ}^2}{(\gamma+1)^2(2-w)(w-1)}\eta^{(2-w)} \tag{28}$$

The characteristic form of the fundamental basic equations for converging shock *i. e.* the form in which equation contains derivatives in only one direction in (r, t) plane is

$$dp - \rho a du + \frac{2\rho a^2 u}{(u-a)}\frac{dr}{r} - \frac{\rho a}{(u-a)}\frac{Gmdr}{r^2} = 0 \tag{29}$$

Equation(29) divided by $\rho_{CJ}u_{CJ}^2$ and using equations(19)-(28) after simplifying, we have

$$\frac{d\xi}{d\eta} = \frac{(\xi^2+1)}{\eta\left[\xi-(\gamma+\phi)(\xi^2+1)\beta\right]}\left[\frac{w}{2} - \frac{\beta\left[\gamma(\gamma+\phi)\left\{(\gamma+1)+(\gamma-1+2\phi)\xi^2\right\}\beta\xi - 2(\gamma+1)^2(w-1)M_{CJ}^{-2}\right]}{\xi(1-\phi)-(\gamma+\phi)\left\{(\gamma+1)+(\gamma-1+2\phi)\xi^2\right\}\beta}\right] \tag{30}$$

where $\beta^2 = \dfrac{(1-\phi)}{2(\gamma+2\phi)\left[\left\{(\gamma+1)+(\gamma-1+2\phi)\xi^2\right\}-\phi(\gamma+1)\left(\xi^2+1\right)\right]}$.

Integrating the differential equation(30) numerically and using jump conditions(19)-(22), the variations of ξ *i.e.* $\frac{u}{u_{CJ}}$, $\frac{U_D}{U_{D_{CJ}}}$, $\frac{p}{p_{CJ}}$ and $\frac{\rho}{\rho_{CJ}}$ with propagation distance η will obtained.

III. RESULTS AND DISCUSSION

The CCW solution for the problem of the propagation of strong detonation waves in a reacting real(non-ideal) gas have been obtained. The expression (30) represents that the parameters of the detonation propagation detonation velocity $\frac{U_D}{U_{D_{CJ}}}$, the pressure behind the detonation wave $\frac{p}{p_{CJ}}$, and the density across the detonation front $\frac{\rho}{\rho_{CJ}}$ just behind the strong cylindrical converging detonation depends upon the ratio of specific heats γ, propagation distance η, parameter of realness of gas ϕ, and density parameter w and Alfven Mach number M_{CJ}.

The software MATLEB have been used for solving equation(30) numerically and calculate the values of all post front values of parameters.

It is also observed from the expressions(30) all the flow variables are not directly depends upon Gravitational constant *G* as in simple problem of strong propagation of shock wave[Yadav and Gangwar (2003)]

Initially taking the flow velocity $\frac{u}{u_{CJ}} = 2$ and $\gamma = 1.2$. The compression of values of flow parameters in real gas with ideal gas in the variation of detonation velocity $\frac{U_D}{U_{D_{CJ}}}$, the pressure across the detonation front $\frac{p}{p_{CJ}}$, and the density across the front $\frac{\rho}{\rho_{CJ}}$ with the propagation distance (η), for different values of γ, w and M_{CJ} have been computed and displayed through Fig. (1)-(6).

From the equation(24) and (26) it is clear that, for finite positive equilibrium pressure, and mass the constant '*w*' must obey the inequality 1<*w*<2[Yadav and Gangwar (2003)].

It is observed from Fig.(1) and Fig. (2) that detonation front velocity $\frac{U_D}{U_{D_{CJ}}}$ increases asymptotically as shock converges in the medium under the self-gravitational effect with power decreasing initial density distribution. It is observed from the figure (1) as the realness parameter $\phi = 0$ (ideal gas) to $\phi = 0.02$ (real gas)increases the detonation front velocity decreases. The effect of the adiabatic index of gas is also analyzed from the figure(3,5 and 6). The variation in detonation velocity with propagation distance decreases for higher value of adiabatic index $\gamma = 1.4$ of gas. It is analyzed from Fig.(3)-(6) that under the effect of self-gravitation the strong pressure across the detonation front $\frac{p}{p_{CJ}}$, and the density across the front $\frac{\rho}{\rho_{CJ}}$ is decreases with shock converges in the medium with power decreasing initial density distribution at adiabatic index of the gas $\gamma = 1.2, 1.4$, $w = 1.1, 1.7$ at $M_{CJ} = 10$. All flow variables are depending upon the Alfven Mach number M_{CJ}. It is noticed that the value of density parameter w increases from 1.1 to 1.7 the slope of the graph of the variation pressure across the detonation front $\frac{p}{p_{CJ}}$, and the density across the front $\frac{\rho}{\rho_{CJ}}$ is decreases at fix value $M_{CJ} = 10$ and $\gamma = 1.2$. It is also observed that strength of detonation wave is also increase with density parameter (ω). The results obtained here are compared with the results for the problem of propagation of strong spherical detonation waves in pure ideal gas including the effect of gravitation and the similar nature of variation in perturbation of flow parameters are observed.[Yadav and Gangwar (2003)].

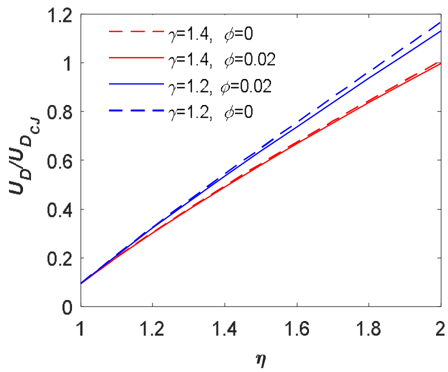

Fig. 1. Variation of detonation front velocity $U_D/U_{D_{CJ}}$ with the propagation distance (η) for M_{CJ} = 10 at w=1.2.

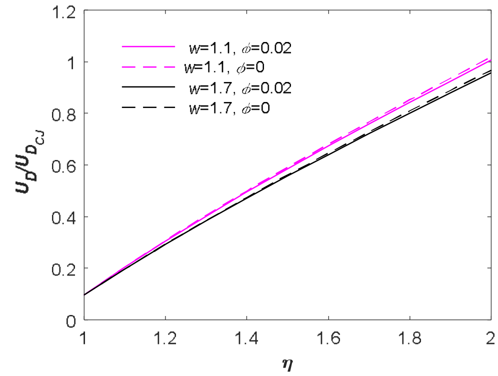

Fig. 2. Variation of detonation front velocity $U_D/U_{D_{CJ}}$ with the propagation distance (η) for γ=1.4 and M_{CJ}=10.

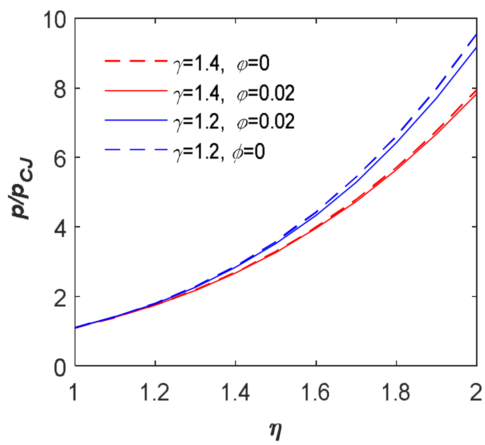

Fig. 3. Variation of pressure p/p_{CJ} with the propagation distance (η) for M_{CJ}= 10 at w=1.2.

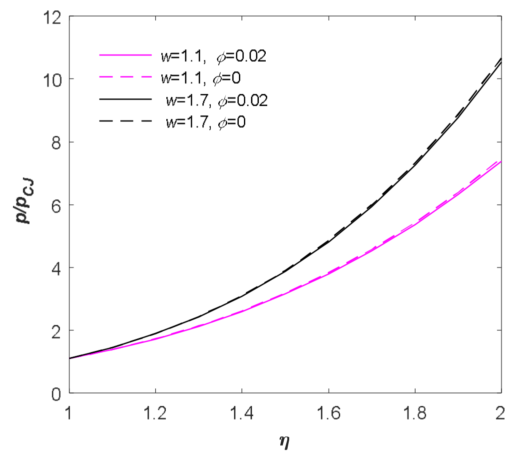

Fig. 4. Variation of pressure p/p_{CJ} with the propagation distance (η) for γ=1.4 and M_{CJ}=10.

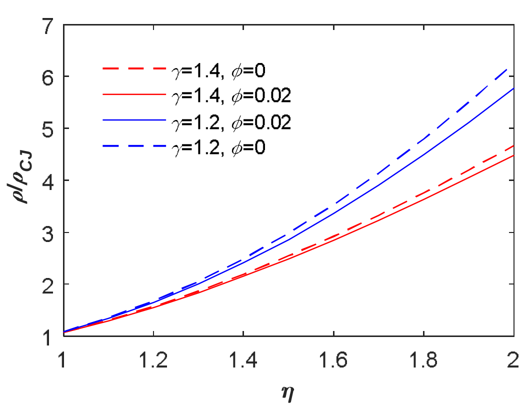

Fig. 5: Variation of density ρ/ρ_{CJ} with the propagation distance (η) for M_{CJ}= 10 at w=1.2.

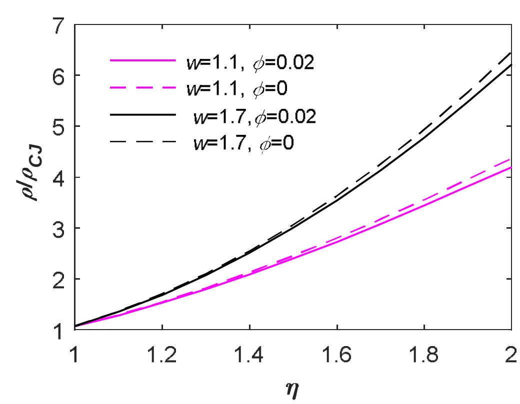

Fig. 5: Variation of density ρ/ρ_{CJ} with the propagation distance (η) for γ=1.4 and M_{CJ}=10.

IV. CONCLUSION

In this study the CCW method for solving the phenomenon of detonation wave propagation in the real reacting gaseous medium have been used. The effect of the realness of the gas have been calculated and discussed through graphs. The post detonation flow variables behind the cylindrical detonation front under the self-gravitating gas with initial power decreasing density have been explored. Impact of the flow behind the shock front has been neglected in this study. It is concluded that the role of realness of the gas is very significant in the phenomenon because it affects the variation of detonation velocity, the pressure across the detonation front, and the density across the front with the convergence of shock front.

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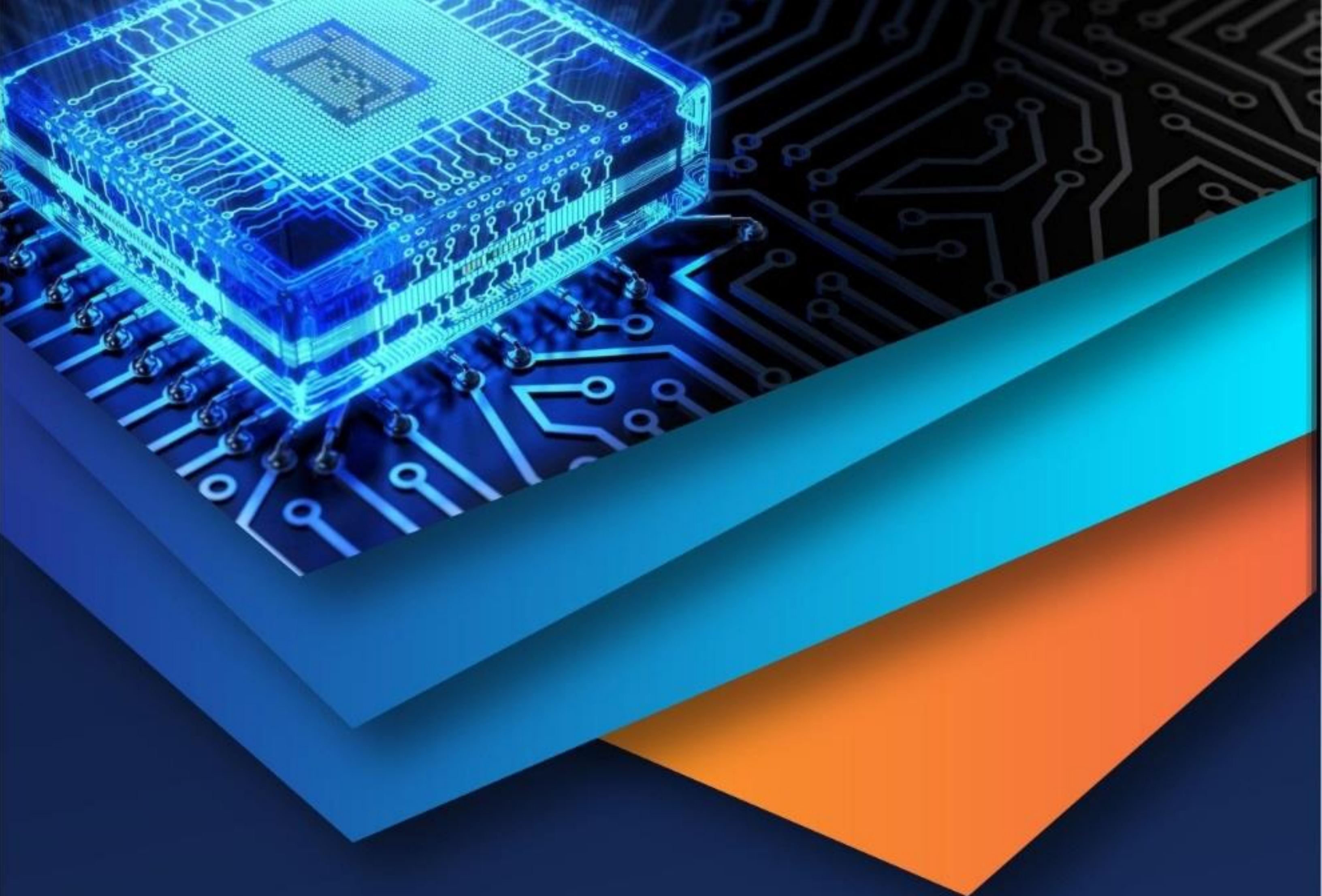

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